

**CLASS XII SESSION 2020-21**

**PRACTICE PAPER IV**

**SUBJECT : MATHEMATICS**

**MARKING SCHEME/VALUE POINTS (THEORY)**

Sr No	Objective type Question Section I Value points	Mark s
1	$\int \frac{\cos \sqrt{x}}{\sqrt{x}} \cdot dx$  Let $\sqrt{x} = t \Rightarrow I = 2\sin \sqrt{x} + C$	1
2	$(1,2) \in R, (2,1) \in R$ but $(1,1) \notin R$ therefore $R$ is not transitive	1
3	1 OR $k=17$	1
4	$A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	1
5	49	1
6	9 OR Scalar component of $\vec{AB} = -7, 6$	1
7	1	1
8	Range $= \{-1, 1\}$	1
9	1	1
10	$27/2$	1
11	Reqd d'cs are, $\left\langle \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}} \right\rangle$	1
12	Dr's of a line parallel to AB = 1, -2.4	1
13	$2/5$	1
14	4 sq units	1
15	0.3	1
16	One- one	1
17	$P(H) = \frac{60}{100} = 0.60, P(E) = 0.40, P(H \cap E) = 0.20$	





22	$I = \int_0^{\pi} \frac{4x}{1+\cos^2 x} dx$ $= \int_0^{\pi} \frac{4(\pi-x)}{1+\cos^2(\pi-x)} dx = 4\pi \int \frac{dx}{1+\cos^2 x} I$ $\Rightarrow 2I = 4\pi \int \frac{dx}{1+\cos^2 x} = 4\pi \cdot 2 \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos^2 x} =$ $\because f(\pi-x) = f(x)$ $\Rightarrow I = 4\pi \cdot \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos^2 x}$ <p>Dividing numerator and denominator by <math>\cos^2 x</math></p> $I = 4\pi \cdot \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{\sec^2 x + 1} = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{\tan^2 x + 2} \text{ put } \tan x = t \Rightarrow \sec^2 x dx dt$ $= 4\pi \int_0^{\infty} \frac{dt}{2+t^2} = 4\pi \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} \Big _0^{\infty} = \frac{4\pi}{\sqrt{2}} [\tan^{-1} \infty - \tan^{-1} 0] = \frac{4\pi}{\sqrt{2}} \cdot \frac{\pi}{2} = \sqrt{2} \pi^2$ <p>OR</p> $I = \int \frac{x+2}{\sqrt{x^2+5x+6}} dx$ $\Rightarrow x+2 = A \frac{d}{dx}(x^2 + 5x + 6) + B \Rightarrow A=1/2, B=-1/2$ $\therefore I = \frac{1}{2} \int \frac{2x+5}{\sqrt{x^2+5x+6}} dx - \frac{1}{2} \int \frac{dx}{\sqrt{x^2+5x+6}}$ $= \sqrt{x^2 + 5x + 6} - \frac{1}{2} \log \left  \left( x + \frac{5}{2} \right) + \sqrt{x^2 + 5x + 6} \right  + C$	1 1 1 1 1 1 1
23	$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ <p>(i) <math>P(\text{Problem is solved}) = 1 - P(\bar{A} \cap \bar{B}) = 1 - P(\bar{A})P(\bar{B}) = 1 - \frac{1}{2} \times \frac{2}{3} = \frac{2}{3}</math></p> <p>(ii) <math>P(\text{Exactly one can solve the problem}) = P(\bar{A} \cap B) + P(A \cap \bar{B})</math></p> $= P(\bar{A})P(B) + P(A)P(\bar{B}) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{2}$	1 1
24	$\vec{a}_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}, \vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ $\vec{a}_2 = 5\hat{i} - 2\hat{j}, \vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$	

