Class: XII Session: 2020-21

Subject: Mathematics

Value points, Practice Paper 3

S. No.	Solutions	Marks
1	Yes Or R is transitive but not symmetric	1
2	Yes	1
3	A and ∅ or 2	1
4	m x n	1
5	3	1
6	405	1
7	$\frac{e^{x^3}}{3}$ + c , where c is arbitrary constant	1
8	32/3 sq units	1
9	order is 2 and degree is 1 Or C = 5	1
10	$\frac{5\sqrt{6}}{3}\ sq\ unit$	1
11	$\sqrt{21}$	1
12	K=4	1

13	$\frac{1}{p^2}$	1
14	($\frac{17}{3}$, 0, $\frac{23}{3}$)	1
15	0.12	1
16	False	1
17	(i) c (ii) b (iii) d (iv) a (v) b	1 1 1 1 1
18	(i) b (ii) a (iii) c (iv) c (v) b	1 1 1 1 1
19	$tan^{-1}(1) + cos^{-1}\left(\frac{-1}{2}\right) + sin^{-1}\left(\frac{-1}{2}\right)$ $= \frac{\pi}{4} + (\pi - cos^{-1}\left(\frac{1}{2}\right)) - sin^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{4} + (\pi - \frac{\pi}{3}) - \frac{\pi}{6} = \frac{3\pi}{4}$	 1 1

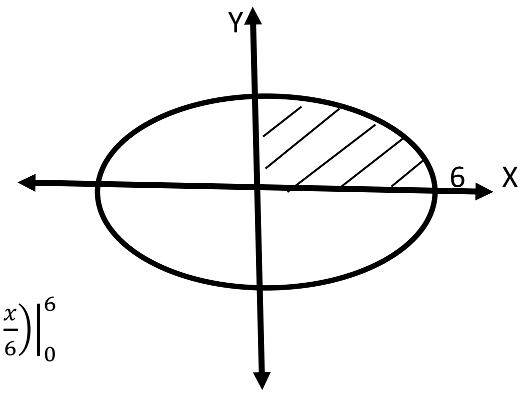
20	$(x \quad -5 \quad -1)\begin{pmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{pmatrix}\begin{pmatrix}x\\4\\1\end{pmatrix} = 0$ $\Rightarrow (x-2 \quad -10 \quad 2x-8)\begin{pmatrix}x\\4\\1\end{pmatrix} = 0$ $\Rightarrow x\,(\,x-2\,) - 40 + 2x - 8 = 0$ $\Rightarrow$ x = $\pm\, 4\sqrt{3}$ OR $2\begin{pmatrix}3 & 4\\5 & x\end{pmatrix} + \begin{pmatrix}1 & y\\0 & 1\end{pmatrix} = \begin{pmatrix}7 & 0\\10 & 5\end{pmatrix}$ $\Rightarrow \begin{pmatrix}7 & 8+y\\10 & 2\,x+1\end{pmatrix} = \begin{pmatrix}7 & 0\\10 & 5\end{pmatrix}$ $\Rightarrow$ x = 2 and y = -8 and x – y = 10	1 1 1 1
21	$f(x) = \begin{cases}kx+1, & if\ x \le \pi\\ cosx, & if\ \ x > \pi\end{cases}$ is continuous at $x = \pi$ If LHL = RHL = f(π) $\Rightarrow$ k $\pi + 1 = \cos\pi$ $\Rightarrow$ k = $\frac{-2}{\pi}$	1 1
22	$x = 1 - asin\theta, y = bcos^2\theta$ $\frac{dx}{d\theta}$ = -a cos θ , $\frac{dy}{d\theta}$ = 2b cos θ (-sin θ) So , slope of the normal = - $\frac{dx}{dy}$ = $\frac{-a}{2b\sin\theta}$ at $\theta = \frac{\pi}{2}$ = $\frac{-a}{2b}$	1 1
23	$\int e^x \frac{sin^4x - cos^4x}{sinx - cosx}\, dx$	

