Directorate of Education, GNCT of Delhi

Solution of Practice Paper

(Term-2)

2021-22

Class – XI

Mathematics (Code: 041)

	SECTION – A
Q. No.	**VALUE POINTS**
1	$\frac{1}{\sqrt{2}}$ Use the formula 2cos A cos B= cos(A+B)+cos (A-B) OR LHS=$\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}$ =$\frac{4}{(\sin 2\theta)^2}$=$4\,cosec^2 2\theta$ $\because cosec^2\phi \geq 1$ so $4\,cosec^2 2\theta \geq 4$
2	Given $\lvert 3x-2\rvert \leq \frac{1}{2}$ $\therefore$ => $\frac{-1}{2}$≤(3x-2)≤$\frac{1}{2}$ ($\because \lvert x\rvert$≤a => -a≤x≤a) =>$\frac{-1}{2}+2$≤3x-2+2≤$\frac{1}{2}+2$ (adding 2 on each term) $\frac{3}{2}$≤3x≤$\frac{5}{2}$=>$\frac{3}{2}$x$\frac{1}{3}$≤3x$\frac{1}{3}$≤$\frac{5}{2}$x$\frac{1}{3}$ (dividing each term by 3) =>$\frac{1}{2}$≤x≤$\frac{5}{6}$ i.e., xϵ[$\frac{1}{2}$,$\frac{5}{6}$]
3	INVOLUTE contains 4 vowels (I,O,U,E) and 4 consonants (N,V,L,T). 3 vowels can be selected out of 4 vowels by $^4C_3$ ways and 2 consonants can be selected out of 4 consonants by $^4C_2$ ways $\therefore$ Number of words formed using 3 vowels and 3 consonants = $^4C_3$ x $^4C_2$ x5!= 4x6x120=2880 Hence total number of words is 2880 which contains 3 vowels and 2 consonants

4	The given equation of parabola is $y^2=8x$=> $y^2=4.2.x$ ∴ a=2, focus =(2,0) Let P(x,y) be any point on the parabola then PS=4 d=$\left\|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right\|$=>$\left\|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right\|$=4 =>$x^2-4x+4+8x$=16 =>$x^2+4x-12=0$=>(x+6)(x-2)=0=> x=-6,x=2=>$y^2=8.2$ $y^2=8.(-6)$not possible =>y=±4 ∴ The point P is (2,4) (2,-4)
5	Let f(x)=$e^x sinx+x^n cosx$ $\therefore f'(x)=\frac{d}{dx}\{e^x sinx+x^n cosx\}=\frac{d}{dx}(e^x sinx)+\frac{d}{dx}(x^n cosx)$ we know$f'(u,v)=vf'(u)\ +uf'(v)$=>$f'(x)=\ sinx\frac{d}{dx}e^x+e^x\frac{d}{dx}\sin x+cosx\frac{d}{dx}x^n+x^n\frac{d}{dx}cosx=sinx.e^x+$ $e^x.cosx+cosx\,n\,x^{n-1}+x^n(-sinx)$ =$e^x(sinx+cosx)+x^{n-1}[n\,cosx-x\,sinx]$
6	Number of ways of drawing 4 cards from 52 cards = $^{52}C_4$ In a deck of 52 cards , there are 13 diamonds and 13 spades. ∴ Number of ways of drawing 3 diamond and one spade is = $^{13}C_3$ x $^{13}C_1$ Thus the probability of obtaining 3 diamond and one spade =$\frac{^{13}C_3\times{}^{13}C_1}{^{52}C_4}=\frac{13\,x\,12\,x\,11\,x\,13\,x\,24}{6\,x\,52\,x\,51\,x\,50\,x\,49}=\frac{286}{20825}$
	SECTION – B
7	Given f(x)=$\frac{\|x-1\|}{x-1}$ **Domain:** Clearly,f(x) is defined for all xεR except x=1 ∴ Domain of f=R-{1} Range :Now f(x)=$\frac{x-1}{x-1}$=1, when x>1 and f(x)=-$\frac{(x-1)}{x-1}$=-1, when x<1 ∴ Range of f={-1,1}
8	Given f(x)=$\frac{cotx}{1+cotx}$ and $\alpha+\beta=\frac{5\pi}{4}$, $\therefore f(\alpha)=\left(\frac{\cot\alpha}{1+\cot\alpha}\right)\left(\frac{\cot\beta}{1+\cot\beta}\right)$ $\frac{\frac{\cos\alpha}{\sin\alpha}}{\left(1+\frac{\cos\alpha}{\sin\alpha}\right)}\left(\frac{\frac{\cos\beta}{\sin\beta}}{1+\frac{\cos\beta}{\sin\beta}}\right)$ $\frac{\cos\alpha.\cos\beta}{(\sin\alpha+\cos\alpha)(\sin\beta+\cos\beta)}=\frac{\cos\alpha.\cos\beta}{\sin\alpha\sin\beta+\sin\alpha\cos\beta+\cos\alpha\sin\beta+\cos\alpha\cos\beta}=$

