# Directorate of Education, GNCT of Delhi

## Solution of Practice Paper 2

## Term -II (2021-22)

## Class – XII

## Mathematics (Code: 041)

| Q. No. | VALUE POINTS |
|---|---|
| | **SECTION – A** |
| 1 | Let I=$\int \frac{1}{\cos^2 x(1-tanx)^2}dx$<br>Put 1-tanx=y so that$-sec^2 x\,dx=dy$<br>I=$\int \frac{-1}{y^2}dy$ = $\int -y^2 dy$ =+$\frac{1}{y}+c$ =$\frac{1}{(1-tanx)}+c$<br>OR<br>SOL$\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}dx$ put $\cos^2 x=t$ => -2cosx sinx dx=dt<br>=> sin2x dx = - dt<br>The given integral =$\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}dx$=$\int \frac{dt}{\sqrt{3^2-t^2}}$ = - $\sin^{-1}\frac{t}{3}+c$=$\sin^{-1}\frac{\cos^2 x}{3}+c$ |
| 2 | Here, $\left(\frac{d^2y}{dx^2}+(1+x)\right)^3=-\frac{dy}{dx}$<br>clearly the highest order is 2 an its degree is three . Hence the sum of the order and degree of the above given differential equation is 2+3=5 |
| 3 | Sol-<br>Using $\vec{a}\times\vec{b}$ = $\lvert\vec{a}\rvert$ .$\lvert\vec{b}\rvert\sin\theta\hat{n}$and putting values of $\vec{a}$ and $\vec{b}$ we get $\theta$=$\frac{\pi}{4}$ |
| 4 | The line AB is given by $\frac{3-x}{-3}$ =$\frac{y+2}{-2}$ =$\frac{z+2}{6}$<br>➔ $\frac{x-3}{3}$ =$\frac{y+2}{-2}$ =$\frac{z+2}{6}$<br>Its direction ratios are 3, -2, 6<br>Hence its dc's are $\frac{3}{\sqrt{3^2+(-2)^2+6^2}}$ ,$\frac{-2}{\sqrt{3^2+(-2)^2+6^2}}$ ,$\frac{6}{\sqrt{3^2+(-2)^2+6^2}}$<br>i.e,$\frac{3}{7}$ ,$\frac{-2}{7}$, $\frac{6}{7}$<br>$\therefore$ Dc's of line parallel to AB are proportional to$\frac{3}{7}$ ,$\frac{-2}{7}$, $\frac{6}{7}$ |

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Sol-
Let X denote number of milk chocolates drawn

| X | P(X) |
|---|---|
| 0 | $\frac{4}{3}\times\frac{3}{5}=\frac{12}{30}$ |
| 1 | $\left(\frac{2}{3}\times\frac{4}{5}\right)\times 2=\frac{16}{30}$ |
| 2 | $\left(\frac{2}{6}\times\frac{1}{5}\right)=\frac{2}{30}$ |

Most likely outcome is getting one chocolate of each type.

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Solution- Here P(A)=0.4, P(B)=0.8 and $P\left(\frac{B}{A}\right)$ =0.6

$\because \quad P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$

=>

$P(B\cap A)=P\frac{B}{A}.P(A)$ =0.6X 0.4=0.24

$\because$ P(AUB)=P(A)+P(B)-$P(B\cap A)$
=0.4+0.8-0.24=1.2-0.24=0.96

## SECTION – B

7

Put $x^2=y$ to make partial fractions

Let I=$\frac{A}{y+2}+\frac{B}{y+3}=\frac{y+1}{(y+2)(y+3)}$
Solving we get A=-1 and B=2 putting these values and integrate

$\int\frac{(x^2+1)}{(x^2+2)(x^2+3)}dx=\int\frac{-1}{(x^2+2)}dx+\int\frac{2}{(x^2+3)}dx$ using formula $\int\frac{1}{(x^2+a^2)}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

$=\frac{-1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

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Comparing with $\frac{dy}{dx}$ +Py =Q we get

