CLASS XII SESSION 2020-21

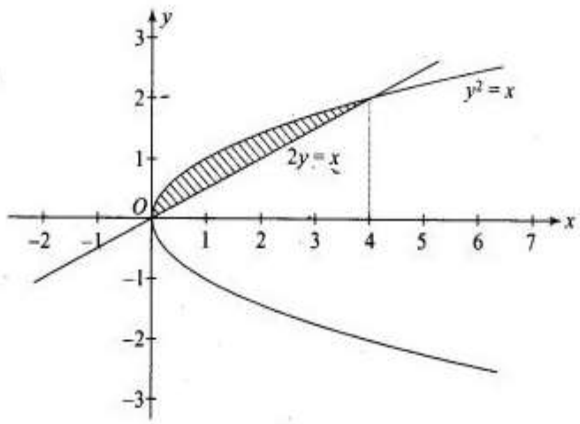
PRACTICE PAPER 5

SUBJECT : MATHEMATICS

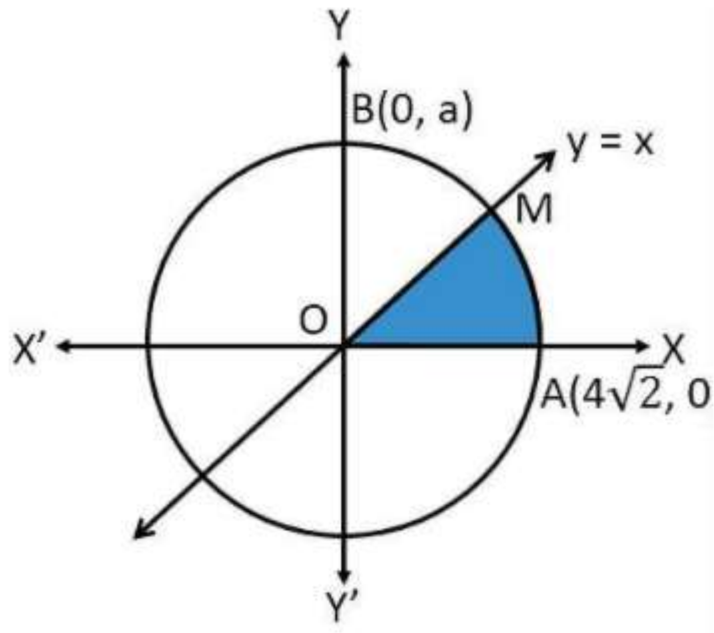
MARKING SCHEME (THEORY)

Sr No	Objective type Question Section I	Marks
1	2^3|A|=8X5 OR x=y	1
2	$A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \emptyset$	1
3	A=2, b=3 OR 1-α^2-$\beta\gamma$=0	1
4	Reflexive Relation	1
5	R is not symmetric	1
6	3 $\times$ p	1
7	$\frac{1}{3}e^{x^3}$+C OR e-1	1
8	$\frac{1}{4}$	1
9	1	1
10	$\overrightarrow{r}$ =(3$\hat{\imath}$ +4$\hat{\jmath}$ +5 $\hat{k}$) + λ (2i +2$\hat{\jmath}$ -3$\hat{k}$) OR distance = $\sqrt{b^2+c^2}$	1
11	Order =2 ,Degree 1	1
12	-1	1
13	P=-2	1
14	Zero OR 3	1
15	2/3 Square Units OR 37/3 Units	1
16	n=3 OR logx	1
17	CASE STUDY –I (i) 2x+2y=36 =>x+y=18=>y=18x	1
	(ii)V(x)=πx^2y= πx^2[18-x]= π[18x^2-x^3]	1
	(iii)V(x)=π(36x-3x^2)=0=>36x=3x^2=>.x=0,x=12 X=12,y=6	1
	(iv)V(x)=$\pi[18.12^2 - 12^3]$ = $\pi 12^2[18-12]$ = $\pi.144.6 = 864\pi$	1
	(V)$\frac{dv}{dx}$= π(36x-3x^2) $\frac{d^2v}{dx^2}$== π(36-6x)=-36 $\pi < 0$	1
18	CASE STUDY -II (i)P(U_1)= P(U_2)= P(U_3)=1/3 Equal probability (ii)E_1=Event of a ball chosen from P(E_1)= P(E_2)= P(E_3)=1/3 (III)P($\frac{E}{E_1}$)=2/5, P($\frac{E}{E_2}$)=3/5, P($\frac{E}{E_3}$)=4/5	1 1

