

COMPREHENSIVE ASSESSMENT FEEDBACK
PRE-BOARD EXAMINATION (2025-26)
CLASS: X
SUBJECT: MATHEMATICS BASIC (241)

SECTION-A

1. The pair of linear equations, $3x-2y=5$ and $2x+y=3$ has:

- (a) No solution (b) One solution
 (c) Two solutions (d) Infinitely many solutions

Sol: Given: $3x-2y=5$ and $2x+y=3$

Comparing the above equations with $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ we have:

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-2}{1} \text{ and } \frac{c_1}{c_2} = \frac{5}{3} \quad \text{Clearly, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

- (a) No solution: Incorrect. This only occurs if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.
 (b) One solution: Correct. Since the ratios of the coefficients of x and y are unequal, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore, this is the correct option.
 (c) Two solutions: Incorrect. Linear equations represent straight lines; two distinct straight lines can never intersect at exactly two points.
 (d) Infinitely many solutions: Incorrect. It requires $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Correct option: (b) one solution.

Suggestive Measures- While attempting such questions, students should

- Know how to compare the given pair of linear equations with the standard form to find the value of a_1, b_1, c_1 and a_2, b_2, c_2 .
- Know how to calculate the ratios of $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$
- Know the conditions of one solution (unique solution), no solution and infinitely many solutions.

2. The ratio of height of a building and its shadow is 1:1, then the angle of elevation of the sun is:

- (a) 30° (b) 60° (c) 45° (d) 90°

Sol: In a right-angled triangle:

The height of the building is the opposite side to the angle of elevation.

The shadow length is the adjacent side to the angle of elevation.

The trigonometric ratio that relates these two sides is tangent:

Given the ratio of height to shadow is 1:1, we can say: $\tan \theta = \frac{1}{1} = 1$

$$\theta = 45^\circ$$

So the only correct option is (c) 45° .

Suggestive Measures- While attempting such questions, students should

- Have the knowledge of the various trigonometric ratios.
- Know the values of trigonometric ratios of important angles like $0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .

3. The end points of diameter of circle are (4, 6) and (-1, 1). The radius of the circle is:

- (a) $\pm \frac{5\sqrt{2}}{2}$ units (b) $\frac{5\sqrt{2}}{2}$ units (c) $5\sqrt{2}$ units (d) $3\sqrt{2}$ units

Sol: To find the radius of the circle when the endpoints of its diameter are given, we first calculate the length of the diameter using the distance formula and then divide it by two.

$$d = \sqrt{(-1 - 4)^2 + (1 - 6)^2} = \sqrt{25 + 25} = 5\sqrt{2} \text{ units}$$

$$r = \frac{d}{2} = \frac{5\sqrt{2}}{2}$$

So the only correct option is (b) $\frac{5\sqrt{2}}{2}$ units

Suggestive Measures- While attempting such questions, students should

- Know how to apply distance formula correctly when coordinates of two points are given.
- Know the relationship between the lengths of diameter and radius of a circle.

4. In triangles ABC and DEF, $\angle A = \angle E = 50^\circ$, $\frac{AB}{AC} = \frac{ED}{EF}$ and $\angle F = 48^\circ$, then $\angle C$ is equal to:

- (a) 82° (b) 98° (c) 48° (d) 50°

Sol: In triangles ABC and DEF, given that $\angle A = \angle E = 50^\circ$, $\frac{AB}{AC} = \frac{ED}{EF}$

Therefore, using SAS similarity rule, $\triangle ABC \sim \triangle EDF$

$\Rightarrow \angle C = \angle F = 48^\circ$ (corresponding parts of similar triangles are equal)

So the only correct option is (c) 48°

Suggestive Measures- While attempting such questions, students should

- Know the various similarity rules.
- Have the knowledge that corresponding angles of similar triangles are equal.

5. The areas of two circles are in the ratio 9:16. The ratio of their circumferences is:

- (a) 9:16 (b) 16:9 (c) 4:3 (d) 3:4

Sol: To find the ratio of the circumferences of two circles when the ratio of their areas is known, we use the relationship between radius, area, and circumference.

Let the radii of two circles be r_1 and r_2 respectively, then: $\frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{9}{16} \Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$

$$\therefore \frac{2\pi r_1}{2\pi r_2} = \frac{2\pi(3)}{2\pi(4)} = \frac{3}{4}$$

So the only correct option is (d) 3 : 4

Suggestive Measures- While attempting such questions, students should

- Know the formulae for circumference and area of a circle.
- Know how to find the ratio of the circumferences of two circles when the ratio of their areas is known.

6. If $\sin(\alpha + \beta) = 0$, then the value of $\sec(\alpha + \beta)$ is equal to :

- (a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{2}$

Sol: Given: $\sin(\alpha + \beta) = 0 \Rightarrow \sin(\alpha + \beta) = \sin 0^\circ \Rightarrow (\alpha + \beta) = 0^\circ$

$$\therefore \sec(\alpha + \beta) = \sec(0^\circ) = 1$$

So the only correct option is (b) 1

Suggestive Measures- While attempting such questions, students should

- Have the knowledge of the various trigonometric ratios.
- Know the values of trigonometric ratios of important angles like 0° , 30° , 45° , 60° and 90° .

7. If 6 is a zero of $x^2 - px - 18$, then the value of p is:
 (a) 3 (b) -3 (c) 4 (d) -4

Sol: If $x = a$ is a zero of a polynomial $p(x)$, then $p(a) = 0$.

$$\begin{aligned} \text{As it is given that 6 is a zero of } x^2 - px - 18 &\Rightarrow (6)^2 - p(6) - 18 = 0 \\ &\Rightarrow 36 - 6p - 18 = 0 \\ &\Rightarrow 6p = 18 \Rightarrow p = 3 \end{aligned}$$

So the only correct option is (a) 3

Suggestive Measures- While attempting such questions, students should

- Have the knowledge of the concept of zeroes of a polynomial.
- Know that if $x = a$ is a zero of a polynomial $p(x)$, then $p(a) = 0$.