	$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 4\hat{j} + 4\hat{k}$ $\vec{b}_1 X \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12-4) - \hat{j}(6-6) + \hat{k}(2-6) = 8\hat{i} - 4\hat{k}$ $S D = \left  \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 X \vec{b}_2)}{ \vec{b}_1 X \vec{b}_2 } \right $ $= \left  \frac{2X8 - 0 - 4X4}{\sqrt{(8)^2 + (-4)^2}} \right  = 0$	1
25	$\vec{a} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ , $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ $\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j} + \hat{k}$ $\vec{a} - \vec{b} = 2\hat{i} + 5\hat{k}$ $\text{Perpendicular vector} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 1 \\ 2 & 0 & 5 \end{vmatrix} = 20\hat{i} - 18\hat{j} - 8\hat{k}$ $ \vec{r}  = \sqrt{400 + 324 + 64} = \sqrt{788} = 2\sqrt{197}$ $\text{unit vector } \hat{r} = \frac{10}{\sqrt{197}}\hat{i} - \frac{9}{\sqrt{197}}\hat{j} - \frac{4}{\sqrt{197}}\hat{k}$	1
26	$(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$ $\Rightarrow \frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{\tan^{-1} x}{1+x^2}$ $P = \frac{1}{1+x^2}$ , $Q = \frac{\tan^{-1} x}{1+x^2}$ $I.F = e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1} x}$ $y \cdot e^{\tan^{-1} x} = \int \frac{\tan^{-1} x}{1+x^2} e^{\tan^{-1} x} dx$ $\text{Put } \tan^{-1} x = t$ $\frac{1}{1+x^2} dx = dt$ $\Rightarrow y \cdot e^{\tan^{-1} x} = \int e^t \cdot t \cdot dt$ $\Rightarrow y \cdot e^{\tan^{-1} x} = t \cdot e^t - \int e^t \cdot dt = t e^t - e^t + C$ $\Rightarrow y \cdot e^{\tan^{-1} x} = (\tan^{-1} x - 1) \cdot e^{\tan^{-1} x} + C$ $\Rightarrow y = \tan^{-1} x - 1 + C e^{-\tan^{-1} x}$	$\frac{1}{2}$ $1/2$ $1$

**OR**

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}$$

$$\frac{dy}{dx} + \frac{2x}{x^2 - 1} \cdot y = \frac{2}{(x^2 - 1)^2}$$

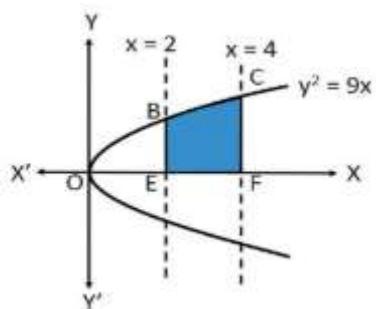
$$P = \frac{2x}{x^2 - 1} \quad Q = \frac{2}{(x^2 - 1)^2}$$

$$\text{I.F. } = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = x^2 - 1$$

$$y(x^2 - 1) = \int \frac{2}{(x^2 - 1)^2} \cdot (x^2 - 1) dx = 2 \int \frac{dx}{x^2 - 1} = 2 \cdot \frac{1}{2} \log \frac{x-1}{x+1} + C$$

$$\Rightarrow y(x^2 - 1) = \log \left| \frac{x-1}{x+1} \right| + C$$

27



Area bounded by  $y^2 = 9x, x = 2,$

$x = 4$  and  $x$  axis in first quadrant

$$\text{Required area} = \int_2^4 3\sqrt{x} dx$$

$$= 3x^{\frac{3}{2}} \Big|_2^4 = 2x^{\frac{3}{2}} \Big|_2^4 = 2(8 - 2\sqrt{2}) = (16 - 4\sqrt{2}) \text{ Sq units}$$

1

1

28	$A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $A - B^T = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$	$\frac{1}{2}$
SECTION IV		
29	<p><math>A = R - \left\{ \frac{2}{3} \right\}</math></p> <p><math>f(x) = \frac{4x+3}{6x-4}</math>, <math>f(x)</math> is defined for <math>x \neq \frac{2}{3}</math></p> <p>For one-one let <math>x_1, x_2 \in A</math> such that <math>f(x_1) = f(x_2) \Rightarrow \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}</math></p> $24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$ $\Leftrightarrow x_1 = x_2 \text{ hence } f(x) \text{ is one- one}$ <p><u>ONTO :</u></p> <p>Let <math>y = \frac{4x+3}{6x-4} \Rightarrow x = \frac{4y+3}{6y-4}</math></p> <p>Here <math>x \neq \frac{2}{3}</math></p> <p>If <math>x = \frac{2}{3}</math> then <math>\frac{2}{3} = \frac{4y+3}{6y-4}</math></p> $\Rightarrow 12y+9=12y-8 \Rightarrow 9=-8 \text{ which is wrong}$ <p>Thus every <math>y</math> has its pre image in <math>A = R - \left\{ \frac{2}{3} \right\}</math></p> <p>for every <math>x \in A</math></p> $\therefore f(x) \text{ is onto}$	$\frac{1}{2}$