P=$C\frac{2xy}{1+x^2}$ and Q=$\frac{1}{(1+x^2)^2}$
then ,I.F=$1+x^2$

Now using solution;
y.(I.F) =$\int Q.(I.F)dx$+c
and solve it.

| | |
|---|---|
| | **OR**<br>Putting y=vx, we get $\frac{dy}{dx}$=v+x$\frac{dv}{dx}$<br>$\therefore$<br>$\frac{dy}{dx}=y\sin\left(\frac{y}{x}\right)-x\ e^{\frac{y}{x}}=y\sin\left(\frac{y}{x}\right)-\frac{xe^{\frac{y}{x}}}{x\sin\left(\frac{y}{x}\right)}$<br>substitute the values of y and $\frac{dy}{dx}$ in it and solve it |
| **9** | Given<br>$\vec{\alpha}=3\hat{i}-\hat{j}$ ,$\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$then we need to express $\vec{\beta}$in the form<br>$\vec{\beta}=\vec{\beta_1}+\vec{\beta_2}$ where $\vec{\beta_1}$ parallel to $\vec{\alpha}$ and $\vec{\beta_2}$is perpendicular to $\vec{\alpha}$<br>Since $\vec{\beta_1}$ parallel to $\vec{\alpha}$therefore $\vec{\beta_1}=\lambda.\vec{\alpha}$where $\lambda$is a scalar--------------(i)<br>since$\vec{\beta}=\vec{\beta_1}+\vec{\beta_2}$<br>$\therefore\ \vec{\beta_2}=\vec{\beta}-\vec{\beta_1}=\vec{\beta}-\lambda.\vec{\alpha}$ by (i)<br>putting given values of $\vec{\alpha}$and $\vec{\beta}$<br>$\vec{\beta_2}=(2\hat{i}+\hat{j}-3\hat{k})-\lambda.(3\hat{i}-\hat{j})=(2-3\lambda)\hat{i}+(1+\lambda)\hat{j}-3\hat{k}$--------(ii)<br>Also $\vec{\beta_2}$is perpendicular to$\vec{\alpha}$(given) $\therefore$ $\vec{\beta_2}.\vec{\alpha}=0$<br>=>$[(2-3\lambda)\hat{i}+(1+\lambda)\hat{j}-3\hat{k}](3\hat{i}-\hat{j})=0$<br>=>$3(2-3\lambda)-(1+\lambda)=0$<br>=>$\lambda=\frac{1}{2}$ putting these values of $\vec{\alpha}$ ,$\vec{\beta}$ and $\lambda$in (i) and (ii) we get<br>$\vec{\beta_1}=\lambda\vec{\alpha}=\frac{1}{2}(3\hat{i}-\hat{j})=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}$<br>and $\vec{\beta_2}=\left(2-\frac{3}{2}\right)\hat{i}+\left(1+\frac{1}{2}\right)\hat{j}-3\hat{k}=\left(\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}\right)$<br>Hence $\vec{\beta}=\vec{\beta_1}+\vec{\beta_2}=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}+\left(\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}\right)$ |
| **10** | Solution-<br>Comparing the given equations of two lines with standard equations<br>$\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $=\vec{a_2}+\mu\vec{b_2}$<br>we have<br>$\vec{a_1}=\hat{i}+2\hat{j}+\hat{k}$<br>$\vec{a_2}=2\hat{i}-\hat{j}-\hat{k}$<br>$\vec{b_1}=(\hat{i}-\hat{j}+\hat{k})$<br>$\vec{b_2}=(2\hat{i}+\hat{j}+2\hat{k})$ shortest distance between two lines is given by<br>SD=$\left\lvert\frac{(\vec{b_1}\times\vec{b_2})(\vec{a_2}-\vec{a_1})}{\lvert(\vec{b_1}\times\vec{b_2})\rvert}\right\rvert$-----------------(i)<br>$(\vec{a_2}-\vec{a_1})=$ $(2\hat{i}-\hat{j}-\hat{k})$ - $(\hat{i}+2\hat{j}+\hat{k})$ $=\hat{i}-3\hat{j}-2\hat{k}$ |

and $\vec{b}_1 x \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$ =(-2-1)$\hat{i}$ - (2-2)$\hat{j}$ +(1+2)$\hat{k}$ $= -3\hat{i}+3\hat{k}$

and $\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-3)^2+(3)^2} = \sqrt{18} = 3\sqrt{2}$