8. If the arithmetic mean of 7, 8, a, 11, 14 is a, then the value of a is:
 (a) 10 (b) 5 (c) 9 (d) 9.5

Sol: It is given that arithmetic mean of 7, 8, 11, 14 is a.

$$\begin{aligned} &\Rightarrow \frac{7 + 8 + a + 11 + 14}{5} = a \\ &\Rightarrow 40 + a = 5a \quad \Rightarrow a = 10 \end{aligned}$$

So the only correct option is (a) 10

Suggestive Measures- While attempting such questions, students should

- Know the formula for calculating the mean of raw data.

9. $5\sqrt{81}$ is:
 (a) A prime number (b) An irrational number
 (c) A rational number (d) A negative integer

Sol: To solve such question, students should first simplify the surd by prime factorization method.

$$\text{Now, } 5\sqrt{81} = 5\sqrt{3 \times 3 \times 3 \times 3} = 5 \times 3 \times 3 = 45$$

Evaluation of Options :

- (a) A prime number: Incorrect. As 45 is a composite number.
- (b) An irrational number: Incorrect. As 45 is not an irrational number.
- (c) A rational number: Correct.
- (d) A negative number: Incorrect. As 45 is a positive integer.

So the only correct option is (c) A rational number.

Suggestive Measures- While attempting such questions, students should

- Know the concept of integers, rational numbers, irrational numbers, prime numbers and composite numbers.
- The idea that every integer is also a rational number.

10. If $p = 2\tan^2\theta$ and $y = 1 + \sec^2\theta$, then $2y - p$ is equal to:
 (a) 1 (b) 4 (c) 3 (d) 5

Sol: To solve such question, students should have the knowledge of trigonometric identities.

It is given that $p = 2\tan^2\theta$ and $y = 1 + \sec^2\theta$,

$$\begin{aligned} \text{Then, } 2y - p &= 2(1 + \sec^2\theta) - 2\tan^2\theta = 2 + 2\sec^2\theta - 2\tan^2\theta \\ &\Rightarrow 2y - p = 2 + 2(\sec^2\theta - \tan^2\theta) = 2 + 2(1) (\because \sec^2\theta - \tan^2\theta = 1) \\ &\Rightarrow 2y - p = 2 + 2 = 4 \end{aligned}$$

So the only correct option is (b) 4.

Suggestive Measures- While attempting such questions, students should

- Have the knowledge of trigonometric identities

11. The perimeter of triangle with vertices (0,12), (0,0) and (5,0) is:

- (a) 30 units (b) 20 units (c) 15 units (d) 17 units

Sol: The distance between two respective points is actually the sides of given triangle and its perimeter is the sum of all these sides.

To find the distance between the two points, we have to apply distance formula.

Here $x_1 = 0$, $x_2 = 0$, $x_3 = 5$, $y_1 = 12$, $y_2 = 0$ and $y_3 = 0$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 0)^2 + (0 - 12)^2} = 12 \text{ units}$$

$$\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} = \sqrt{(5 - 0)^2 + (0 - 0)^2} = 5 \text{ units}$$

$$\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} = \sqrt{(5 - 0)^2 + (0 - 12)^2} = 13 \text{ units}$$

So perimeter = $12 + 5 + 13 = 30$ units

Therefore, the correct option is (a) 30 units.

Suggestive Measures - While solving such type of questions students should:

- define perimeter
- know the how to write x_1 , x_2 , y_1 and y_2 from the given points
- know the distance formula correctly

12. XY is drawn parallel to base EF of $\triangle DEF$ intersecting DE at X and DF at Y. If $DE = 4EX$ and $FY = 2$ cm, then DY is equal to:

- (a) 2 cm (b) 8 cm (c) 10 cm (d) 6 cm

Sol : Since $XY \parallel EF$, using BPT $\frac{DX}{EX} = \frac{DY}{YF}$

$$\text{Given } DE = 4EX \Rightarrow \frac{DE}{EX} = 4$$

$$\Rightarrow \text{But } \frac{DE}{EX} = \frac{DX + YF}{EX} = \frac{DX}{EX} + 1$$

$$\Rightarrow \frac{DX}{EX} = 4 - 1 = 3$$

$$\text{From } \frac{DX}{EX} = \frac{DY}{YF}, \quad DY = 3 \times 2 = 6 \text{ cm}$$

So the only correct option is (d).

Suggestive Measures – While solving such type of questions students should:

- have the understanding of ratio and proportion
- know how to apply BPT
- perform correct calculations

13. If 15, p, q, -3 are in A.P, then p - q is equal to:

- (a) 6 (b) -6 (c) 12 (d) -12

Sol : Since 15, p, q, and -3 are in A.P

$$p - 15 = q - p = -3 - q \Rightarrow 2p = q + 15$$

$$q - p = -3 - q \Rightarrow 2q = p - 3$$

Solving and $p = 9$ & $q = 3$

$$\text{Hence } p - q = 9 - 3 = 6$$

Therefore, the only correct option is (a).

Suggestive Measures– While solving such type of questions students should

- Correctly identify terms and common difference in an AP
- Know how to solve linear equation sin two variables.

14. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 13 cm. Then, the length of PQ is :

- (a) 8 cm (b) 5 cm (c) 12 cm (d) 13 cm

Sol : Let O be the center of the circle and P be the point of contact.

