COMPREHENSIVE ASSESSMENT FEEDBACK
PRE-BOARDEXAMINATION (2024-25)
CLASS: X
SUBJECT: MATHEMATICS (STANDARD)(041)

SECTION-A

1. **One card is selected at random from a well shuffled deck of 52 cards. The probability that the selected card is a red even number card is:**

   (a) $\frac{5}{13}$ **(b)** $\frac{5}{32}$

   (c) $\frac{5}{26}$ **(d)** $\frac{3}{13}$

Sol. Red even number cards are: Heart - 2,4,6,8,10 Diamond - 2,4,6,8,10

Total no. of Red even number of cards are = 5+5=10

Total no. of cards are = 52

P (red even cards) = $\frac{10}{52} = \frac{5}{26}$

(a) In the options (a) and (b), the numerator is correct but denominator are wrong, so both of these options are incorrect.

(b) In the option (c) we have 26 in the denominator i.e half of 52 and 5 in the numerator i.e half of 10 so it is correct option.

(c) In the option (d), both numerator and denominator are wrong, so it is incorrect option.

Suggestive Measures: While attempting the question students should

- **Be aware of the composition of playing cards**
- **know about odd and even numbers**
- **Perform accurate calculations using the correct formula**

2. **The coordinates of a point which lies on the perpendicular bisector of line segment joining the points A(-4,-5) and B(4,5) is:**

   (a) (-4, 0) **(b) (5, -5)**

   (c) (-4, -4) **(d) (0, 0)**

Sol. By using midpoint formula the coordinates of midpoint of AB is :($\frac{-4+4}{2}$,$\frac{-5+5}{2}$) = (0,0)

The option (a) has correct y coordinate but incorrect x coordinate, so this is wrong option.
Both the coordinates in option (b) and (c) are incorrect, so these two are wrong options.
The option (d) is (0, 0) which has 0 as x and y coordinate so it is the correct option.

Suggestive Measures: While solving such type of questions students should

- **Perpendicular bisector of a line segment is its mid-point**
- **apply mid point formula**

3. **If α and β are the zeroes of the polynomial $5x^2$-7x+2 then the sum of their reciprocal is:**

   (a) $\frac{7}{2}$ **(b)** $\frac{7}{5}$ **(c)** $\frac{-7}{5}$ **(d)** $\frac{2}{5}$

Sol. Sum of reciprocal of α and β is = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

Here, $\alpha + \beta = -\frac{b}{a} = \frac{7}{5}$ and $\alpha\beta = \frac{c}{a} = \frac{2}{5}$

so sum of reciprocal of α and β is = $\frac{7}{5} \times \frac{5}{2} = \frac{7}{2}$

(a) $\frac{7}{2}$ is the correct option.

(b) $\frac{7}{5}$ is the sum of the zeroes but we are asked about sum of reciprocal of zeroes, so (b) $\frac{7}{5}$ is the incorrect option.

(c) $-\frac{7}{5}$ is the negative of sum of the zeroes but we are asked about sum of reciprocal of zeroes, so (c)$-\frac{7}{5}$ is the incorrect option.

(d) $\frac{2}{5}$ is the product of the zeroes but we are asked about sum of reciprocal of zeroes, so (d) $\frac{2}{5}$ is the incorrect option.

Suggestive Measures: While solving such type of questions students should

- **Know the relation between coefficients of x^2, x and constant term**
- **Know formula for the sum and product of zeroes of a polynomial**

4. The value of x for which 2x, x+10 and 3x+2 are in A.P. is:

(a) 5 (b) 2 (c) 6 (d) 3

Sol. Since 2x, x+10 and 3x+2 are in A.P

$2(x+10) = 3x + 2 + 2x \Rightarrow 3x = 18 \Rightarrow x=6$

(a) If x = 5 then terms will be 10,15,17 in which the common difference is not same so they are not in A.P , so the option (a) is incorrect.

(b) If x = 2 then terms will be 4,12,8 in which the common difference is not same so they are not in A.P and the option (b) is incorrect.

(c) If x = 6 then terms will be 12, 16, 20 in which the common difference is same so they are in A.P and the option (c) is correct.

(d) If x = 3 then terms will be 6,13,11 in which the common difference is not same so they are not in A.P and the option (a) is incorrect.

Suggestive Measures: While solving such type of questions students should

- **Correctly identify first term, common difference, number of terms of given series**
- **Be aware of the fact that twice the middle term is equal to sum of first and third term**

5. **The perimeter of a circle is equal to that of a square then the ratio of their areas is:**

(a) 7:22 (b) 22:7 (c)11:14 (d)14:11

Sol. Perimeter of circle = perimeter of square $\Rightarrow 2\pi r = 4a$

$\Rightarrow \left(\frac{\pi r}{a}\right)^2 = 2^2$

$\Rightarrow \pi.\left(\frac{\pi r^2}{a^2}\right) = 4$

$\Rightarrow \left(\frac{\pi r^2}{a^2}\right) = \frac{4}{\pi}$ = 14:11 = ratio of their areas

(a) 7:22 is the reciprocal of the value of π, so option (a) is incorrect.

(b) 22:7 is the value of π, so option (b) is incorrect.

(c) 11:14 is the reciprocal of correct value, so option (c) is incorrect.

(d) 14:11 is the only correct option as the ratio of their areas is 4: π .

Suggestive Measures: While solving such type of questions students should

- **Know the perimeter and area of circle and square**
- **understand meaning of ratio and how to use it.**

6. If H.C.F of 90 and 144 is expressed in the form of 17m-16 then the value of m is:

(a) 18 **(b)** 2 **(c)** 17 **(d)** 34

Sol. LCM of 90 and 144 is 720.