	$sin^4x - cos^4x$ = $(sin^2x + cos^2x)$ $(sin^2x - cos^2x)$ = $(\sin x + \cos x)$ $(\sin x - \cos x)$ ($sin^4x - cos^4x$) / $(\sin x - \cos x)$ = $(\sin x + \cos x)$ So , $\int e^x \frac{sin^4x - cos^4x}{sinx - cosx}\ dx = \int e^x$ (sin x + cos x) dx = e^x sin x + c OR $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7x\ dx$ = 0 (being an odd function)	1 1 1 + 1
24	Area = $\int_0^2 \sqrt{4 - x^2}$ dx = = $\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}$ $Putting\ the\ upper\ and\ lower\ limits\ , we\ get,\ Area = \pi\ sq\ units$	1 1
25	. $\frac{dy}{dx} = x^3 cosec\ y$, given that $f(0) = 0$ $\int \sin y\ dy = \int x^3\ dx + c$ -cos y = $\frac{x^4}{4}$ + c As, $f(0) = 0$ so c = - 1 $\frac{x^4}{4}$ + cos y = 1	1 ½ ½
26	The vector equation of a plane passing through A(2, 5, -3), B(-2,-3, 5) and C(5, 3, -3) is given by ($\vec{r}$ - $\vec{a}$) [($\vec{b}$ - $\vec{a}$) x ($\vec{c}$ - $\vec{a}$)] = 0 ($\vec{r}$ – (2$\hat{\imath}$ + 5$\hat{\jmath}$ - 3$\hat{k}$)[(-4$\hat{\imath}$ -8$\hat{\jmath}$ +8$\hat{k}$)x (3$\hat{\imath}$ -2$\hat{\jmath}$)] = 0	1 1
27	$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ And $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$. Since, the drs of the above lines are in same ratios	

	So, lines are parallel S.D = $\left\|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}}{\|\vec{b}\|}\right\|$ Here ($\overrightarrow{a_2} - \overrightarrow{a_1}$) = $2\hat{\imath} + \hat{\jmath} - \hat{k}$, $\|\vec{b}\| = 7$ = $\frac{\sqrt{293}}{7}$ units	½ + 1 ½
28	$\sum p_i = 1$ $\Longrightarrow$ 6k = 1 $\Longrightarrow k = 1/6$ P(X $\leq$ 2) = 6k = 1	1 1
29	A=$\{x \in Z : \ 0 \leq x \leq 12\}$, given by R=$\{(a, b) : \|a - b\| \; is\ a\ multiple\ of\ 4\}$ **R is reflexive** Let a ∈ A , a R a ∀ a ∈ A as | a – a | = 0 = 0 x 4 clearly a multiple of 4 . **R is symmetric** Let a , b ∈ A such that a R b i.e. | a – b | is a multiple of 4 . Since | a – b | = | b – a | So , | b – a | is a multiple of 4 So , b R a **R is transitive** Let a , b , c ∈ A such that a R b and b R c $\Longrightarrow$ | a – b | = 4m and | b – c | = 4n where m , n ∈ N $\Longrightarrow$ a – b = $\pm$ 4m and b - c = $\pm$ 4n $\Longrightarrow$ a – c = (a – b) + (b – c) = $\pm$ 4 (m + n) $\Longrightarrow$ | a – c | = 4 (m + n) , clearly a multiple of 4 . So , a R c Hence R is an equivalence relation .	½ 1 1

		½
30	$y = x^a + x^{\sin x}$ = u + v $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ Here , $\frac{du}{dx}$ = a x^{a-1} V = $x^{\sin x}$ log v = sin x log x $\frac{1}{v}\frac{dv}{dx} = \frac{\sin x}{x}$ + log x cos x $\frac{dv}{dx} = x^{\sin x}$ ($\frac{\sin x}{x}$ + log x cos x) So, $\frac{dy}{dx}$ = a x^{a-1} + $x^{\sin x}$ ($\frac{\sin x}{x}$ + log x cos x)	½ ½ 1 ½ ½
31	. y=$3e^{2x} + 2e^{3x}$ $\frac{dy}{dx}$ =$6e^{2x} + 6e^{3x}$ $\frac{d^2y}{dx^2}$=$12e^{2x} + 18e^{3x}$ $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 12e^{2x} + 18e^{3x} - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x}) = 0$ OR We have x=a(cosθ+θsinθ) , y=a(sinθ-θcosθ) $\frac{dy}{d\theta}$=a(cosθ+θsinθ-cosθ)=aθsinθ $\frac{dx}{d\theta}$= a(-sinθ+θcosθ+sinθ.1)=aθcosθ $\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta}$ = tanθ	1 1 1 1 1