$$\frac{\cos\alpha.\cos\beta}{(\cos\alpha\cos\beta+\sin\alpha\sin\beta)+(\sin\alpha\cos\beta+\cos\alpha\sin\beta)} = \frac{\cos\alpha.\cos\beta}{\cos(\alpha-\beta)+\sin(\alpha+\beta)} = \frac{2\cos\alpha.\cos\beta}{2[\cos(\alpha-\beta)+\sin(\alpha+\beta)]} =$$

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2[\cos(\alpha-\beta)+\sin(\alpha+\beta)]} = \frac{\cos\left(\frac{5\pi}{4}\right)+\cos(\alpha-\beta)}{2\left[\cos(\alpha-\beta)+\sin\left(\frac{5\pi}{4}\right)\right]} = \frac{\frac{-1}{\sqrt{2}}+\cos(\alpha-\beta)}{2\left[\cos(\alpha-\beta)-\frac{1}{\sqrt{2}}\right]} = \frac{1}{2}$$

OR

$\tan 70^0 = \tan(50^0+20^0)$

=>

$\tan 70^0 = \frac{\tan 50^0+\tan 20^0}{1-\tan 50^0\tan 20^0}$ [because $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

=>$\tan 70^0(1-\tan 50^0\tan 20^0)\tan 50^0+\tan 20^0$

=>$\tan 70^0-\tan 70^0\tan 50^0\tan 20^0=\tan 50^0+\tan 20^0$

=>$\tan 70^0-\tan(90^0-20^0)\tan 50^0\tan 20^0=\tan 50^0+\tan 20^0$

=>$\tan 70^0-\cot 20^0\tan 50^0\tan 20^0=\tan 50^0+\tan 20^0$ [because $\tan(90^0-A)$=cotA

=>$\tan 70^0=\tan 50^0+\tan 20^0+\tan 50^0$

=>$\tan 70^0=\tan 20^0+2\tan 50^0$ hence proved

9

A committee of 7 has to be formed from 9 boys and 4 girls.

(i) exactly 3 girls= ${}^9C_4\times{}^4C_3$

$=\frac{9!}{4!5!}\times\frac{4!}{3!1!}=\frac{9x8x7x6x5!}{3x2x5!}$=72x7=504

(ii) at most 3 girls

(a)No girl and 7 boys

(b) 1 girl and 6 boys

(c)2 girls and 5 boys

(d) 3 girls and 4boys

∴ Committee consisting of atmost 3 girls

= ${}^4C_0\times{}^9C_7+{}^4C_1\times{}^9C_6+{}^4C_2\times{}^9C_5+{}^4C_3\times{}^9C_4$