	E-White ball	1
	(iv)$P(\frac{E_2}{E_1})=\frac{P(E_2)\,P(\frac{E}{E_2})}{P(E_1).P(\frac{E}{E_1})+P(E_2).P(\frac{E}{E_2})+P(E_3).P(\frac{E}{E_3})}$ $=\frac{\frac{1}{3}.\frac{3}{5}}{\frac{1}{3}.\frac{2}{5}+\frac{1}{3}.\frac{3}{5}+\frac{1}{3}.\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{2}{15}+\frac{3}{15}+\frac{4}{15}} = \frac{3}{9} = \frac{1}{3}$	1
	(v) $P(E_2)$=1/3 $P(E/E_2)$=3/5 $P(E_2)\ P(E/E_2)$=1/3X3/5=1/5	1
	PART B **SECTION III 2 M**	
19	$tan^{-1}\left[\frac{\cos x}{1-sinx}\right]=tan^{-1}\left[\frac{\sin(\frac{\pi}{2}-x)}{1-\cos(\frac{\pi}{2}-x)}\right]=tan^{-1}\left[\frac{2\sin(\frac{\pi}{4}-\frac{x}{2})cos(\frac{\pi}{4}-\frac{x}{2})}{2sin^2(\frac{\pi}{4}-\frac{x}{2})}\right]$ $=tan^{-1}\left[\cot((\frac{\pi}{4}-\frac{x}{2})\right]=tan^{-1}\left[\tan(\frac{\pi}{2}-(\frac{\pi}{4}-\frac{x}{2})\right]$ $=\frac{\pi}{4}+\frac{x}{2}$	1 1
20	2A-3B+5C=0 ⇨ 2A=3B-5C=3$\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}-5\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$ =>2A- $\begin{bmatrix}-16 & 6 & 10\\ -26 & -2 & -18\end{bmatrix}$=>A= $\begin{bmatrix}-8 & 3 & 5\\ -13 & -1 & -9\end{bmatrix}$ OR $\begin{bmatrix}2x\\ x\end{bmatrix}+\begin{bmatrix}3y\\ 5y\end{bmatrix}+\begin{bmatrix}-8\\ -11\end{bmatrix}=0$ 2x+3y-8=0 X+5y-11=0 solving x=1, y=2	½ 1 ½ ½ 1 1/2
21	K=6	2
22	$\frac{dy}{dx}=\frac{-16x}{9y}$ slope of tangent at(2,3)=-32/27 , Slope of normal=27/32 Equation of tangent=32x+27y=145 , Equation of normal=27x-32y=-4	1+1
23	$\int\frac{1-2sin^2x+2sin^2x}{cos^2x}dx=\int sec^2x dx=$ tanx+C OR $\frac{1}{4}\int_0^a\frac{dx}{\frac{1}{4}+x^2}=\frac{\pi}{8}$ $=>\frac{1}{4}\int_0^a\frac{dx}{(\frac{1}{2})^2+x^2}=\frac{\pi}{8}$ $=>\frac{1}{4}\left[\frac{\tan^{-1}2x}{1/2}\right]_0^a=\frac{\pi}{8}$ $=>\frac{1}{4}[\tan^{-1}2a-\tan^{-1}0]=\frac{\pi}{8}$=>a=1/2	2 ½ 1+1/2