We know that the radius of the circle will be perpendicular to the tangent at the point of contact. Therefore, we have $\triangle OPQ$ as a right triangle.

$$\Rightarrow PQ = \sqrt{13^2 - 5^2} \text{ (Pythagoras Theorem)}$$

$$\Rightarrow PQ = 12 \text{ cm}$$

Therefore, length of PQ is 12 cm and hence the correct option is (c) 12 cm.

Suggestive Measures – While solving such type of questions students should:

- know that the radius of the circle will be perpendicular to the tangent at the point of contact
- know how to apply Pythagoras theorem

15. The area of circle that can be inscribed in a square of side 6 cm is:

- (a) 18 cm^2 (b) $36\pi \text{ cm}^2$ (c) $9\pi \text{ cm}^2$ (d) $72\pi \text{ cm}^2$

Sol: If a circle is inscribed in a square, then its diameter is of the same length as the side of a square.

$$\text{So, diameter} = \text{side of square} = 6 \text{ cm} \quad \Rightarrow \quad \text{radius } r = \frac{6}{2} = 3 \text{ cm}$$

$$\text{Hence area of circle} = \pi r^2 = \pi \times 3 \times 3 = 9\pi \text{ cm}^2$$

Therefore, the correct option is (c) $9\pi \text{ cm}^2$

Suggestive Measures – While solving such type of questions students should:

- Identify that side of a square inscribed in a circle is same as the diameter of that circle.
- Calculate area of the circle

16. In triangles LMN and XYZ, $\angle Y = \angle L$, $\angle Z = \angle M$ and $XY = 2LN$ then the two triangles are:

- (a) Congruent but not similar (b) Similar but not congruent
(c) Neither congruent nor similar (d) Congruent as well as similar

Sol : For angles to be congruent, all the sides and angles must be equal. Here two angles are equal but sides are in the ratio therefore triangles cannot be congruent.

Hence option (a) and (b) are eliminated.

Since angles are equal and sides are in the ratio therefore triangles can be similar.

So the only correct option is (b).

Suggestive Measures – While solving such type of questions students should:

- know about the similarity criteria
- know about the condition of congruency
- be able to distinguish between similarity and congruency of triangles

17. 15 defective bulbs are accidentally mixed with 150 good bulbs. It is not possible to just look at a bulb and tell whether or not it is defective. One bulb is taken out at random from this lot. The probability that the bulb taken out is a defective one is :

- (a) $\frac{1}{10}$ (b) $\frac{11}{10}$ (c) $\frac{10}{11}$ (d) $\frac{1}{11}$

Sol : Here total outcomes = $150 + 15 = 165$

Favourable outcomes = 15

$$P(\text{having defective bulb}) = \frac{15}{165} = \frac{1}{11}$$

Therefore, the correct option is (d) $\frac{1}{11}$.

Suggestive Measures–While solving such type of questions students should:

- Know about finding total outcomes of an event
- apply the formula of probability

18. The length of tangent drawn from a point 25 cm away from the centre of a circle of radius 24 cm is:

- (a) 16 cm (b) 10 cm (c) 7 cm (d) $3\sqrt{7}$ cm

Sol: It is known that the radius of a circle is perpendicular to the tangent of a circle at its point of contact.

So, by joining the other end of the tangent with the centre of the circle forms a right angled triangle as :

Length of tangent as BASE

radius of a circle as PERPENDICULAR

and line joining the other end of the tangent with the centre of the circle as HYPOTENUSE

$$\text{By Pythagoras theorem, } 24^2 + (\text{BASE})^2 = 25^2 \Rightarrow \text{BASE} = 7 \text{ cm}$$

Therefore, the correct option is (c) 7 cm

Suggestive Measures: While solving such type of questions students should

- aware about properties of tangents with respect to centre and radius of the circle
- apply Pythagoras theorem

Direction for question number 19 and 20:

There is one Assertion (A) and one Reason (R). Choose the correct answer of these questions from the four options (a), (b), (c) and (d) given below:

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A): The value of $\cos \theta = \frac{5}{3}$ is not possible.

Reason (R): Hypotenuse is the longest side in a right angled triangle.

Sol: The trigonometric ratio $\cos \theta = \frac{5}{3}$ corresponds to $\frac{\text{Base}}{\text{Hypotenuse}}$

$$\Rightarrow \text{Base} = 5 \text{ units} \quad \text{and} \quad \text{Hypotenuse} = 3 \text{ units}$$

$\Rightarrow \text{Hypotenuse} < \text{Base}$ which is not possible since hypotenuse is the longest side in a right angled triangle.

Therefore, the correct option is (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the assertion (A).

Suggestive Measures: While solving such type of questions students should

- know about trigonometric ratios
- be aware about facts related to sides of right angled triangle

20. Assertion (A): The point which divides the line segment joining the points P(3, -6) and R(6, -4) internally in the ratio 2:1 is $(5, \frac{-14}{3})$.

Reason (R): The coordinates of the point which divides the line segment joining points (x_1, y_1) , (x_2, y_2) in the ratio $m_1 : m_2$ are $[\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}]$.

Sol : Here the coordinates of points are $x_1 = 3$, $x_2 = 6$, $y_1 = -6$, $y_2 = -4$

The given ratio is 2:1 $\Rightarrow m_1 = 2$ and $m_2 = 1$

The coordinates of required point are given by $[\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}]$

Hence the point is $[\frac{1 \times 3 + (2 \times 6)}{2 + 1}, \frac{1 \times -6 + 2 \times -4}{2 + 1}] = (5, \frac{-14}{3})$

Therefore, the correct option is (c) Assertion (A) is true but reason (R) is false.