We know that LCM (a, b) × HCF (a, b) = a×b

(17m-16) × 720 = 90 ×144 ⇒ (17m-16) =18

17m = 34 ⇒ m = 2

(a) The option (a) is 18 which is not possible if 17m = 34, so it is incorrect option

(b)The option (b) is 2 which is only correct option if 17m =34

(c) The option (c) is 17 which is not possible if 17m =34, so it is incorrect option

(d) The option (d) is 34 which is not possible if 17m =34, so it is incorrect option

Suggestive Measures: While solving such type of questions students should

- **know that LCM (a, b) × HCF(a, b)) = a×b**
- **know how to solve linear equation in one**

7. The polynomial having zeroes $\frac{1}{3}$ and ($\frac{-3}{4}$) is:

(a) $k(12x^2+5x-3)$ **(b)** $k(12x^2-5x-3)$

(c) $k(12x^2+13x+3)$ **(d)** $k(12x^2-13x-3)$

Sol. Sum of zeroes = $\frac{1}{3} + (\frac{-3}{4}) = \frac{-5}{12}$, product of zeroes = $\frac{1}{3} \times \frac{-3}{4} = \frac{-1}{4}$

Since any polynomial is represented as $k[x^2$- (sum of zeroes) x+ product of zeroes]

(a) The option (a) is $k(12x^2+5x-3)$in which the sum of zeroes is $\frac{-5}{12}$and the product of zeroes is $\frac{-1}{4}$ which is correct for the given polynomial.

(b) The option (b) is $k(12x^2-5x-3)$ which could be possible only in the case where the sum of zeroes is $\frac{5}{12}$, so it is incorrect option.

(c) The option (c) is $k(12x^2+13x+3)$ which could be possible only in the case where the sum of zeroes is $\frac{-13}{12}$, so it is incorrect option.

(d) The option (d) is $k(12x^2-13x-3)$ which could be possible only in the case where the sum of zeroes is $\frac{13}{12}$, so it is incorrect option.

Suggestive Measures: While attempting the question students should

- **Know the sum of zeroes and product of zeroes**
- **Have the understanding of standard form of polynomial**

8. If the difference of mode and median of a data is 36 then the difference of median and mean is:

(a) 12 **(b)** 18 **(c)** 24 **(d)** 36

Sol. Mode –Median = 36

We know that mode = 3 median – 2 mean

⇨ median + 36 = 3 median – 2 mean

⇨ 2(median – mean) = 36

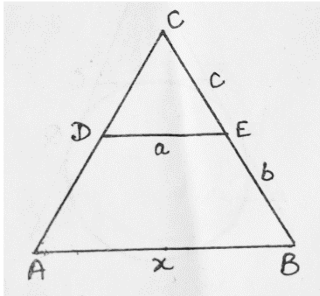
⇨ median – mean = 18

Hence (b**)** 18 is the only correct option and all other options (a),(c) and (d) are incorrect.

Suggestive Measures: While attempting the question students should

- **Have the understanding of mean, mode and median**
- **Know the Relation between mean, mode and median**

9. In the given figure, x expressed in terms of a, b & c, when DE||AB is:



(a) $\frac{ab}{a+c}$ **(b)** $\frac{ac}{b+c}$

(c) $\frac{a(b+c)}{c}$ **(d)** $\frac{ab}{c}$

Sol. DE||AB ⇒ ΔDEC ∽ ΔABC

⇒ sides of these triangles are proportional

$\Rightarrow \frac{DE}{AB} = \frac{CE}{BC}$

$\Rightarrow \frac{a}{x} = \frac{c}{b+c} \Rightarrow x = \frac{a(b+c)}{c}$

Hence (c) $\frac{a(b+c)}{c}$ is the only correct option and all other options (a),(b) and (d) are incorrect.

Suggestive Measures: While attempting the question students should
- **be aware of the properties of parallel lines**
- **apply the concept of similarity in triangles**
- **perform accurate calculations using the correct values**

10. A linear equation in two variables is of the form ax + by +c, where:

(a) a=0, b≠0 **(b) a≠0,b=0**

(c) a≠0,c=0 **(d) b=0,c≠0**

Sol. ax + by +c is a linear equation in two variables only if it has both a and b non-zero so **(c)a≠0,c=0** is the only correct answer as in option (a) a=0 (having one variable y left) and in options (b) and (d) b=0 (having one variable x left)

Suggestive Measures: While attempting the question students should
- **have the understanding of constants and variables**
- **understand linear equations in one and two variables.**

11. If the probability of the letter chosen at random from the letters of the word 'Mathematics' to be a vowel is $\frac{2}{2x+1}$ then x is equal to :

(a) $\frac{4}{11}$ **(b)** $\frac{9}{4}$ **(c)** $\frac{11}{4}$ **(d)** $\frac{4}{9}$

Sol: Number of vowels in word mathematics = 4

P (letter chosen to be a vowel) $= \frac{4}{11}$

$\frac{2}{2x+1} = \frac{4}{11} \Rightarrow x = \frac{9}{4}$

Suggestive Measures–While attempting the question students should:
- **be aware about the vowels**
- **perform accurate calculations using the correct formula**

12. The 20th term from the last term of an A.P. 253, 248, 243, 8, 3 is:

(a) 92 **(b)**158 **(c)** 98 **(d)**101

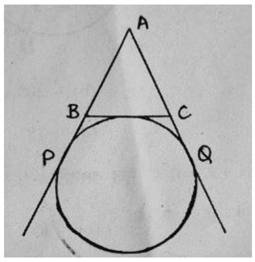
Sol. Using a =3, d = 8 – 3 =5 i.e finding 20th term from the last term of A.P

a_{20} = a+19d = 3+19×5 = 98

so a_{20} cannot be **(a)**92 ,**(b)**158 and **(d)**101, so only correct option is **(c)**98

Suggestive Measures: While solving such type of questions students should
- **know the nth term formula of A.P**
- **be aware of nth term of A.P from last term**

13. In the given figure AP, AQ and BC are tangents to the circle.
If AB=5 cm, AC=6 cm and BC=4 cm, then the length of AP (in cm) is :



(a) 4.5 **(b)** 7.5
(c) 5 **(d)** 5.5

Sol. Let CQ = x so BP = 4-x as BC = 4 and the length of tangents drawn from an external point to a circle are equal.

$6 + 4 - x = 5 + x$

$10 - 5 = 2x \Rightarrow x = 2.5$

AP = AB + BP = 5 + 2.5 = 7.5 so **(b)** 7.5 is the only correct option and all other options (a) 4.5 ,(c) 5 and (d)5.5 are incorrect.