24	We have $y^2 = x$ and $2y = x$ Solving, we get $y^2 = 2y$ $\Rightarrow y = 0,2$ When y = 0, x=0 and when y = 2, x = 4 So, points of intersection are (0,0) and (4,2) Graphs of parabola $y^2 = x$ and $2y = x$ are as shown in the following figure.  From the figure, area of the shaded region, $A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right]dx$ $= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}\cdot\frac{x^2}{2}\right]_0^4 = \frac{2}{3}\cdot 4^{3/2} - \frac{16}{4} - 0 = \frac{16}{3} - 4 = \frac{4}{3}$ sq. units	1+1
25	$\frac{dy}{dx} + 2y\,tanx = sinx$ P=2tanx Q=sinx IF= $e^{\int 2tanxdx}$=$e^{2\int tanxdx}$ =$e^{2logsecx}$=sec^2x Solution=ysec^2x=$\int sinx\ sec^2xdx$=$\int \sec x\ tanxdx$ => y$sec^2x = secx + C$ at x=$\frac{\pi}{3}$, y=0 => 0=sec$\frac{\pi}{3}$+C =>C=-2 PS= y$sec^2x = secx - 2$	1+1
26	$\overrightarrow{PQ}$ =$\hat{\imath}$-4$\hat{\jmath}$-2$\hat{k}$ $\overrightarrow{PR}$=-3$\hat{\imath}$+12$\hat{\jmath}$+6$\hat{k}$ =-3[$\hat{\imath}$-4$\hat{\jmath}$-2$\hat{k}$] ⇨ $\overrightarrow{PR}$ =-3$\overrightarrow{PQ}$ ⇨ P,Q,R are collinear	1+1
27	Normal vector =$\vec{n}$ Through (1,0,0) ie i $(\vec{r}$ -$\hat{\imath}$). $\vec{n}$ =0 Plain contains line $\vec{r}$=0+$\lambda\hat{\jmath}$ $(\vec{r}$ -$\hat{\imath}$). $\hat{k}$=0 i.n=0 $\vec{n}$ =$\hat{k}$ r.$\hat{k}$=0 j.n=0	1+1

28	$P(\frac{\overline{E}}{\overline{F}})= \frac{P(\overline{E}\cap\overline{F})}{P(\overline{F})} = \frac{P(\overline{E \cup F})}{P(\overline{F})} = \frac{1-P(EUF)}{1-P(F)}$ P(EUF)=P(E)+P(F)-P(E$\cap F$) =0.8+0.7-0.6=0.9 $=>P\left(\frac{\overline{E}}{\overline{\overline{F}}}\right)=\frac{1-0.9}{1-0.7} = \frac{0.1}{0.3}=\frac{1}{3}$ OR P(A$\cap B$)=$\frac{7}{10}$, P(B)=$\frac{17}{20}$ $P(\frac{A}{B})=\frac{P(A\cap B)}{,P(B)}$ $=\frac{\frac{7}{10}}{\frac{17}{20}}=\frac{14}{17}$	1+1 1 1
	SECTION IV	
29	For $x_{1\,,}x_2 \in A$ (i)f($x_{1\,,}$)= f($x_{2\,,}$)=> $\frac{4x_1+3}{6x_1-4}=\frac{4x_2+3}{6x_2-4}$ =>34x_1=34x_2 =>x_1=x_2 f is one-one (ii) y=$\frac{4x+3}{6x-4}$ =>x=$\frac{4y+3}{6y-4}$ f is onto	1+1+1
30	Point of intersection=x=y^2 xy=k (1) y^2y=k=>y=$k^{\frac{1}{3}}$,x=$k^{\frac{2}{3}}$ (2)$m_1=\frac{1}{2y}$ $m_2=\frac{-y}{x}$ (3) $m_1 m_2$=-1 =$\frac{1}{2y}(\frac{-y}{x})$=-1=>2x=1 2$k^{\frac{2}{3}}$=1 cubing 8$k^2 = 1$ OR Y=$\tan^{-1}\sqrt{\frac{1-cosx}{1+cosx}}$=> Y=$\tan^{-1}\sqrt{\frac{sin^2\frac{x}{2}}{cos^2\frac{x}{2}}}$ => Y=$\tan^{-1}(tan\frac{x}{2})$ =>y=$\frac{x}{2}$=>$\frac{dy}{dx}=\frac{1}{2}$	1+1+1 1 1 1

31	 $\therefore$ Required Area $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$ ...(1) $\downarrow$ I_1 $\downarrow$ I_1 Taking I_1 i.e. $I_1 = \int_0^4 x.dx$ $= \left[\frac{x^2}{2}\right]_0^4$ $= \frac{(4)^2 - 0}{2}$ $= \frac{16}{2}$ $= 8$	1+1+1

Now solving I_2

$$I_2 = \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\ dx$$

It is of form

$$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$$

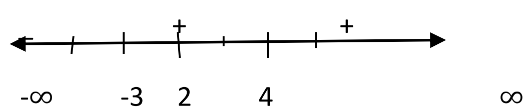
Replacing a by $4\sqrt{2}$, we get

$$I_2 = \left[\frac{x}{2}\sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= \frac{4\sqrt{2}}{2}\sqrt{(4\sqrt{2})^2 - (4\sqrt{2})^2} + \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}}$$

$$-\frac{4}{2}\sqrt{(4\sqrt{2})^2 - (4)^2} - \frac{(4\sqrt{2})^2}{2}\sin^{-1}\frac{4}{4\sqrt{2}}$$