Suggestive Measures: While solving such type of questions students should

- Identify correct values of x_1, x_2, y_1, y_2 etc.
- Do proper calculation using correct formula

SECTION-B

21. If 2 is a root of the quadratic equation $px^2 + 3x - 2 = 0$, then find the value of p.

Sol: Given: 2 is a root of the quadratic equation $px^2 + 3x - 2 = 0$

$$\Rightarrow p(2)^2 + 3(2) - 2 = 0$$

$$\Rightarrow 4p + 6 - 2 = 0 \quad \Rightarrow p = -1$$

Suggestive Measures- While attempting such questions, students should

- Have the knowledge of the concept of zeroes of a quadratic polynomial and roots of a quadratic equation
- Know that if $x = a$ is a root of a quadratic equation $p(x) = 0$, then $p(a) = 0$.

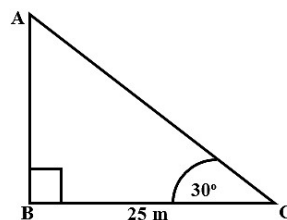
22. The angle of elevation of the top of a tower from a point on the ground, which is 25 m away from the foot of the tower, is 30° . Find the height of the tower.

Sol: Let AB is a tower and C is the point the ground such that BC = 25 m.

$$\text{Now, } \frac{AB}{BC} = \tan 30^\circ \Rightarrow \frac{AB}{25} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{25}{\sqrt{3}} = \frac{25\sqrt{3}}{3}$$

\therefore The height of the tower is $\frac{25\sqrt{3}}{3}$ m.



Suggestive Measures – While solving such type of questions students should:

- draw correct figure according to information given in the question.
- identify angle of elevation and depression.
- apply correct trigonometric ratios and calculate the unknown side.
- write the final answer with proper units.

23. Check whether 12^n can end with digit zero for n being a natural number.

Sol: For a number to end with the digit 0, it must be divisible by 10. Since the prime factors of 10 are 2 and 5, any number ending in zero must have both 2 and 5 in its prime factorization.

$$\text{Now } 12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$\Rightarrow 12^n = 2^{2n} \times 3^n$$

The only prime factors are 2 and 3. There is no prime factor 5 in the factorization.

Therefore, 12^n cannot end with the digit zero for any natural number n .

Suggestive Measures – While solving such type of questions students should:

- Know how to find the prime factorization of a given number.
- Know that to end in 0, a number must be divisible by 10, requiring both 2 and 5 as prime factors.
- Invoke the Fundamental Theorem of Arithmetic; prime factors of a number are unique and do not change regardless of the exponent n .

OR

Check whether $2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 + 51$ is a prime number or a composite number.

Give reason in support of your answer.

Sol: A composite number is a positive integer greater than 1 that has more than two distinct factors.

$$\text{Now, } 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 + 51 = 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 + 3 \times 17$$

$$3 \times (2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 + 17) = 3 \times 2789$$

Since the number 3×2789 has more than two distinct factors (namely 1, 3, 2789 and the number itself), therefore, $2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11 + 51$ is a composite number.

Suggestive Measures – While solving such type of questions students should:

- Know definition of prime and composite number.
- Know the prime factorization of a given number.
- Always check if the "added" number (constant) shares a prime factor with the "multiplied" part of the expression.

24. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol: Given: A circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P (see Fig.)

To prove: $PQ = PR$.

Construction: Join OP, OQ and OR.

Proof: In triangles OQP and ORP

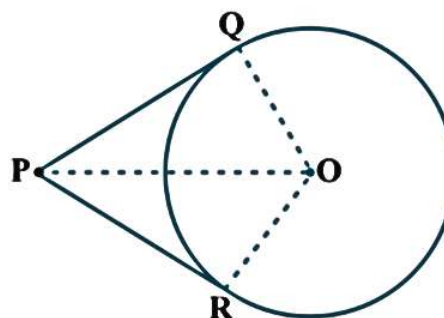
$$OQ = OR \text{ (Radii of the same circle)}$$

$$OP = OP \text{ (Common)}$$

$$\angle OQP = \angle ORP \text{ (each } 90^\circ)$$

Therefore, $\triangle OQP \cong \triangle ORP$ (RHS)

This gives $PQ = PR$ (CPCT)



Suggestive Measures – While solving such type of questions students should:

- Draw the correct figure.
- Know that the radius is perpendicular to the tangent at the point of contact.
- Know about the congruency of two triangles and corresponding parts of congruent triangles are equal.

25. If $\operatorname{cosec} \theta = \frac{13}{5}$, then find the value of $\cot \theta$.

Sol: $\operatorname{cosec} \theta = \frac{13}{5} = \frac{H}{P}$

Let $H = 13k$, $P = 5k$, $\Rightarrow B = 12k$ (using Pythagoras Theorem)

$$\Rightarrow \cot \theta = \frac{B}{P} = \frac{12}{5}$$

Suggestive Measures- While attempting such questions, students should

- Know how to find third side of a right triangle when two sides are known using Pythagoras Theorem.
- Have the knowledge of the various trigonometric ratios.

OR

Evaluate $\frac{\tan^2 60^\circ + \sin^2 45^\circ}{\operatorname{cosec} 30^\circ + \sec 30^\circ}$

Sol: Here, $\tan 60^\circ = \sqrt{3}$, $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\operatorname{cosec} 30^\circ = 2$, $\sec 60^\circ = 2$

$$\frac{\tan^2 60^\circ + \sin^2 45^\circ}{\operatorname{cosec} 30^\circ + \sec 30^\circ} = \frac{(\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2}{2+2} = \frac{3+\frac{1}{2}}{4} = \frac{7}{8}$$

Suggestive Measures- While attempting such questions, students should

- Have the knowledge of the various trigonometric ratios.
- Know the values of trigonometric ratios of important angles like 0° , 30° , 45° , 60° and 90° .

SECTION-C

26. Prove that the tangents at end points of any chord make equal angles with the chord.

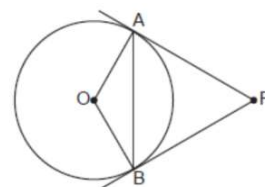
Sol: In figure, PA and PB are tangents at end points of the chord AB.