Suggestive Measures: While solving such type of questions students should
- **Know that tangents drawn from an external point to a circle are equal.**
- **Apply the correct concept and do proper calculations**

14. The roots of quadratic equation $x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ are

(a) Irrational and distinct **(b)** Not Real
(c) Rational and Distinct **(d)** Real and Equal

Sol. D = b^2-4ac = $(-3)^2$ -$4\sqrt{2} \times \sqrt{2}$= 9-8 =1 which is greater than 0 so roots are real and distinct Hence options (b), (d) are not possible and on finding roots found irrational so only correct option is **(a)**Irrational and distinct.

Suggestive Measures: While solving such type of questions students should
- **Know discrimination formula**
- **Do proper calculation using correct formula**

15. The value of m and n for which the pair of linear-equations have infinitely many solutions is:

$$mx + 3y - 6 = 0$$
$$ny - 12x + 12 = 0$$

(a) m = -1, n = 2 **(b) m = -1, n = 3**
(c) m =6, n=-8 **(d) m=6, n=-6**

Sol. For infinitely many solutions, the condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{m}{-12} = \frac{3}{n} = \frac{-6}{12}$ $\Rightarrow$ n = -6 and m=6

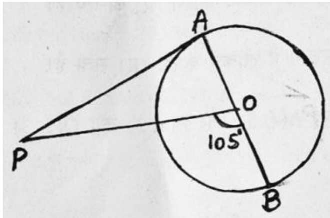
Only option (d) m = 6, n = - 6 where $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{-1}{2}$ is correct.

In other options (a), (b) and (c) the condition of $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is not fulfilled, so all other options are wrong.

Suggestive Measures: While solving such type of questions students should
- **be aware of the condition for infinitely many solutions**
- **do proper calculation**

16. In the given figure, PA is tangent from an external point P to a circle with centre O. If ∠POB =105°, then measure of ∠APO is:

(a) 35° **(b)** 20° **(c)**15° **(d)** 75°



Sol. ∠APO +∠PAO = ∠POB (exterior angle property)

∠APO = 105° − 90° = 15° (OA is perpendicular to AP)

So only option (c)15° is correct and (a) 35° , (b) 20° and (d) 75° are incorrect.

Suggestive Measures: While solving such type of questions students should
- **know about exterior angle property and relation between tangent and radius of a circle**
- **do proper calculations**

17. If the quadratic equation x^2 - 8x + K= 0 has real roots then:

(a) K<16 **(b)** K≤16 **(c)** K>16 **(d)** K≥16

Sol. The roots of quadratic equations are real when D = b^2-4ac ≥ 0

⇒ $(-8)^2-4\times1\times k \geq 0$

⇒ 64 − 4k ≥0 ⇒ K≤16

So only correct option is (b) K≤16 and other options (a) K<16 ,(c) K>16 and (d) K≥16 are wrong.

Suggestive Measures: While solving such type of questions students should
- **Know discrimination formula**
- **Do proper calculation using correct formula**

18. The point of intersection of the line represented by 3x-y =3 and y-axis is :

(a) (0,-3) **(b) (0,3)** **(c) (2,0)** **(d) (-2,0)**

Sol: For y-axis intersection putting x=0 in 3x-y =3 we get y = -3 so intersection is (0,-3)

Hence option (a) is only correct answer

(a) The option (b) (0,3) is incorrect as it has 3 as y coordinate instead off -3.

(b) In options (c) and (d) y coordinate is zero which can't be correct as given line intersects at y-axis only.

Suggestive Measures: While solving such type of questions students should
- **know x-coordinate and y-coordinate**
- **be aware that intersection with y-axis only means x-coordinate is 0**

Directions for Q 19 & 20:

There is one Assertion (A) and one Reason (R). Choose the correct answer of these questions from the four options (a), (b), (c) and (đ) given below:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false

(d) Assertion (A) is false but Reason (R) is true

19. **Assertion (A): If the points A (4, 3) and B (x, 3) lies on a circle with the centre O(2, 3) then the value of x is 2.**

Reason (R): The midpoint of the line segment joining the points P(x_1, x_2,) and Q(y_1, y_2) is $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$.

Sol: The midpoint of A(4, 3) and B(x, 3) using midpoint formula

$$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} = \left(\frac{4+x}{2}, \frac{3+3}{2}\right)$$

since Centre O has coordinates (2, 3) so $2 = \frac{4+x}{2} \Rightarrow x = 0$

but in Assertion (A) the value of x is 2 which is incorrect but Reason (R) is true so **option (d) Assertion (A) is false but Reason (R) is true** is correct.

Suggestive Measures: While solving such type of questions students should
- **Identify correct values of x_1, x_2, y_1, y_2 etc.**
- **Do proper calculation using correct formula**

20. Assertion (A): The probability of getting a bad pen in a lot of 200 is 0.035. The number of good pens in a lot is 193.

Reason (R): If the probability of an event is p, the probability of complementary event will be 1+p.

Sol. Total number of pens = 200

The number of good pens in a lot = 193

The number of bad pen pens in a lot = 200 – 193 = 7

The probability of getting a bad pen in a lot of 200 = $\frac{7}{200}$ = 0.035

If the probability of an event is p, the probability of complementary event will be 1 – p.

Hence Assertion (A) is true but Reason (R) is false so (c) is the only correct option

Suggestive Measures–While attempting the question students should:
- **know the results of probability**
- **know about the complement of probability**
- **perform accurate calculations using the correct values**

SECTION-B

21. Find the middle term of A. P. 6, 13, 20....... 230.

Sol: Here a = 6, d = 7 and a_n = 230

$a_n = a + (n-1)d$

$\Rightarrow \quad 216 = 6 + (n-1)7 \quad \Rightarrow \quad n = 31$

So, total no. of terms = 31

Middle term = $\frac{31+1}{2}$ = 16^{th} term $\quad \Rightarrow \quad a_{16} = a+(16-1)\times 7$

$= 6+(15\times 7) = 111$

Therefore, the middle term of the given AP is 111.

OR

Which term of an A. P. 3, 14, 25, 36..… will be 99 more than its 25^{th} term?

Sol: Here, a = 3, d= 11 and let a_n be the required term.