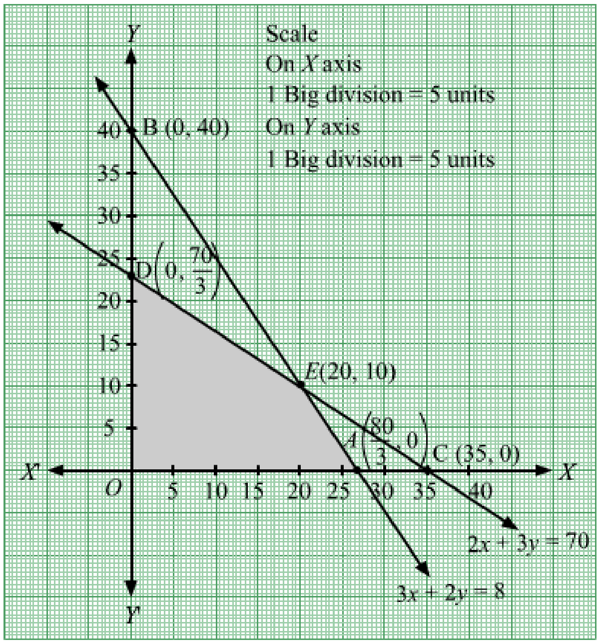
$$= 0 + \frac{16 \times 2}{2}\sin^{-1}(1) - 2\sqrt{32 - 16} - \frac{16 \times 2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

	$= 16\sin^{-1}(1) - 2\sqrt{16} - 16\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $= 16\left[\sin^{-1}(1) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right] - 8$ $= 16\left[\frac{\pi}{2} - \frac{\pi}{4}\right] - 8$ $= 16\left[\frac{4\pi - 2\pi}{4 \times 2}\right] - 8$ $= \frac{16}{8}[2\pi] - 8$ $= 2[2\pi] - 8$ $= 4\pi - 8$ Putting the value of I_1 & I_2 in (1) Area $= 8 + 4\pi - 8$ $= 4\pi$ $\therefore$ Required Area $= 4\pi$ Square units	
32	$f'(X)=x^3 - 3x^2 - 10x + 24$ =(x-2)(x-4)(x+3)=0 =>x=-3,2,4 Strictly ↑ =(-3,2)U(4,∞) Strictly ↓ =(-∞, 3)$U(2,4)$  	3
33	Putting Sinx=t =>$\int \frac{2dt}{(1-t)(1+t^2)}$ $\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$ A=1,B=1,C=1 $\int \frac{1}{1-t}$dt $+\frac{1}{2}\int \frac{2t}{1+t^2}\,dt + \int \frac{1}{1+t^2}$ =>-log(1-sinx)+$\frac{1}{2}$log(1+$sin^2 x + \tan^{-1}(sinx)$ +C OR $\frac{1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$	3 ½