<p>30 <math>y = \log(x + \sqrt{x^2 + a^2})</math></p> $\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}} \times [1 + \frac{2x}{2\sqrt{x^2 + a^2}}] = \frac{1}{\sqrt{x^2 + a^2}}$ $\Rightarrow \sqrt{x^2 + a^2} \frac{dy}{dx} = 1$ $\Rightarrow \sqrt{x^2 + a^2} \frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{x^2 + a^2}} \frac{dy}{dx} = 0$ $\Rightarrow (x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$	$1\frac{1}{2}$
<p>31 Let <math>y = x^{\sin x} + (\sin x)^{\cos x}</math></p> <p><math>y = u + v</math> where</p> <p><math>u = x^{\sin x}, \Rightarrow \log u = \sin x \log x</math>      <math>v = (\sin x)^{\cos x}</math></p> $\Rightarrow \frac{1}{u} \frac{du}{dx} = \cos x \log x + \frac{\sin x}{x}$ $\Rightarrow \frac{du}{dx} = x^{\sin x} (\cos x \log x + \frac{\sin x}{x})$ <p>Now <math>v = (\sin x)^{\cos x}</math></p> $\Rightarrow \log v = \cos x \log \sin x$ $\Rightarrow \frac{1}{v} \frac{dv}{dx} = -\sin x \log \sin x + \frac{\cos x}{\sin x} \cdot \cos x$ $\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} [-\sin x \log \sin x + \cot x \cos x]$ <p>Now <math>y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}</math></p> $= (x)^{\sin x} \left[ \cos x \log x + \frac{\sin x}{x} \right] + (\sin x)^{\cos x} [-\sin x \log \sin x + \cot x \cos x]$ <p>OR</p> <p><math>x = a(\theta - \sin \theta), y = a(1 + \cos \theta)</math></p> $\frac{dx}{d\theta} = a(1 - \cos \theta), \frac{dy}{d\theta} = -a \sin \theta$ $\Rightarrow \frac{dy}{dx} = \frac{-a \sin \theta}{a(1 - \cos \theta)} = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = -\cot \frac{\theta}{2}$ $\frac{d^2y}{dx^2} = \frac{1}{2} \operatorname{cosec}^2 \frac{\theta}{2} \cdot \frac{d\theta}{dx} = \frac{1}{2a(1 - \cos \theta)} \operatorname{cosec}^2 \frac{\theta}{2}$	$1\frac{1}{2}$

	$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{4a} \frac{\cosec^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}} = \frac{1}{4a} \cosec^4 \frac{\theta}{2}$	$1\frac{1}{2}$
32	<p><math>(3xy + y^2)dx + (x^2 + xy)dy = 0</math></p> $\Rightarrow \frac{dy}{dx} = -\frac{(3xy + y^2)}{(x^2 + xy)}$ <p>Put <math>y=vx</math></p> $\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\Rightarrow v + x \frac{dv}{dx} = -\frac{(3x.vx + v^2 x^2)}{(x^2 + x.vx)} \Rightarrow x \frac{dv}{dx} = \frac{-3v - v^2}{1+v} - v$ $= \frac{-3v - v^2 - v - v^2}{1+v} = \frac{-4v - 2v^2}{1+v}$ $\frac{1+v}{2v^2 + 4v} dv = \frac{-1}{x} dx$ <p>Integrating both sides we get</p> $\frac{1}{4} \log v^2 + 2v  = -\log x  + c$ $\log v^2 + 2v  = -\log x^4 + 4c$ $\log v^2 + 2v  + \log x^4 = 4c$ $\frac{y^2 + 2xy}{x^2} \cdot x^4 = C$ $(y^2 + 2xy)x^2 = C$	$1\frac{1}{2}$