Together they form a $\triangle PAB$.

We have to prove that $\angle PAB = \angle PBA$.

In $\triangle PAB$, $PA = PB$ (tangents from external point)

$\Rightarrow \angle PAB = \angle PBA$ hence proved



Suggestive Measures: While solving such type of questions students should

- Know that tangents drawn from an external point to a circle are equal.
- Know that angles opposite to equal sides are also equal in a triangle

OR

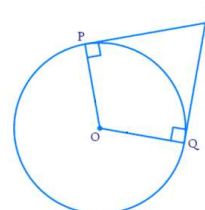
Show that the quadrilateral formed by the tangents from an external point to a circle and the radii at the point of contact is cyclic.

Sol: In figure, TP and TQ are tangents from an external point T.

OP and OQ are radii of the circle.

$\Rightarrow OP \perp TP$ and $OQ \perp TQ$

$\Rightarrow \angle P = \angle Q = 90^\circ \Rightarrow \angle P + \angle Q = 180^\circ$



In quadrilateral TQOP, $\angle P$ and $\angle Q$ are opposite angles

$$\text{and } \angle T + \angle Q + \angle O + \angle P = 360^\circ$$

$$\Rightarrow \angle T + \angle O = 180^\circ$$

Since sum of opposite angles of quadrilateral TQOP is 180° .

Hence the quadrilateral is cyclic.

Suggestive Measures - While solving such type of questions students should:

- know the concept of the parallel tangents of a circle
- know that the radius is half of the diameter
- know about exterior angle property and relation between tangent and radius of a circle
- do proper calculations

27. If A(-4, 3) B(x, 2) C(6, y) and D(3, 4) are the vertices of a parallelogram ABCD, then find the values of x and y. Also, find the length of the diagonals.

Sol: Diagonals of parallelogram bisect each other

Mid-point of diagonal AC = Mid-point of diagonal BD

$$\Rightarrow \left(\frac{-4+6}{2}, \frac{3+y}{2} \right) = \left(\frac{x+3}{2}, \frac{2+4}{2} \right)$$

$$\Rightarrow x = -1 \text{ \& } y = 3$$

Using distance formula, we have $AC = \sqrt{(6+4)^2 + (3-3)^2} = 10$ units

$$BD = \sqrt{(3+1)^2 + (4-2)^2} = 2\sqrt{5} \text{ units}$$

Suggestive Measures – While solving such type of questions students should:

- be aware of the fact that the diagonals of a parallelogram bisect each other at a point
- know how to equate the coordinates of two points
- know the distance formula and do proper calculations

28. Find the nature of the roots of the quadratic equation $6x^2 + 6\sqrt{3}x + 3 = 0$. If the real root exist, find them.

Sol: $6x^2 + 6\sqrt{3}x + 3 = 0$ is a quadratic equation in the standard form $ax^2 + bx + c = 0$,

Here, $a = 6$, $b = 6\sqrt{3}$, $c = 3$

$$D = b^2 - 4ac = 36$$

$\Rightarrow D > 0$, hence equation has real roots

$$\text{Roots are } \frac{-b \pm \sqrt{D}}{2a} \Rightarrow \frac{-6\sqrt{3} \pm 6}{6} = (1 - \sqrt{3}) \text{ and } -(1 + \sqrt{3})$$

Suggestive Measures – While solving such type of questions students should:

- know how to write a given quadratic equation in the standard form.
- know how to find the coefficients of each term of a quadratic equation written in the standard form.
- know the formula of calculating the discriminant of a quadratic equation.
- know how to decide the nature of roots of a quadratic equation according to the value of its discriminant.
- know how to factorize a quadratic polynomial.

29. If α, β are the zeroes of the polynomial $5x^2 - 4 - 8x$ then find the value of $\alpha^2 + \beta^2$.

Sol: Here $a = 5$; $b = -8$; $c = -4$

$$\text{Sum of zeroes} = \alpha + \beta = -\frac{b}{a} = \frac{8}{5}$$

$$\text{Product of zeroes} = \alpha\beta = \frac{c}{a} = \frac{-4}{5}$$

$$\begin{aligned}(\alpha + \beta)^2 &= \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \\ &= \left(\frac{8}{5}\right)^2 - 2 \times \left(\frac{-4}{5}\right) = \frac{104}{25}\end{aligned}$$

Suggestive Measures – While solving such type of questions students should:

- know how to find the values of coefficients of terms of polynomial written in the standard form
- know the relationship between zeroes and coefficients of a polynomial
- apply the required algebraic identities to get to the answer.

OR

Find a quadratic polynomial, sum of whose zeroes is -4 and product of whose zeroes is 12. Also, find the zeroes of the polynomial.

Sol: Given Sum = -4; product = 12

$$\text{quadratic polynomial is } K[x^2 - (\text{sum})x + \text{product}] \Rightarrow K[x^2 + 4x + 12]$$

We can give any value to K and form the required polynomial.

$$\text{To find zeroes : } x^2 + 4x + 12 = 0$$

We have to find two such numbers whose sum or difference is 4 and product is 12.

There are no such numbers whose sum or difference is 4 and product is 12.

To find zeroes using quadratic formula, we will find discriminant.

$$D = b^2 - 4ac = 16 - 48 = -32 < 0$$

$\Rightarrow D < 0$, hence equation has no real roots and cannot be found.