	Getting A=1/5,B=-1/5,C=2/5 Finally a=-1/10 ,b= -2/10	1+1/2 1
34	Xdy-ydx=$\sqrt{x^2+y^2}$ dx ⇨ Xdy=($\sqrt{x^2+y^2}$+y)dx ⇨ $\frac{dy}{dx}=\frac{(\sqrt{x^2+y^2}+y}{x}$ homogeneous differential equation y=v(x) ⇨ v+x$\frac{dv}{dx}$=$\frac{(\sqrt{x^2+v^2x^2}+vx}{x}$ =$\sqrt{1+v^2}+v$ ⇨ x$\frac{dv}{dx}=\sqrt{1+v^2}$ ⇨ $\int\frac{dv}{\sqrt{1+v^2}}$=$\int\frac{dx}{x}$ ⇨ Log|$\sqrt{1+v^2}+v$|=logx +logC ⇨ $\sqrt{1+v^2}+v$ =Cx ⇨ $\frac{y}{x}$+$\sqrt{1+\frac{y^2}{x^2}}$ =Cx	
35	**log(x^2+y^2) =2tan^{-1}($\frac{y}{x}$)** $\frac{1}{x^2+y^2}\left[2x+2y\frac{dy}{dx}\right]$ =$\frac{2}{1+\frac{y^2}{x^2}}\left[\frac{x\frac{dy}{dx}-y.1}{x^2}\right]$ **=>**$\frac{x+y\frac{dy}{dx}}{x^2+y^2}$ **=**$\frac{x^2}{x^2+y^2}\left[\frac{x\frac{dy}{dx}-y.1}{x^2}\right]$ **=>x+y**$\frac{dy}{dx}$ =**x**$\frac{dy}{dx}-y$ **=>**$\frac{dy}{dx}=\frac{x+y}{x-y}$	
	SECTION V	
36	$\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$ =AB=6 I =$A^{-1}=\frac{1}{6}B$ =>$\begin{bmatrix}x\\y\\z\end{bmatrix}$=$\frac{1}{6}\begin{bmatrix}12\\-6\\24\end{bmatrix}$=$\begin{bmatrix}2\\-1\\4\end{bmatrix}$ OR 3x+0y+3z=8+2y =>3x-2y+3z=8 2x+ 1y+0z=1+z =>2x+y-z=1 4x+0y +2z=4+3y=>4x_3y+2z=4 Ax=B =>$\begin{bmatrix}3&2&-3\\2&1&-1\\4&-3&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$=$\begin{bmatrix}8\\1\\4\end{bmatrix}$ A^{-1}=$\frac{1}{17}\begin{bmatrix}1&5&1\\8&6&-9\\10&-1&7\end{bmatrix}$ X=$A^{-1}B$ => X=$\begin{bmatrix}1\\2\\3\end{bmatrix}$	**5** **1.5** **½** **1.5** **1.5**

37

First we will convert the given inequations into equations, we obtain the following equations:
3x+2y=80=0,2x+3y=70,x=0 and y=0



The corner points of the feasible region are O(0, 0), $A\left(\frac{80}{3}, 0\right)$, $E(20, 10)$ and $D\left(0, \frac{70}{3}\right)$.

The values of Z at these corner points are as follows.

Corner point	Z = 15x + 10y
O(0, 0)	15 × 0 + 10 × 0 = 0
$A\left(\frac{80}{3}, 0\right)$	$15 \times \frac{80}{3} + 10 \times 0 = 400$
$E(20, 10)$	15 × 20 + 10 × 10 = 400
$D\left(0, \frac{70}{3}\right)$	$15 \times 0 + 10 \times \frac{70}{3} = \frac{700}{3}$

We see that the maximum value of the objective function Z is 400 which is at $A\left(\frac{80}{3}, 0\right)$ and $E(20, 10)$.
Thus, the optimal value of Z is 400.

OR

Plane=0x+3y+4z=6
$\vec{x}$=0.$\hat{\imath}$+3$\hat{\jmath}$+4$\hat{k}$
$\overrightarrow{OP}$=x.$\hat{\imath}$+y$\hat{\jmath}$+z$\hat{k}$
$\frac{x_1}{0} = \frac{y_1}{3} = \frac{z_1}{4}$=r =>$x_1$=0, y_1=3r, z_1=4r
(x,y,z) lies on plane=>0+3(3r)+4(4r)=6
=>r=6/25
Foot of perpendicular=(0, 18/25 ,24/25)

5

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$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = \begin{vmatrix} x-2 & y-2 & z+1 \\ 3-2 & 4-2 & 2+1 \\ 7-2 & 0-2 & 6+1 \end{vmatrix}$ => 20x+8y-12z-68=0

5x+2y-3z-17=0

{ $\vec{r}$-(4$\hat{\imath}$+3$\hat{\jmath}$+$\hat{k}$) }, (20$\hat{\imath}$ +8$\hat{\jmath}$-12$\hat{k}$), (5$\hat{\imath}$ +2$\hat{\jmath}$-3$\hat{k}$)

OR

(i)

Corner points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12

$Max \quad Z = 12 \; at \; E(4,0)$

$Min \quad Z = -32 \; at \; A(0,8)$

(ii) Since maximum value of Z occurs at B(4,10) and C(6, 8)

$\therefore 4p + 10q = 6p + 8q$

$\Rightarrow 2q = 2p$

$\Rightarrow p = q$

Number of optimal solution are infinite

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$1\frac{1}{2}$

1

2

$\frac{1}{2}$

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