According to question, $a_n = 99 + a_{25}$

$\Rightarrow a + (n – 1)d = 99 + a + 24d$

$\Rightarrow (n – 1)11 = 99 + (24\times 11)$

$\Rightarrow n = 34$

Therefore, 34^{th} term is 99 more than the 25^{th} term.

Suggestive Measures – While solving such type of questions students should:
- **know how to find out the common difference of an AP**
- **be aware of the fact that the required term should be taken as nth term**
- **know how to transform the given condition into a mathematical equation**
- **know how to calculate the middle term when number of terms is even or odd**

22. 7560 = $2^3 \times 3^p \times q \times 7$, then find the value of p + q.

Sol: Given 7560 = $2^3 \times 3^p \times q \times 7$

Also 7560 = $2^3 \times 3^3 \times 5 \times 7$

Therefore, $2^3 \times 3^3 \times 5 \times 7 = 2^3 \times 3^p \times q \times 7$

It gives us p = 3 and q = 5

Therefore, p + q = 3 +5 = 8

Suggestive Measures – While solving such type of questions students should:
- **know about the prime factorization of a composite number**
- **know how to compare the exponents when the bases are equal**

23. Find the mean of the following frequency data:

Class-interval	0-20	20-40	40-60	60-80	80-100
Frequency	7	11	10	9	13

Sol:

Class-interval	0-20	20-40	40-60	60-80	80-100	
x_i	10	30	50	70	90	**Total**
Frequency f_i	7	11	10	9	13	**50**
$f_i x_i$	70	330	500	630	1170	**2700**

Now, Mean $= \frac{\sum f_i x_i}{\sum f_i}$ (Direct Method)

$= \frac{2700}{50} = 54$

Suggestive Measures – While solving such type of questions students should:
- **know how to calculate the class mark of each given class-interval**
- **know how to decide which method to be used to calculate the mean as per the given data**
- **know the formula of calculating mean as per the method applied**

24. If x = p secθ + q tanθ and y = p tanθ + q secθ, then prove that $x^2 – y^2 = p^2 – q^2$.

Sol: x + y = (psecθ + qtanθ) + (ptanθ + qsecθ)

= p(secθ + tanθ) + q(secθ + tanθ) = (p + q) (secθ + tanθ) …(i)

x – y = (psecθ + qtanθ) – (ptanθ + qsecθ)

= p(secθ–tanθ) – q(secθ–tanθ) = (p – q) (secθ – tanθ) … (ii)

$x^2 – y^2$ = (x + y)(x – y)

= (p + q) (secθ + tanθ) (p – q) (secθ–tanθ) [using (i) and (ii)]

= $(p^2 – q^2)(\sec^2\theta – \tan^2\theta)$

= $(p^2 – q^2)$ [∵ $\sec^2\theta – \tan^2\theta = 1$]

$x^2 – y^2 = p^2 – q^2$ Hence Proved

OR

Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=4$, where $\theta \le 90^o$.

Sol: Given $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=4$

$$\Rightarrow \quad \frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=4$$

$$\Rightarrow \quad \frac{\cos\theta+\cos\theta\sin\theta+\cos\theta-\cos\theta\sin\theta}{(1-\sin^2\theta)}=4$$

$$\Rightarrow \quad \frac{2\cos\theta}{\cos^2\theta}=4$$

$$\Rightarrow \quad \frac{2}{\cos\theta}=4 \quad \Rightarrow \quad \cos\theta=\frac{1}{2}$$

$$\therefore \boldsymbol{\theta = 60^o}$$

Suggestive Measures – While solving such type of questions students should:
- **know the basic trigonometric ratios and trigonometric identities**
- **know relationship between various t-ratios**
- **know the value of trigonometric ratios for the various standard angles (0^o, 30^o, 45^o, 60^o and 90^o)**

25. One card is drawn at random from a bag of cards numbered from 3 to 49. Find the probability that the card drawn has: (a) a prime number (b) a multiple of 6.

Sol: Total number of cards = (49 – 3) + 1 = 47

(i) Prime number = 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

P (Prime number) = $\frac{14}{47}$

(ii) Multiple of 6 = 6, 12, 18, 24, 30, 36, 42, 48

P (Multiple of 6) = $\frac{8}{47}$

Suggestive Measures – While solving such type of questions students should:
- **know how to find out the total number of cards from the given information**
- **be aware about the concepts of prime numbers and multiples**
- **be aware of the fact that probability of an event cannot be greater than 1**

<u>SECTION-C</u>

26. Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p and q are primes.

Sol: Let us assume $\sqrt{p}+\sqrt{q}$ is rational. Then,

$\sqrt{p}+\sqrt{q} = a$ (where a is a non-zero rational number)

$\sqrt{p} = a-\sqrt{q}$

Squaring both sides, we get $(\sqrt{p})^2 = (a-\sqrt{q})^2$

$\Rightarrow \quad p = a^2+q-2a\sqrt{q}$

$\Rightarrow \quad \sqrt{q} = \frac{a^2+q-p}{2a}$

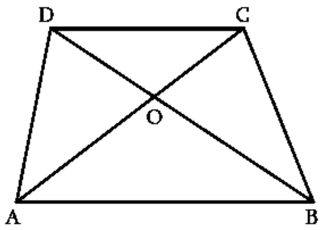
This contradicts our assumption, as the right-hand side is a rational number whereas left-hand side is an irrational number. Hence, $\sqrt{p}+\sqrt{q}$ is irrational.

Suggestive Measures – While solving such type of questions students should:

- **know about the concepts of rational and irrational numbers**
- **be aware of the fact that sum, difference, product or quotient of two non-zero rational numbers is a rational number**
- **know that if p is a prime number then $\sqrt{p}$ is an irrational number**
- **be aware about the technique of proof by the method of contradiction**

27. ABCD is a trapezium in which AB ∥ CD. Show that $\frac{OA}{OB} = \frac{OC}{OD}$.

Sol: In ΔAOB and ΔCOD



∠AOB = ∠COD (vertically opposite angles)

∠BAO = ∠DCO (alternate interior angles)

⇒ ΔAOB ~ ΔCOD (AA criterion)

Hence $\frac{OA}{OB} = \frac{OC}{OD}$

OR

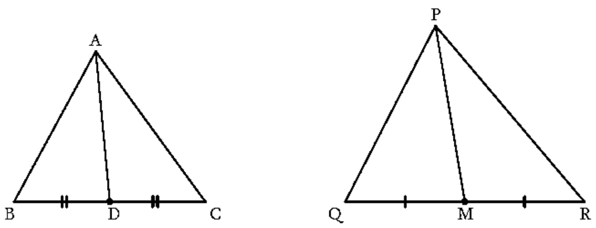
The sides AB, BC and median AD of a triangle ABC are respectively proportional to sides PQ, QR, and median PM of another triangle PQR. Show that ΔABC ~ΔPQR.