Suggestive Measures – While solving such type of questions students should:

- know the relationship between zeroes and coefficients of a polynomial
- express the polynomial in the standard form
- know the method of factorization of given polynomial to find two zeroes

30. The length of a minute hand of a wall clock is 21 cm. Find the area swept by the minute hand in 20 minutes.

Sol: Minute hand completes full circle degree in 60 minutes.

$$\text{Angle swept by minute hand in 60 minutes} = 360^\circ$$

$$\text{Angle swept by the minute hand in 20 minutes} = \frac{60 \times 20}{360} = 120^\circ$$

$$\text{Therefore, } \theta = 120^\circ$$

Length of minute hand = $r = 21$ cm

Area swept by minute hand in 21 minutes = Area of sector = $\frac{\theta}{360^\circ} \times \pi \times r^2$

Area swept by the minute hand in 21 minutes = $\frac{120^\circ}{360^\circ} \times \frac{22}{7} \times (21)^2 = 462 \text{ cm}^2$

Hence the area swept by the minute hand in 21 minutes is 462 cm^2 .

Suggestive Measures – While solving such type of questions students should:

- know that degree measure of a circle is 360 degrees.
- know that minute hand of a clock takes one revolution in 60 minutes.
- know how to find the central angle when minute hand moves certain amount of time.
- know that the length of the minute hand is equal to the radius of the circle.
- know the correct formula of calculating the area of sector of a circle.

31. If $\sqrt{3}$ is irrational, prove that $4\sqrt{3} + 3$ is an irrational number.

Sol: Let us assume that $4\sqrt{3} + 3$ is a rational number with p and q as co-prime integers and $q \neq 0$

$$\Rightarrow 4\sqrt{3} + 3 = \frac{p}{q}$$

$$\Rightarrow \sqrt{3} = \frac{p-3q}{4q} \text{ is a rational number}$$

Given that $\sqrt{3}$ is irrational

These two contradict each other.

\Rightarrow our assumption that $4\sqrt{3} + 3$ is a rational number is wrong

Hence $4\sqrt{3} + 3$ is an irrational number

Suggestive Measures – While solving such type of questions students should:

- know about the concepts of rational and irrational numbers
- be aware of the fact that sum, difference, product or quotient of two non-zero rational numbers is a rational number
- know that if p is a prime number then \sqrt{p} is an irrational number
- be aware about the technique of proof by the method of contradiction

SECTION-D

32. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter of 56 m, the height of the cylindrical part is 10 m and the total height of the tent is 31 m, find the area of canvas used in making the tent and total cost of canvas, if it is purchased at a rate of 7 per m^2 .

Sol: radius of cylindrical part = radius of conical part = $r = 28$ m

Height of conical part = $31 - 10 = 21$ m

$$\ell = \text{Slant height of cone} = \sqrt{(21)^2 + (28)^2} = 35 \text{ m}$$

$$\begin{aligned} \text{Area of canvas} &= \text{curved surface area of cylinder} + \text{curved surface area of cone} \\ &= 2\pi rh + \pi r\ell = 4840 \text{ m}^2 \end{aligned}$$

$$\text{Cost of canvas} = \text{area} \times \text{rate} = 4840 \times 7 = ₹ 33,880$$

OR

A solid toy in the form of a hemisphere surmounted by a right circular cone. The height of Cone is 24 cm and the diameter of the base is 20 cm. Determine the volume of toy. If this toy is placed in a cylindrical box of same height and diameter as of the toy, then find the volume of the liquid that can be filled in the cylindrical box (Use $\pi = 3.14$)

Sol: In the toy, radius of hemisphere = radius of base of cone $= r = 10$ cm

Height of cone Height of cone (h) = 24 cm

$$\begin{aligned}\therefore \text{Volume of toy} &= \text{Volume of the hemispherical part} + \text{Volume of the conical part} \\ &= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi r^2 (2r + h) = 4605.33 \text{ cm}^3 (\text{approx})\end{aligned}$$

When the solid is submerged in the cylindrical tub, the volume of water that flows out of the cylinder is equal to the volume of the solid.

Therefore, Volume of water filled in the cylindrical box

$$\begin{aligned}&= \text{Volume of cylinder} - \text{Volume of solid} \\ &= (3.14 \times 10 \times 10 \times 34 - 4605.33) \\ &= (10676 - 4605.33) = 6070.67 \text{ cm}^3 \text{ approx}\end{aligned}$$

Suggestive Measures – While solving such type of questions students should:

- draw correct figure according to the information given in question
- identify which part of solid will get hidden while combining two solids
- know the difference between curved and total surface area
- know the correct formulae of volumes and surface areas of solids

33. Six years hence Kush will be three times the age of his daughter and three years ago he was nine times as old as his daughter. Find their present ages.

Sol: Let present age of Kush and his daughter be x years and y years respectively.

$$\text{A.T.Q.} \quad x - 3y = 12 \dots(1) \quad \& \quad x - 9y = -24 \dots(2)$$

$$\text{Solving} \quad x = 30, y = 6$$

Present age of Kush = 30 years and of his daughter = 6 years

Suggestive Measures – While solving such type of questions students should:

- be aware how to solve linear equations in two variables
- perform correct calculations

34. State and prove Basic Proportionality Theorem.

Sol: Statement : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

$$\text{To Prove: } \frac{AD}{DB} = \frac{AE}{EC}$$

Construction: Let us join BE and CD and then draw $DM \perp AC$ and $EN \perp AB$.

$$\text{Proof: } \text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EN$$

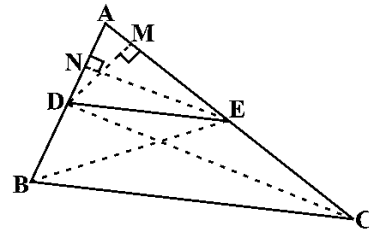
$$\text{ar}(\triangle BDE) = \frac{1}{2} \times DB \times EN$$

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AE \times DM$$

$$\text{ar}(\triangle DEC) = \frac{1}{2} \times EC \times DM$$

$$\text{Therefore, } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \dots (1)$$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \dots (2)$$