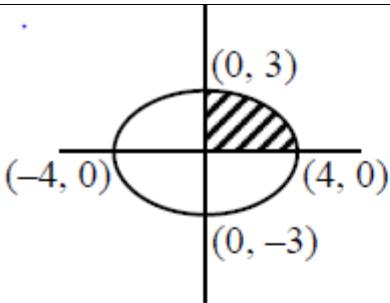
OR

$$(1+x^2)dy + 2xydx = \cot x dx, \quad x \neq 0$$

	$\frac{dy}{dx} = \frac{\cot x}{1+x^2} - \frac{2x}{1+x^2} y \Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{\cot x}{1+x^2}$ $P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$ $I.F = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$ $y(1+x^2) = \int \frac{\cot x}{1+x^2} (1+x^2) dx = \int \cot x dx = \log \sin x + C$ $\Rightarrow y(1+x^2) = \log \sin x + C$	$1\frac{1}{2}$
33	$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ $f'(x) = \cos x - \sin x = \sqrt{2}(\cos x \sin \frac{\pi}{4} - \sin x \cos \frac{\pi}{4}) = -\sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$ $0 < x < 2\pi \Rightarrow 0 - \frac{\pi}{4} < x - \frac{\pi}{4} < 2\pi - \frac{\pi}{4}$ $\frac{\pi}{4} < x - \frac{\pi}{4} < \frac{7\pi}{4}$ <u><math>f(x)</math> is strictly increasing if <math>f'(x) &gt; 0</math></u> $\Rightarrow -\sqrt{2} \sin\left(x - \frac{\pi}{4}\right) > 0$ $\Rightarrow \sin\left(x - \frac{\pi}{4}\right) < 0 \Rightarrow \frac{-\pi}{4} < x - \frac{\pi}{4} < 0$ <u>Or</u> $\pi < x - \frac{\pi}{4} < \frac{7\pi}{4}$ Or $0 < x < \frac{\pi}{4}$ or $5\frac{\pi}{4} < x < 2\pi$ $\Leftrightarrow f(x)$ is strictly increasing when $x \in [0, \frac{\pi}{4}] \cup (\frac{5\pi}{4}, 2\pi]$ <u>For strictly decreasing:</u> $-\sqrt{2} \sin\left(x - \frac{\pi}{4}\right) < 0$ $\sin\left(x - \frac{\pi}{4}\right) > 0$ $\Rightarrow 0 < x - \frac{\pi}{4} < \pi \Rightarrow \frac{\pi}{4} < x < \frac{5\pi}{4} \quad x \in (\frac{\pi}{4}, \frac{5\pi}{4})$ $f(x)$ is strictly decreasing when $x \in (\frac{\pi}{4}, \frac{5\pi}{4})$ <u>OR</u> $x = y^2$	$1\frac{1}{2}$

	$1=2y \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{1}{2y}$ $xy=k \Rightarrow x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ curve intersect at right angle if $m_1 m_2 = -1$ $\Rightarrow (\frac{1}{2y})(-\frac{y}{x}) = -1 \Rightarrow \frac{1}{2x} = 1 \Rightarrow x = \frac{1}{2}$ $x = y^2 = \frac{1}{2}$ $xy=k \Rightarrow x^2 y^2 = k^2$ $(1/4)(1/2) = k^2 \therefore 8k^2 = 1$	1 1 1 1
34	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \dots \dots \dots \text{(i)}$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\frac{\pi}{2} - x)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \dots \dots \dots \text{(ii)}$ <p>Adding (i) &amp; (ii) we get</p> $2 I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $2 I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx = x \Big _{\frac{\pi}{6}}^{\frac{\pi}{3}} = 2 I = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$	2 2 1

35



1

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \quad \Rightarrow \quad y^2 = 9 \left(1 - \frac{x^2}{16}\right)$$

Required Area

$$= 4 \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} dx$$

1

$$= 3 \int_0^4 \sqrt{16 - x^2} dx = 3 \left[ \frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \frac{x}{4} \right]_0^4$$

$$= 3 \left[ 8 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq. units}$$

1

36

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2

½

$$\Rightarrow |A| = -1 \quad A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