	$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}(tan\theta)= sec^2\theta\,\frac{d\theta}{dx}=sec^2\theta$x$\frac{1}{a\theta\cos\theta}$ $=\frac{sec^3\theta}{a\theta}$	1
32	. f(x)= $4x^3$- $6x^2$-72x+30 $f'(x)$=$12x^2$-12x-72 $12x^2$-12x-72=0 12(x^2-x-6)=0 X=-2,3 Interval Sign of $f'(x)$ Nature of f(x) (-∞,-2) + f is strictly increasing (-2,3) - f is strictly decreasing (3, ∞) + f is strictly increasing	1 ½ 1 ½
33	Let I= $\int\frac{x^2}{(x^2+1)(x^2+4)}$dx Put x^2= t $\frac{x^2}{(x^2+1)(x^2+4)}=\frac{t}{(t+1)(t+4)}=\frac{A}{t+1}+\frac{B}{t+4}$ A= -1/3 , B=4/3 $=\frac{-1}{3(x^2+1)}+\frac{4}{3(x^2+4)}$ I = $\int\frac{-1}{3(x^2+1)}dx+\int\frac{4}{3(x^2+4)}$ dx	1 1

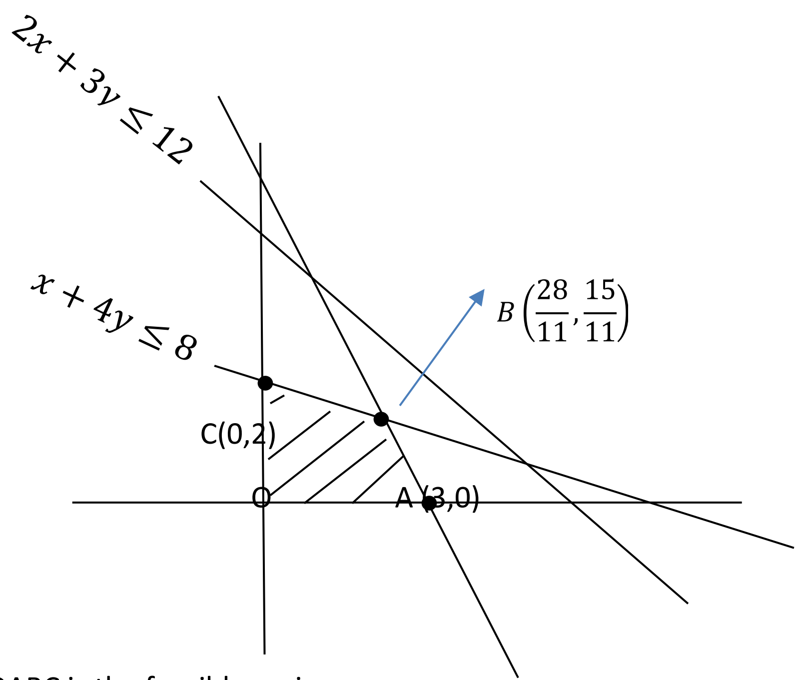
	I= $\frac{-1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2}$ +c	1
34	$x^2 + 9y^2 = 36$ $\frac{x^2}{36} + \frac{y^2}{4} = 1$ A = $4\int_0^6 y\,dx$ A= $\frac{4}{3}\int_0^6 \sqrt{6^2 - x^2}$  A= $\left\| \frac{4}{3}\left(\frac{x}{2}\sqrt{36 - x^2} + \frac{36}{2} sin^{-1}\frac{x}{6}\right) \right\|_0^6$ A= $\frac{4}{3}[18sin^{-1}(1) - 18sin^{-1}(0)]$ A= $\frac{4}{3}$ X 18 X $\frac{\pi}{2}$ = 12π	1 1 1
35	$$e^x tany\,dx + (1 - e^x)\,sec^2y\,dy = 0$$ $\frac{e^x}{1-e^x}$dx + $\frac{sec^2y}{tany}$dy =0 $\int\frac{e^x}{1-e^x}$dx +$\int\frac{sec^2y}{tany}$dy =0 -log|1-e^x| + log |tany|= c tany = c(1-e^x) **OR** x logx $\frac{dy}{dx} + y = \frac{2}{x}$ logx	1 1 ½ ½

	$\frac{dy}{dx}+\frac{1}{xlogx}y=\frac{2}{x^2}$ I.F. $=e^{\int\frac{1}{xlogx}dx}$ $= e^{\log(logx)dx}$ = logx y logx $=\int\frac{2}{x^2}logxdx = 2\int(logx)x^{-2}$dx +c $=2[\frac{-logx}{x}+\int x^{-2}$dx] +c ylogx $=\frac{-2}{x}$ (1+ logx) +c	
36	x+2y-3z=-4 2x+3y+2z=2 3x-3y-4z=11 $\begin{pmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}-4\\ 2\\ 11\end{pmatrix}$ A X = B X= A^{-1}B $A^{-1}=\frac{adj\ A}{\|A\|}$ $\|A\|=\begin{vmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{vmatrix}$ = 1(-12+6)-2(-8-6)-3(-6-9) $=67\neq 0$ Adj A = $\begin{pmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{pmatrix}$ $A^{-1}=\frac{1}{67}\begin{pmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{pmatrix}$	1/2 1 ½ 1 ½