Note that $\triangle BDE$ and $\triangle DEC$ are on the same base DE and between the same parallels BC and DE .

$$\text{So, } \text{ar}(\triangle BDE) = \text{ar}(\triangle DEC) \dots (3)$$

$$\text{Therefore, from (1), (2) and (3), we have } \frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

Suggestive Measures – While solving such type of questions students should:

- **draw correct figure according to information given in the question.**
- **know the formula of calculating the area of a triangle.**
- **know that the triangles on the same base and between same parallels have equal areas.**

35. Three unbiased coins are tossed together. Find the probability of getting :

- (i) all tails (ii) exactly two tails (iii) exactly one head
(iv) atleast two heads (v) atleast two tails

Sol: Total outcomes when 3 coins are tossed = $2^3 = 8$

(i) Favourable Outcomes = (TTT)

$$P(\text{all tails}) = \frac{1}{8}$$

(ii) Favourable Outcomes = (HTT), (THT), (TTH)

$$P(\text{exactly two tails}) = \frac{3}{8}$$

(iii) Favourable Outcomes = (HTT), (THT), (TTH)

$$P(\text{exactly one head}) = \frac{3}{8}$$

(iv) Favourable Outcomes = (HHT), (HTH), (THH), (HHH)

$$P(\text{atleast two heads}) = \frac{4}{8} = \frac{1}{2}$$

(v) Favourable Outcomes = (HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH)

$$P(\text{atmost two tails}) = \frac{7}{8}$$

OR

In a single throw of pair of different dice, what is the probability of getting:

- (i) a prime number on each die (ii) a sum of 9 (iii) a sum of 7 or 11
(iv) a doublet (v) a product of 12

Sol: Total outcomes on throwing a pair of die = $6^2 = 36$

(i) Favourable Outcomes = (2,3) (2,5), (3,5), (3,2), (5,2), (5,3), (2,2), (3,3), (5,5)

$$P(\text{a prime number on each dice}) = \frac{9}{36} = \frac{1}{4}$$

(ii) Favourable Outcomes = (3,6), (4,5), (6,3), (5,4)

$$P(\text{a sum of 9}) = \frac{4}{36} = \frac{1}{9}$$

(iii) Favourable Outcomes = (2,5), (5,2), (6,1), (1,6), (5,6), (4,3), (6,5), (3,4)

$$P(\text{a sum of 7 or 11}) = \frac{8}{36} = \frac{2}{9}$$

(iv) Favourable Outcomes = (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

$$P(\text{a doublet}) = \frac{6}{36} = \frac{1}{6}$$

(v) Favourable Outcomes = (2,6), (6,2), (3,4), (4,3)

$$P(\text{a product of 12}) = \frac{4}{36} = \frac{1}{9}$$

Suggestive Measures—While solving such type of questions students should:

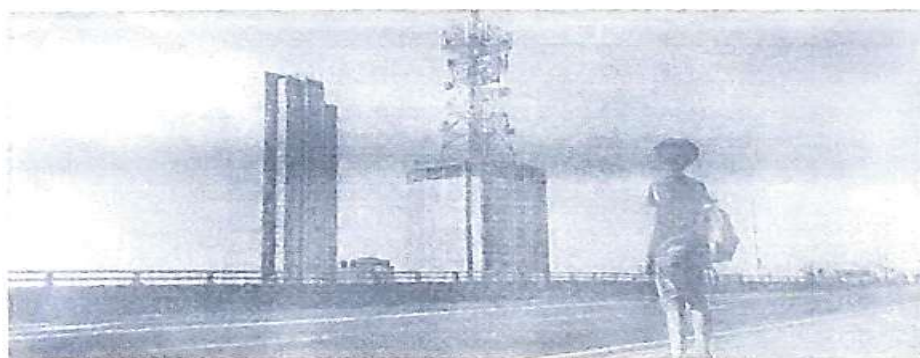
- Be aware about the outcomes of tossing a coin & throwing a die
- know the results of probability
- perform accurate calculations using the correct values

SECTION-E

While solving case study based questions students should:

- Carefully note the given information
- relate the given information with the mathematical concept studied so far
- apply the correct formula and perform accurate calculations

36. A mobile tower is installed on a 70 m high building. From a point P on the ground the angle of elevation of the foot of the tower is 45° , whereas from the same point the angle of elevation of the top of the tower is found to be 60° . A technician working at the top of the mobile tower observes a car at an angle of depression of 30° .



On the basis of above information, answer the following questions:

- How far is point P from the building?
- Find the height of the mobile tower.
- How far is car from the building?

OR

A hawk hovers just 23 m above the technician. How high above the ground was the hawk flying? (Use $\sqrt{3}=1.73$)

Sol.

- (i) To find the distance of the point P from the building.
Let AB is the mobile tower and BC is 70 m high building. Further, let AB = h, CD = x and DP = y.

$$\text{In } \triangle BCP, \frac{BC}{CP} = \tan 45^\circ$$

$$\Rightarrow \frac{70}{x+y} = 1 \Rightarrow x + y = 70$$

This means point P is at a distance of 70 m from the building.

- (ii) To find the height of the mobile tower

$$\text{In } \triangle ACP, \frac{AC}{CP} = \tan 60^\circ$$

$$\Rightarrow \frac{70+h}{70} = \sqrt{3} \Rightarrow h = 70(\sqrt{3} - 1) = 51.1$$

This means height of the mobile tower is 51.1 m (approx).