2

$$x=0, y=5, z=3$$

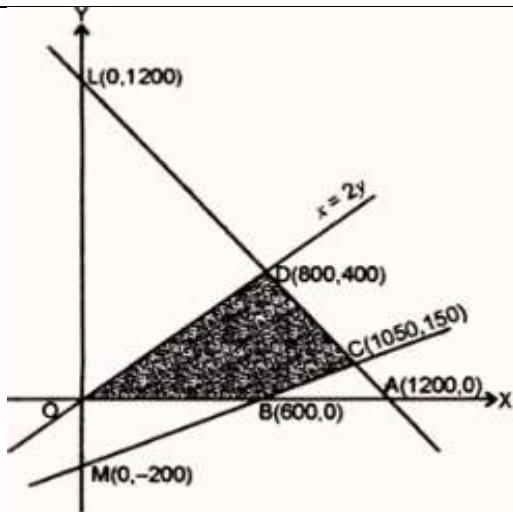
½

OR

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

	$\text{adj}A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$ $ A  = 4, \quad A^{-1} = \frac{1}{ A } \text{adj } A$ $X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $x=2, y=1, z=3$	2 $\frac{1}{2} + \frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
37	<p>Equation of plane through given 3 points</p> $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$ $\Rightarrow \begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$ $\Rightarrow 2x + 3y + 4z = 7$ <p>Distance = <math>\frac{ ax_1 + by_1 + cz_1 + d }{\sqrt{a^2 + b^2 + c^2}} = \frac{2(7) + 3(2) + 4(-5) - 7}{\sqrt{4 + 9 + 16}} = \frac{29}{\sqrt{29}} = \sqrt{29}</math></p> <p>OR</p> <p>Solving the equation of line and plane, we get</p> $(2+3\lambda).1 - (-1+4\lambda) + (-2+2\lambda) = 5$ $\Rightarrow \lambda = 4$ <p>Therefore point of intersection of line and plane is (14, 15, 6)</p> <p>Thus Distance between the point (-1, -5, -10) and (14, 15, 6) is</p> $\sqrt{225 + 400 + 256} = \sqrt{881}$	2 1 2 1 2

38



2

Let  $x$  is the number of dolls of type A and  $y$  is the number of dolls of type B that are manufactured.

Now according to question

$$\text{Maximize } Z = 12x + 16y$$

subject to constraints

$$y \leq x/2$$

$$\Rightarrow 2y \leq x$$

$$x \leq 3y + 600$$

$$\Rightarrow x - 3y \leq 600 \dots 3$$

$$x \geq 0, y \geq 0 \dots 4$$

Now draw the graph using equation 1 to 4 as shown in figure.

The shaded portion in the graph depicts the feasible region.

Point  $Z = 12x + 16y$

A(800, 400) 7200

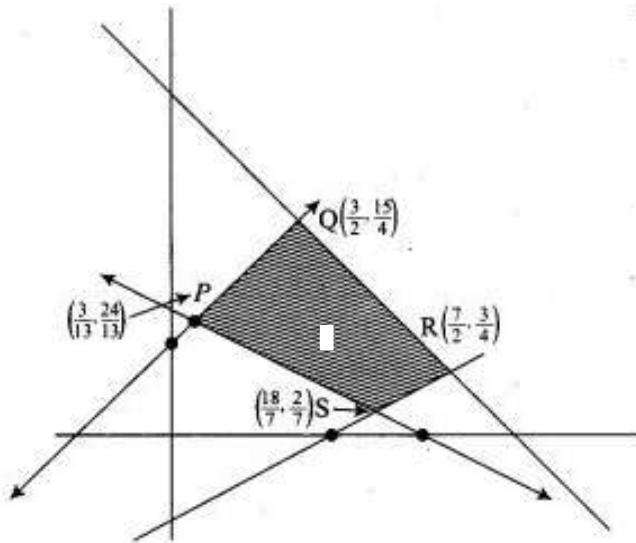
B(1050, 150) 15000

C(600, 0) 16000

The maximum profit is 16000.

OR

3



**Sol.** From the figure we have bounded region with corner points as

$$P\left(\frac{3}{13}, \frac{24}{13}\right), Q\left(\frac{3}{2}, \frac{15}{4}\right), R\left(\frac{7}{2}, \frac{3}{4}\right), S\left(\frac{18}{7}, \frac{2}{7}\right)$$

Also  $Z = x + 2y$ .

Corner points	Corresponding value of Z
$\left(\frac{3}{13}, \frac{24}{13}\right)$	$\frac{51}{13} = 3\frac{12}{13}$
$\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{22}{7} = 3\frac{1}{7}$ (Minimum)
$\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{20}{4} = 5$
$\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{36}{4} = 9$ (Maximum)

Hence, the maximum and minimum value of Z are 9 and  $3\frac{1}{7}$ , respectively

3

(ii)  $Z = px + qy$

$$Z = \frac{3p}{2} + \frac{15q}{4} \text{ at } \left(\frac{3}{2}, \frac{15}{4}\right)$$

$$Z = \frac{7p}{2} + \frac{3q}{4} \text{ at } \left(\frac{7}{2}, \frac{3}{4}\right)$$

Both values of Z are maximum.  $\therefore \frac{3p}{2} + \frac{15q}{4} = \frac{7p}{2} + \frac{3q}{4}$

$$\frac{4p}{2} = \frac{12q}{4} \Rightarrow 2p = 3q$$

No of maximum solutions are infinite lying in the line joining Q and R.

2