Sol: In ΔABC and ΔPQR,

AB/PQ = BC/QR = AD/PM [given]

AD and PM are medians of ΔABC and ΔPQR respectively

⇒ BD/QM = (BC/2)/(QR/2) = BC/QR



Now, in ΔABD and ΔPQM

AB/PQ = BD/QM = AD/PM

⇒ ΔABD ~ ΔPQM [SSS]

Now, In ΔABC and ΔPQR

AB/PQ = BC/QR [given in the statement]

∠ABC = ∠PQR [∵ ΔABD ~ ΔPQM]

⇒ ΔABC ~ ΔPQR [SAS criterion]

Suggestive Measures – While solving such type of questions students should:

- **Know about the concepts like trapezium and medians.**
- **Know about the various similarity criteria and when to use them as per the requirement of the question.**

28. Solve for x:

$$x^2 - (2b - 1)x + (b^2 - b - 20) = 0$$

Sol: The given quadratic equation is in the standard form. Therefore, using quadratic formula,

we get $x = \frac{-[-(2b-1) \pm \sqrt{(2b-1)^2 - 4(b^2-b-20)}}{2 \times 1}$

$$= \frac{(2b-1) \pm \sqrt{(4b^2+1-4b-4b^2+4b+80)}}{2}$$

$$= \frac{(2b-1) \pm 9}{2}$$

$$\Rightarrow x = b + 4, b - 5$$

Suggestive Measures – While solving such type of questions students should:

- **Know how to write a quadratic equation in the standard form and how to find the value of a, b and c**
- **Know the quadratic formula correctly.**
- **Know that there are two roots of a quadratic equation.**

29. If point A (-2, 1), B (a, 0), C (4, 1) and D (1, 2) are the vertices of a parallelogram ABCD, then find the value of a. If P divides the side AB in ratio 3:2 and Q divides the side CD in ratio 4 : 1, then find the co-ordinates of P and Q.

Sol: We know that the diagonals of a parallelogram bisect each other.

Therefore, coordinates of mid-point of AC = coordinates of mid-point of BD

$$\Rightarrow \quad \left(\frac{-2+4}{2}, \frac{1+1}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$$

$$\Rightarrow \quad \frac{-2+4}{2} = \frac{a+1}{2}$$

$$\Rightarrow \quad a = 1$$

Further, it is given that P divides AB in the ratio 3 : 2, therefore, using section formula

$$P\left(\frac{3-4}{5}, \frac{0+2}{5}\right) \quad \Rightarrow \quad P\left(\frac{-1}{5}, \frac{2}{5}\right)$$

Also, it is given that Q divides AB in the ratio 4 :1, therefore, using section formula

$$Q\left(\frac{4+4}{5}, \frac{8+1}{5}\right) \quad \Rightarrow \quad Q\left(\frac{8}{5}, \frac{9}{5}\right)$$

Suggestive Measures – While solving such type of questions students should:

- **know the mid-term and section formula of internal division**
- **be aware of the fact that the diagonals of a parallelogram bisect each other at a point**
- **know how to equate the coordinates of two points**

30. Find the zeroes of quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Sol: The given polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ is in the standard form of $ay^2 + by + c$.

Here $a = 7$, $b = -\frac{11}{3}$, $c = -\frac{2}{3}$

Let $\quad 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$$\Rightarrow \frac{1}{3}(21y^2 - 11y - 2) = 0$$

$$\Rightarrow 21y^2 - 14y + 3y - 2 = 0$$

$$\Rightarrow 7y(3y - 2) + 1(3y - 2) = 0$$

$$\Rightarrow (3y - 2)(7y + 1) = 0$$

$$\Rightarrow (3y - 2) = 0 \text{ and } (7y + 1) = 0$$

$$\Rightarrow y = \frac{2}{3} \quad \text{and} \quad y = -\frac{1}{7}$$

$\Rightarrow \quad \alpha = \frac{2}{3}$ and $\beta = -\frac{1}{7}$

Now, sum of zeroes $\alpha+\beta = \frac{2}{3} + (-\frac{1}{7}) = \frac{11}{21} = -\frac{b}{a}$

product of zeroes $\alpha\beta = \frac{2}{3} \times (-\frac{1}{7}) = -\frac{2}{21}$

Also $\frac{c}{a} = \frac{\left(-\frac{2}{3}\right)}{7} = -\frac{2}{21}$

$\therefore \quad \alpha\beta = \frac{c}{a}$

OR

If α and β are the zeroes of the polynomial $3x^2 - 13x - 10$, then find the value of $(3\alpha + 1)(3\beta + 1)$.

Sol: The given polynomial $3x^2 - 13x - 10$ is in the standard form $ax^2 + bx + c$.