$\therefore$ ( $\vec{b}_1 x \vec{b}_2$ ) $(\vec{a}_2 - \vec{a}_1)$= $(3\hat{i}+3\hat{k})$ $(\hat{i}-3\hat{j}-2\hat{k})$ =-3-6=-9

substituting these values in (i) we have

SD= $\left|\frac{-9}{3\sqrt{2}}\right| = \frac{3\sqrt{2}}{2}$ units

**OR**

Vector equation of plane is

- $\vec{r}.(-8\hat{i}+3\hat{j}+21\hat{k})$=-4

## SECTION – C

**11**

Let I = $\int_0^{\frac{3}{2}} |x\cos\pi x| dx$

here

$\int_0^{\frac{3}{2}} |x\cos\pi x| dx$

x cosπx =0 means

=>

x=0 or cosπx =0

=>x=o or cosπx=cos$\frac{\pi}{2}$ =cos$\frac{3\pi}{2}$

=>x=o or $\pi x = \frac{\pi}{2}$ or $\pi x = \frac{3\pi}{2}$

=>x=o or $x = \frac{1}{2}$ or $\pi x = \frac{3}{2}$

0-----1/2 --------3/2---

0<x<$\frac{1}{2}$ ; x>0 and cos $\pi x$>0 , then

$\therefore x\cos\pi x < 0$

$$|x\cos\pi x| = \begin{cases} x\cos\pi x, for\ 0 < x \le \frac{1}{2} \\ -x\cos\pi x, for\ \frac{1}{2} < x \le \frac{3}{2} \end{cases}$$

Now $\int_0^{\frac{3}{2}} |x\cos\pi x| dx = \int_0^{\frac{1}{2}} x\cos\pi x\, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} (-x\cos\pi x) dx$

$\Big( \qquad \Big)^{1/2} \qquad \Big( \qquad \Big)^{3/2}$

$$= x\frac{\sin\pi x}{\pi} - \int x\sin\frac{\pi x}{x}dx \qquad - \; x\frac{\sin\pi x}{\pi} - \int x\sin\frac{\pi x}{x}dx$$ after solving we get

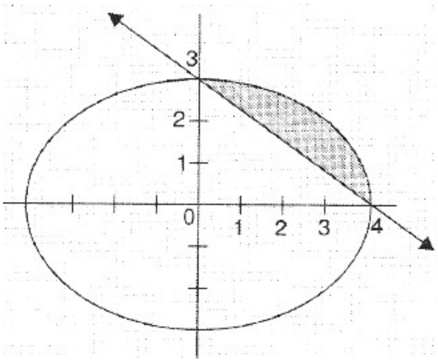
0 1/2

$$\int_0^{\frac{3}{2}} |x\cos\pi x|\,dx = \frac{5}{\pi} - \frac{1}{\pi^2} \text{ Ans}$$

**12**

Getting points of intersection (4,0) (0,3)

Required area $= \int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx - \frac{1}{4}\int_0^4 (12-3x)\,dx$



$$\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx - \frac{1}{4}\int_0^4 (12-3x)\,dx$$

$$\left.\frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+8\sin^{-1}\frac{x}{4}\right] - \frac{1}{4}\left(12x-\frac{3x^2}{2}\right)\right]_0^4$$

=

$$\frac{3}{4}\left(8\frac{\pi}{2}-6\right) = (3\pi-6)\,sq\,units$$

OR

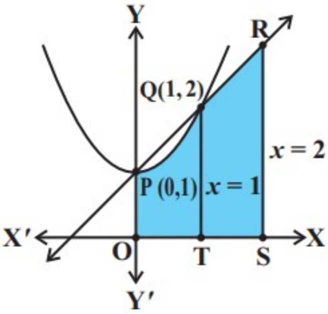
This region is the intersection of the following regions.