	X= A^{-1}B $=\frac{1}{67}\begin{pmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{pmatrix}\begin{pmatrix}-4\\ 2\\ 11\end{pmatrix}$ $=\frac{1}{67}\begin{pmatrix}201\\ -134\\ 67\end{pmatrix} = \begin{matrix}3\\ -2\\ 1\end{matrix}$ X=3, y=-2, z=1 **OR** A=$\begin{pmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{pmatrix}$ B=$\begin{pmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{pmatrix}$ BA=$\begin{pmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{pmatrix}$ $=\begin{pmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{pmatrix}$ = 6I y+2z=7 x-y =3 2x+3y+4z=17 Let's rearrange the equations x-y = 3 2x+3y+4z = 17 Y+2z= 7	1 ½ 1 ½ 1

	$= \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix}$ $= \frac{1}{6} \begin{pmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{pmatrix}$ $= \frac{1}{6} \begin{pmatrix} 12 \\ -6 \\ 24 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ $\therefore x = 2, \quad y = -1, \quad z = 4$	2 ½
37	$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$ = k (say) X=-2k+4 , y=6k, z=-3k+1 D.R.'s are (-2k+4-2 , 6k-3, -3k+1+8) = (-2k+2 , 6k-3, -3k+9) (-2k+2)(-2) +(6k-3)(6)+(-3k+9)(-3)= 0 4k-4+36k-18+9k-27=0 49k-49=0 K=1 X=-2x1+4 , y=6x1 , z=-3x1+1 X=2 , y=6 , z=-2	1 ½ ½ 1 ½ ½ 1

	Distance $=\sqrt{(2-2)^2+(6-3)^2+(-2+8)^2}=\sqrt{9+36}=\sqrt{45}=3\sqrt{5}$	
	OR	
	The equation of plane passing through the intersection of two given planes is,	
	(x+3y-6)+t(3x-y-4z)=0	1
	(1+3t)x +(3-t)y -4tz-6=0	
	Distance of this plane from the origin (0,0,0,) is unity.	
	So,	1
	$1=\left\lvert \frac{-6}{\sqrt{(1+3t)^2+(3-t)^2+(-4t)^2}} \right\rvert$	
	$1=\left\lvert \frac{-6}{\sqrt{10+26t^2}} \right\rvert$	
	$10+26t^2$=36	1
	t=±1	
	putting t=1 in (A) we get,	
	4x+2y-4z-6=0	1
	putting t=-1 in (A) we get,	1
	-2x+4y+4z-6=0	

38	Max $z = x + y$	
	s,t $x + 4y \leq 8$	
	$2x + 3y \leq 12$	
	$3x + y \leq 9$	
	x , y ≥ 0	
		
	Region OABC is the feasible region	3
	At O(0,0) Z=0+0=0	
	At A(3,0) , Z=3+0=3	1
	At B$(\frac{28}{11}, \frac{15}{11})$, Z=$\frac{28}{11} + \frac{15}{11}$ =$\frac{43}{11}$,	
	At C(0,2) , Z=0+2=2	1
	Therefore, Optimal solution is $(\frac{28}{11}, \frac{15}{11})$ and maximum value of the function is $\frac{43}{11}$	
	OR	
	1) Points z = x + 2y	

	P($\frac{3}{13}$, $\frac{24}{13}$) $\frac{51}{13}$ Q($\frac{3}{2}$, $\frac{15}{4}$) 9 max R ($\frac{7}{2}$,$\frac{15}{4}$) 5 S ($\frac{18}{7}$,$\frac{2}{7}$) $\frac{22}{7}$ min Max z = 9 at Q($\frac{3}{2}$, $\frac{15}{4}$) and min z is $\frac{22}{7}$ $at\ S$ ($\frac{18}{7}$,$\frac{2}{7}$) 2) Z= px+qy If max Z occurs at Q ($\frac{3}{2}$,$\frac{15}{4}$) and R ($\frac{7}{2}$,$\frac{3}{4}$), then $\frac{3p}{2}+\frac{15q}{4} = \frac{7p}{2}+\frac{3q}{4}$ Simplifying, we get, 2p=3q This is the required condition. Also, the number of optimal solutions in this case will be infinite solutions lying on the line segment QR.	1 ½ 1 2 ½