Here, a = 3, b = −13 and c = −10

sum of zeroes $\alpha+\beta = -\frac{b}{a} = -\frac{(-13)}{3} = \frac{13}{3}$

product of zeroes $\alpha\beta = \frac{c}{a} = -\frac{10}{3}$

Now $(3\alpha+ 1)(3\beta+ 1) = 9\alpha\beta + 3(\alpha+\beta) + 1$

$= -30 + 13 + 1 = -16$

Suggestive Measures – While solving such type of questions students should:

- **know how to write a given polynomial in standard form**
- **know how to find the values of coefficients of terms of polynomial written in the standard form**
- **know the method of factorization of given polynomial to find two zeroes**
- **know the relationship between zeroes and coefficients of a polynomial**

31. Compute the median for the following data:

Class Interval	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90	Less than 100
No. of Days	0	4	16	30	46	66	82	92	100

Sol: Converting the given less than frequency distribution table into grouped data

Class-Interval	***f_i***	***cf***
10-20	0	0
20-30	4	4
30-40	12	16
40-50	14	30
50-60	16	46
60-70	20	66
70-80	16	82
80-90	10	92
90-100	8	100
Total	**100**	

Here, N = 100

Therefore $\frac{N}{2} = 50$

So, median class is 60 – 70 and $l = 60$, $f = 20$, $cf = 46$ and $h = 10$

$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$

$$= 60 + \left(\frac{50-46}{20}\right) \times 10 = 60 + 2 = 62$$

Therefore, Median = 62 days

Suggestive Measures – While solving such type of questions students should:
- **know how to convert less than frequency table into standard frequency distribution table.**
- **be aware how to calculate the cumulative frequencies**
- **be well verse with the idea of finding median class, class-size (*h*), *f* and *cf* from the frequency distribution table**
- **know the correct formula of median of grouped data**
- **write the proper unit with the final answer**

SECTION-D

32. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then show that $\cot^2\theta + \tan^2\theta = \frac{10}{3}$

Sol: $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

$\Rightarrow \sqrt{3}\cot^2\theta - 3\cot\theta - 1\cot\theta + \sqrt{3} = 0$

$\Rightarrow \sqrt{3}\cot\theta(\cot\theta - \sqrt{3}) - 1(\cot\theta - \sqrt{3}) = 0$

$\Rightarrow (\cot\theta - \sqrt{3})(\sqrt{3}\cot\theta - 1) = 0$

$\Rightarrow \cot\theta - \sqrt{3} = 0$ or $\sqrt{3}\cot\theta - 1 = 0$

$\Rightarrow \cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30^o$ or $\theta = 60^o$

Case I: When $\theta = 30^o$

$$\cot^2\theta + \tan^2\theta = \cot^2(30^o) + \tan^2(30^o) = \left(\sqrt{3}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 = 3 + \frac{1}{3} = \frac{10}{3}$$

Case II: When $\theta = 60^o$

$$\cot^2\theta + \tan^2\theta = \cot^2(60^o) + \tan^2(60^o) = \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 3 = \frac{10}{3}$$

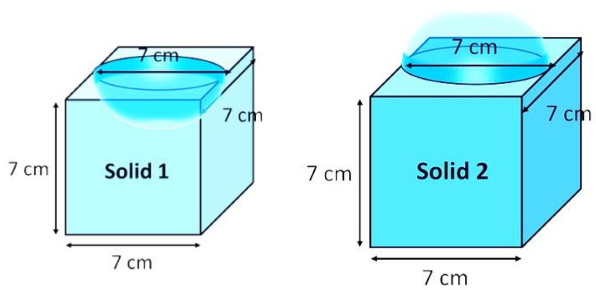
Hence Proved

Suggestive Measures – While solving such type of questions students should:
- **know the basic trigonometric ratios and trigonometric identities**
- **know relationship between various t-ratios**
- **know the value of trigonometric ratios for the various standard angles (0^o, 30^o, 45^o, 60^o and 90^o)**
- **know how to factorize a given quadratic equation to find its roots.**

33. There are two identical solid cubical boxes of side 7 cm. From the top face of first cube, a hemisphere of diameter equal to the side of cube is scooped out. This hemisphere is inverted and placed on the top of second cube's surface to form a dome. Find:

(i) The ratio of the total surface area of two new solids formed.

(ii) Volume of each new solid formed.



(Image Courtesy: Teachoo.com)

Sol: (i) Side of Cube = a = 7 cm

Radius of Hemisphere = $\frac{7}{2}$ cm

Total Surface Area of Solid $= 6a^2 + 2\pi r^2 - \pi r^2 \quad = 6a^2 + \pi r$

Now, Total Surface Area of Solid 2 $= 6a^2 + 2\pi r^2 - \pi r^2 \quad = 6a^2 + \pi r^2$

Therefore, Ratio of their Surface Area = 1 : 1

(ii) Volume of Solid 1 = Volume of Cube − Volume of hemisphere

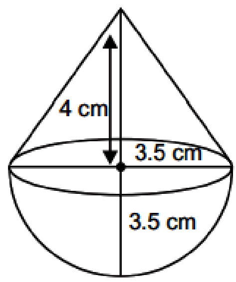
$= 343 - 89.83 \quad = \quad 253.17 \text{ cm}^3$

Volume of Solid 2 = Volume of Cube + Volume of hemisphere

$= 343 + 89.83 \quad = \quad 432.83 \text{ cm}^3$

OR

A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that whole solid is submerged in water. If the radius of the cylinder is 5 cm and height is 10.5 cm, find the volume of water left in cylindrical tub. (Take π = 3.14)



Soln: In the given solid, radius of hemisphere = radius of base of cone

(r) = 3.5 cm   Height of cone (h) = 4 cm

∴ Volume of the solid

= Volume of the hemispherical part + Volume of the conical part

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2(2r + h) \quad = 141.03 \text{ cm}^3$

When the solid is submerged in the cylindrical tub, the volume of water that flows out of the cylinder is equal to the volume of the solid.

Therefore, Volume of water left in the cylinder = Volume of cylinder – Volume of solid

$= (3.14 \times 5 \times 5 \times 10.5 - 141.03)$

$= (824.25 - 141.17) \quad = 683.22 \text{ cm}^3$

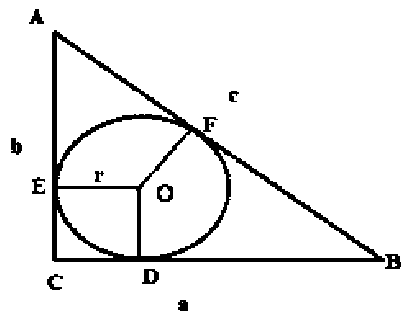
Suggestive Measures – While solving such type of questions students should:
- **draw correct figure according to the information given in question**
- **identify which part of solid will get hidden while combining two solids**
- **know the difference between curved and total surface area**
- **know the correct formulae of volumes and surface areas of solids**

34. a, b and c are the sides of a right-angled triangle and c is the hypotenuse. A circle of radius r, touches the sides of the triangle. Prove that

r = S – c

where S is the semi-perimeter of the triangle.