$A_1 = \{(x, y) : 0 \le y \le x^2 + 1\}$,

$A_2 = \{(x, y) : 0 \le y \le x + 1\}$ and

$A_3 = \{(x, y) : 0 \le x \le 2\}$

The points of intersection of $y = x^2 + 1$ and $y = x + 1$ are points P(0, 1) and Q(1, 2). From the Fig the required region is the shaded region O P Q R S T O whose area = area of the region O T Q P O + area of the region T S R Q T $= \int_0^1 (x^2+1)\,dx + \int_1^2 (X+1)\,dx$



$$\int_0^1 (x^2+1)\,dx + \int_1^2 (X+1)\,dx$$

$$= \left[\left(\frac{x^3}{3}+x\right)\right]_0^1 + \left[\left(\frac{x^2}{2}+x\right)\right]_1^2$$

$$= \left[\left(\frac{1}{3}+1\right)-0\right] + \left[(2+2)-\left(\frac{1}{2}+1\right)\right] = \frac{23}{6}$$

**13**

let the equation of the plane passing through (3 ,2 ,0) is

=>a(x-3) +b (y-2)+c(z-0)=0

Since the plane contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$

so the point (3,6,4)satisfy the equation of the plane

∴ a(3-3) +b(6-2)+c(4-0)=0........(i)

=>0+4b+4c=0

=>b+c=0-=>b=-c------------(ii)

As the plane contains the line therefore normal to the plane is perpendicular to line ie.,

$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ where $a_1 = a$ ,$b_1 = b$ ,$c_1 = c$ and $a_2 = 1$ , $b_2 = 5$ $c_2 = 4$

| | |
|---|---|
| | ∴ a+5b+4c=0-----(iii)<br>on solving (ii) and (iii)we get a=c<br>on putting values of a,b ,c in (i)<br>x-y+z=0 is the required equation |
| **14** | Let E be the event =Group A completes the task<br>F be the event =Group B completes the task<br>G be the event =Group C completes the task<br>H be the event =Group D completes the task<br>$P(E)=\frac{1}{3}=>P(\bar{E})=\frac{2}{3}$<br>$P(F)=\frac{1}{4}=>P(\bar{F})=\frac{3}{4}$<br>$P(G)=\frac{1}{5}=>P(\bar{G})=\frac{4}{5}$<br>$P(H)=\frac{2}{3}=>P(\bar{H})=\frac{1}{3}$<br>(i)The required probability=P(EUFUGUH)= 1-$P(\bar{E})XP(\bar{F})XP(\bar{G})XP(\bar{H})$<br>$=1-\frac{2}{3}X\frac{3}{4}X\frac{4}{5}X\frac{4}{5}=\frac{13}{15}$<br><br>(ii)The required probability= $P(\bar{E})xP(\bar{F})XP(\bar{G})XP(\bar{H})+P(E)XP(\bar{F})XP(\bar{G})XP(\bar{H})+P(\bar{E})XP(F)X$<br>$P(\bar{G})XP(\bar{H})+P(\bar{E})XP(\bar{F})XP(G)XP(\bar{H})+P(\bar{H})XP(\bar{F})XP(\bar{G})XP(H)$<br>$=\frac{2}{3}X\frac{3}{4}X\frac{4}{5}X\frac{1}{3}+\frac{1}{3}X\frac{3}{4}X\frac{4}{5}X\frac{1}{3}+\frac{2}{3}X\frac{1}{4}XXX\frac{1}{3}+\frac{2}{3}X\frac{3}{4}X\frac{1}{5}X\frac{1}{3}+\frac{2}{3}X\frac{3}{4}X\frac{4}{5}X\frac{2}{3}$<br>$=\frac{2}{15}+\frac{1}{15}+\frac{2}{45}+\frac{1}{30}+\frac{4}{15}$<br>$=\frac{7}{15}+\frac{1}{30}+\frac{2}{45}=\frac{42+3+4}{90}=\frac{49}{90}$ |