- (iii) To find the distance of the car from the building

$$\text{In } \triangle ACD, \frac{AC}{CD} = \tan 30^\circ$$

$$\Rightarrow \frac{70 + 51.1}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = 121.1 \times 1.73 = 209.5$$

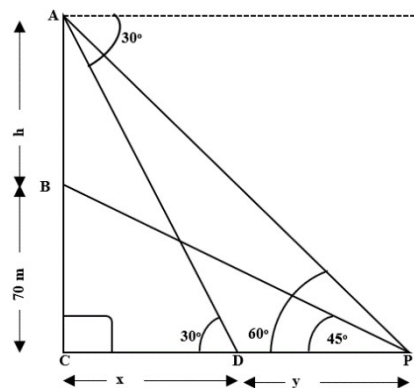
This means car is at a distance of 210 m (approx.) from the building.

OR

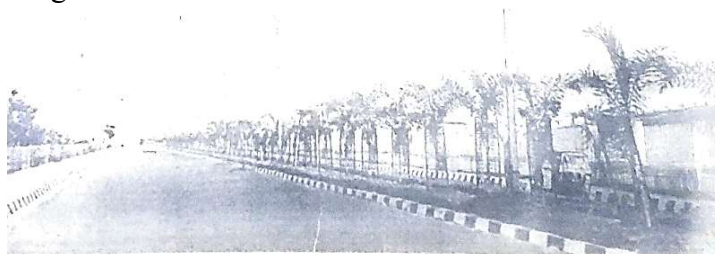
The height of the hawk above the ground is $121.1 + 23 = 144.1$ m (approx.)

Suggestive Measures – While solving such type of questions students should:

- draw correct figure according to information given in the question.
- identify angle of elevation and depression.
- apply correct trigonometric ratios and calculate the unknown side.
- write the final answer with proper units.



37. In order to increase the green cover an NGO chose a plantation site along a road. Gaur and Kanak were made incharges for plantation and maintenance. They planted 50 trees maintaining a gap of 12 m between every two adjacent trees. For watering they build a tank at a distance of 5 m from the first tree. They fill a bucket from the tank and water one tree at a time, then they return back to fill the bucket again and water the next tree.



On the basis of above information, answer the following questions:

- Find the distance between water tank and fifth tree.
- Find the distance between water tank and seventh tree.
- Find the total distance to cover to water first 10 trees.

OR

Find the total distance to cover to water first 20 trees.

Sol. Clearly, the given problem is of arithmetic progressions.

Here $a = 5$ and $d = 12$

- (i) To find the distance between water tank and fifth tree.

$$a_5 = a + 4d = 5 + 48 = 53$$

Therefore, the distance between water tank and fifth tree is 53 m.

- (ii) To find the distance between water tank and seventh tree.

$$a_7 = a + 6d = 5 + 72 = 77$$

Therefore, the distance between water tank and seventh tree is 77 m.

- (iii) To find the total distance to cover to water first 10 trees.

$$S_{10} = \frac{10}{2} [2(5) + (10 - 1)12] = 590$$

Therefore, the total distance to cover to water first 10 trees is $2 \times 590 = 1180$ m.

OR

To find the total distance to cover to water first 20 trees.

$$S_{20} = \frac{20}{2} [2(5) + (20 - 1)12] = 2380$$

Therefore, the total distance to cover to water first 10 trees is $2 \times 2380 = 4760$ m.

Suggestive Measures – While solving such type of questions students should:

- Understand from the given question that it belongs to the chapter arithmetic progression.
- Know how to identify first term and common difference of the AP.
- Differentiate clearly where to apply a_n and S_n .
- Know the correct formula of a_n and S_n .

38. In metro cities pollution and nutrition deficiency is being a major threat to health conditions of residents. In order to study the scenario, a health agency conducted a survey on the age and number of patients admitted in a week in a hospital of a city. The data gathered by the agency is as given below:

| Age (in years) | 5-15 | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 |
|-----------------|------|-------|-------|-------|-------|-------|
| Number of cases | 6 | 11 | 21 | 23 | 14 | 5 |

On the basis of above information, answer the following questions:

- What is the median class of the given data?
- Find the modal class of the given data.
- Find the mean age of the patients admitted.

OR

Find the modal age of the patients admitted.

Sol. (i) To find the median class of the given data

| Age (in years) | 5-15 | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 |
|-----------------|------|-------|-------|-------|-------|-------|
| Number of cases | 6 | 11 | 21 | 23 | 14 | 5 |
| Cf | 6 | 17 | 38 | 61 | 75 | 80 |

$$\text{Here } N = 80 \text{ and } \frac{N}{2} = 40$$

So, the median class of the given data is 35-45.

- (ii) The modal class is the class with the highest frequency.

Clearly, the class with the highest frequency is 35-45.

Therefore, the modal class of the given data is 35-45.

(iii) To find the mean age of the patient admitted

| Age (in years) | x_i | f_i | $f_i x_i$ |
|----------------|-------|-------|-----------|
| 5-15 | 10 | 6 | 60 |
| 15-25 | 20 | 11 | 220 |
| 25-35 | 30 | 21 | 630 |
| 35-45 | 40 | 23 | 920 |
| 45-55 | 50 | 14 | 700 |
| 55-65 | 60 | 5 | 300 |
| | Total | 80 | 2830 |

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.37$$

Therefore, the mean age of the patient admitted is 35.37 years.

OR

Clearly, the highest frequency is 23 and the corresponding class is 35-45.

Therefore, the modal class is 35-45.

$$\text{So, } l = 35, h = 10, f_0 = 21, f_1 = 23, f_2 = 14$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$\text{Mode} = 35 + \left(\frac{23 - 21}{46 - 21 - 14} \right) \times 10 = 36.81$$

Therefore, the modal age of the patient admitted is 36.81 years.

Suggestive Measures – While solving such type of questions students should:

- Know how to calculate the class mark & cumulative frequencies of each given class-interval.
- Choose requisite method to calculate the mean as per the given data.
- Know the formula of calculating mode.
- Know how to find median class and modal class.
- Write the final answer with proper units.