Sol: Let the circle touches the sides BC, CA, AB of the right triangle ABC at D, E and F respectively, where BC = a, CA = b and AB = c.



Since length of tangents drawn from an external point are equal ⇒AE=AF and BD=BF

OE⊥AC, OD⊥CD [∵ radius ⊥ tangent]

OECD is a square. So, CE=CD=r

and b−r =AF, a−r = BF

Therefore AB=AF+BF

$\Rightarrow \quad c = (b - r) + (a - r)$

$\Rightarrow \quad 2r = a + b - c$

Adding 2c on both sides

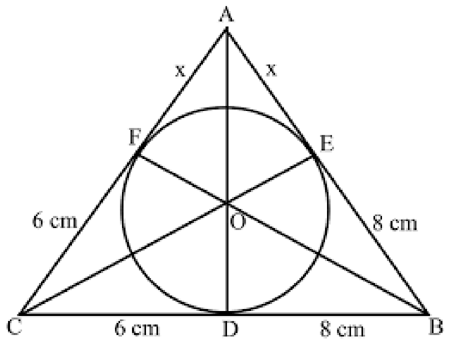
$\Rightarrow \quad 2r + 2c = a + b - c + 2c$

$\Rightarrow \quad r + c = S$

$\Rightarrow \quad r = S - c$ Hence Proved

OR

A triangle ABC is drawn to circumscribe a circle of radius 4 cm. The side BC is divided by the point of contact D into BD and DC of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



Sol: Length of two tangents drawn from the same point to the circle are equal.

So, CF = CD = 6 cm

and BE = BD = 8 cm

AE = AF = x

We observed that AB = AE + EB = x + 8

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = 6 + x

Let perimeter of triangle ABC be s.

$\Rightarrow 2s = AB + BC + CA$

$= x + 8 + 14 + 6 + x = 28 + 2x$

$\Rightarrow s = 14 + x$

Now, Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(14+x)(14+x-14)(14+x-x-6)(14+x-x-8)}$

$= \sqrt{(14+x)(x)(8)(6)}$

$= \sqrt{(14+x)48x}$ ....(i)

Also, Area of Δ ABC = 2 × (area of ΔAOF+ΔCOD+ΔDOB)

$= 2 \times [(\frac{1}{2}\times OF\times AF)+(\frac{1}{2}\times CD\times OD)+(\frac{1}{2}\times DB\times OD)]$

$= 2 \times \frac{1}{2}(4x+24+32) = 56+4x$ ....(ii)

From equations (i) and (ii) we get, $\sqrt{(14+x)48x} = 56 + 4x$

Squaring both sides,

$48x(14+x)=(56+4x)^2$

$\Rightarrow 48x = [4(14+x)]^2 \div (14+x)$

$\Rightarrow 48x = 16(14+x)$

$\Rightarrow 48x = 224+16x$

$\Rightarrow 32x = 224 \quad \Rightarrow x = 7$ cm

So AB = x + 8 = 7 + 8 = 15 cm and CA = 6 + x = 6 + 7 = 13 cm

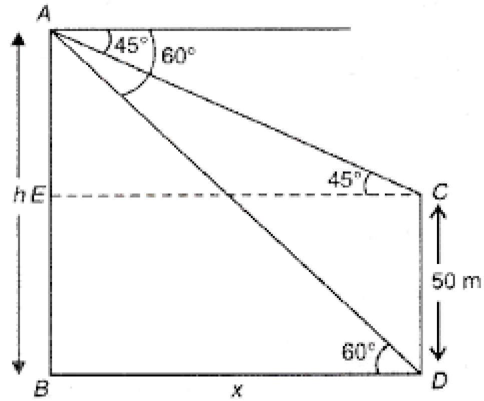
Suggestive Measures – While solving such type of questions students should:

- **draw correct figure according to the information given in question**
- **know about the theorems to be applied in the question**
- **be aware about the fact that the area of a triangle could be calculated in two different ways**

35. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45º and 60º respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Take $\sqrt{3}$ = 1.73)

Sol: Let AB be the tower of height h metres and CD be the building of height 50 m.

Draw CE ⊥ AB.



Now, CE = BD = x m

CD = BE = 50 m

AE = (h – 50) m

In ΔAEC, $\tan 45° = \frac{AE}{EC}$

$1 = \frac{h-50}{x}$

x = h – 50 …(i)

In ΔABD, $\tan 60° = \frac{AB}{BD}$

$\sqrt{3}x = h$

$\sqrt{3}(h-50) = h$ … (from equation i)

$50\sqrt{3} = h(\sqrt{3}-1)$

$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$h = 25\sqrt{3}(\sqrt{3}+1) = 75 + 25\sqrt{3} = 118.25$

Put the value of h in equation (i), we get x = 118.25– 50 = 68.25

So, the height of the tower is 118.25 m and the distance between the tower and the building is 68.25 m.

Suggestive Measures – While solving such type of questions students should:
- **draw figure according to information given in question**
- **identify angle of elevation and depression**
- **apply correct trigonometric ratios and calculation**
- **write the final answer with proper units**

SECTION-E

While solving case study based questions students should:
- **Carefully note the given information**
- **relate the given information with the mathematical concept studied so far**
- **apply the correct formula and perform accurate calculations**

36. Rajni went to a fair in her village. She wanted to enjoy rides on Giant wheel and play dart game. The number of times she played dart game is half the number of rides she had on giant wheel. If each ride costs ₹ 5 and dart game costs ₹ 4 then she spent ₹28.

Based on the above information, answer the following questions:

(i) Represent the given situation algebraically.

Soln: Let number of rides = x

Number of dart games played = y

According to question $y = \frac{1}{2}x \Rightarrow x - 2y = 0$

and $5x + 4y = 28$

(ii) Which type of solution is shown by these equations?

Soln: $x - 2y = 0$ ………………….equation 1

$5x + 4y = 28$ ………………….equation 2

Here $a_1 = 1$, $a_2 = -2$, $b_1 = 5$, $b_2 = 4$, $c_1 = 0$, $c_2 = -28$

$\frac{a_1}{a_2} = \frac{-1}{2}$ and $\frac{b_1}{b_2} = \frac{5}{4}$

We notice that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Hence these equations have unique solution.

(iii) How many times does Rajni ride the giant wheel and how many times does she play dart game?

Soln: $x - 2y = 0$ ………………….equation 1

$5x + 4y = 28$ ………………….equation 2

Multiplying equation 1 by 2 to make the coefficients of y equal

$2x - 4y = 0$

$5x + 4y = 28$

Since variable y has opposite signs, so we add both the equations to cancel y variable, we get

$7x = 28 \Rightarrow x = 4$

Putting value of x in equation 1, we get

$2y = 4 \Rightarrow y = 2$

So Number of rides = 4 and Number of dart games = 2

OR

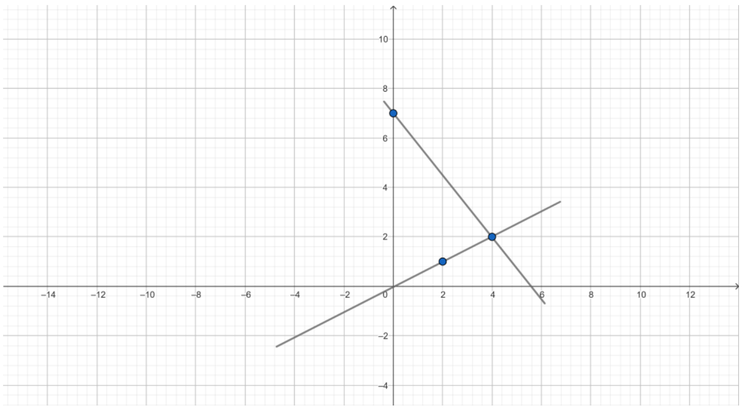
Draw the graph for the pair of equations of part 1.

Soln: For equation $x - 2y = 0$

x	2	4
y	1	2

For equation $5x + 4y = 28$

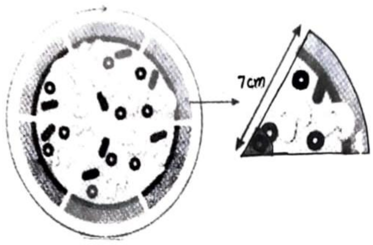
x	4	0
y	2	7



Suggestive Measures :

- **Q 36 relates to the chapter 'Linear Equation in two variables'**
- **Form the equation from the given information**
- **Identify the type of solution by checking the condition**
- **Find solution of these equations by any method and plot the graph**

37. **Rama and her two friends went to a pizza shop. They ordered a medium sized pizza whose radius is 7 cm. They divided the pizza in such a way that each one of them get two equal slices.**



Based on the above information, answer the following questions:

(i) Find the central angle of pizza slice.

Soln: No. of friends = 3

Each friend get number of pieces = 2

Pizza divided into slices = 3 × 2 = 6

These slices are sector of pizza shaped circle.

Central angle of pizza slice $\theta = \frac{360^\circ}{6} = 60^\circ$

(ii) Find the area of pizza slice.

Soln: Area= $\frac{\theta}{360^\circ} \pi r^2 = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{3}$ cm^2

(iii) Find the perimeter of pizza slice.

Soln: Perimeter of Pizza slice = arc-length of pizza slice + 2r

$= \frac{\theta}{360^\circ} 2\pi r + 2r = \frac{64}{3}$ cm

OR

If the shopkeeper gives 2 pizzas, one of 3 cm and other of 4 cm radius instead of 1 pizza of 7 cm radius at same price, what would Rama prefer?

Soln: Area of first pizza = $\pi r^2 = \pi(3)^2 = 9\pi$ cm^2

Area of second pizza = $\pi(4)^2 = 16\pi$ cm^2

Area of two pizza = 25π cm^2

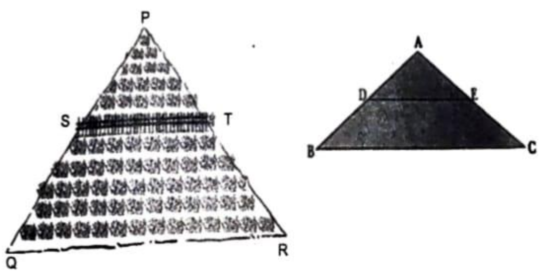
Area of pizza with radius 7cm = 49π cm^2

Rama will prefer pizza with radius 7 cm

Suggestive Measures :

- **Q 37 relates to the chapter 'Area related to circles'**
- **Identify sector/segment and its central angle, area and perimeter**
- **apply relevant formula and do proper calculations**

38. **A farmer had a triangular piece of land. He put a fence, parallel to one of the sides of the field as shown in figure. ABC is a scale drawing of the field which is made on the scale of 1:8.**



Based on the above information, answer the following questions:

(i) By Thales theorem, which sides of the field are proportional?

Soln: The fence ST is parallel to side QR.

By Thales theorem $\frac{PS}{SQ} = \frac{PT}{TR}$

So other sides PQ and PR are proportional.

(ii) If side PQ is 640 cm and PS is 160 m then the sides PQ and PR are divided into which ratio by the fence ST?

Soln: QS= 640 – 160 = 480 m

Ratio = PS:QS = 160 :480=1: 3

(iii) If point D is 20 m away from A and AB and AC are 80 m and 100 m respectively, then find the value of PT.

Soln: $\frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{20}{80} = \frac{AE}{100} \Rightarrow AE = 25$ m

Given AE: PT= 1:8 ⇒ PT = 25 × 8 = 200 m

OR

If AD = x+1, DB = 3x – 1, AE = x+3 and EC = 3x+4 then find the value of x.

Soln: $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x+1}{3x-1} = \frac{x+3}{3x+4}$

$\Rightarrow (x+1)(3x+4) = (x+3)(3x-1)$

$\Rightarrow 3x^2 + 7x + 4 = 3x^2 + 8x - 3$

$\Rightarrow x = 7$

Suggestive Measures :

- **Q 38 relates to the chapter 'Triangles'**
- **Be aware about Thales theorem and how to apply it**
- **apply the theorem and do proper calculations**