=1x36+4x84+6x126+126x4=36+336+756+504

=1632

10

1:3 externally

OR

To show ABCD is a parallelogram we need to show opposite sides are equal

Note that

AB=

$\sqrt{(-1-1)^2+(-2-2)^2+(-1-3)^2}=\sqrt{4+16+16}=6$

BC=$\sqrt{(2+1)^2+(3+2)^2+(2+1)^2}=\sqrt{9+25+9}=\sqrt{43}$

CD=$\sqrt{(4-2)^2+(7-3)^2+(6-2)^2}=\sqrt{4+16+16}=6$

DA=$\sqrt{(1-4)^2+(2-7)^2+(3-6)^2}=\sqrt{9+25+9}=\sqrt{43}$

since AB=CD and BC=AD, ABCD is a parallelogram

Now it is required to prove that ABCD is not a rectangle . For this we need to shoe that diagonals are unequal, we have

AC= $\sqrt{(2-1)^2+(3-2)^2+(2-3)^2}=\sqrt{1+1+1}=\sqrt{3}$

BD=$\sqrt{(4+1)^2+(7+2)^2+(6+1)^2}=\sqrt{25+81+49}=\sqrt{155}$

since AC≠BD ,ABCD is not a rectangle

SECTION – C

11

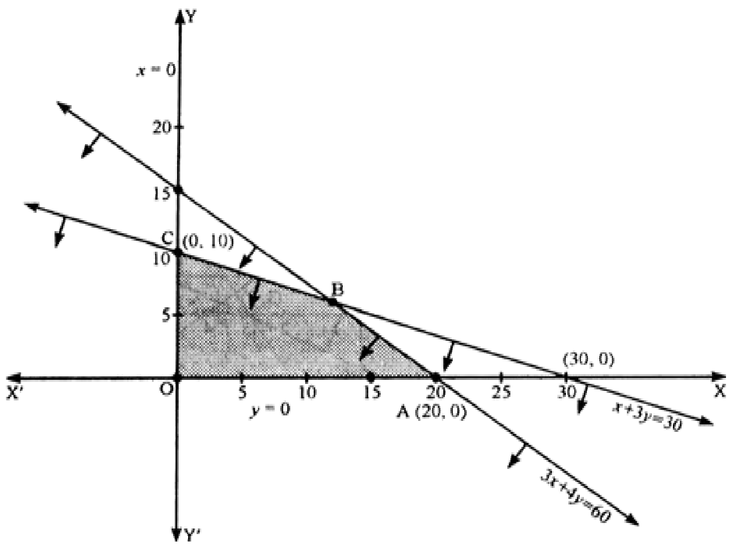
We have the following inequalities

$3x+4y \leqslant 60$ ----(i)

$x+3y \leqslant 30$ ---------(ii)

$x \geqslant 0$ ---------------(iii)

$y \geqslant 0$ ----------------(iv)



Take inequality (i)

$3x+4y \leqslant 60$

In equation form it can be written as $3x+4y=60$ make table and plot graph. Similarly for line $x+3y \leqslant 30$. $x \geqslant 0 \geqslant 0$ plot graph.

Here the common shaded region represent the solution region for system of inequalities

OR

Let x litres of 3% solution be added to 460 litres of 9% solution of acid

∴ Total quantity of mixture=(460+x)litres

Total acid content in the (460+x)litres of mixture = $(460x \frac{9}{100}+x\frac{3}{100})$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid

Therefore

5% of $(460+x)<460x \frac{9}{100}+x\frac{3}{100})<$ 7% of $(460+x)$

$=> \frac{5}{100}$ x $(460+x)<460x \frac{9}{100}+x\frac{3}{100} < \frac{7}{100}$ x $(460+x)$

$=>5 X (460+x)<460X 9+3x <7 x (460+x)$ [multiplying by 100]

$=>2300 +5x<4140+3x<3220+7x$

Taking first two inequalities ,

$2300 +5x<4140+3x$

$=>5x-3x<4140-2300=>x<920$-----------(i)

Taking last two inequalities

$4140+3x<3220+7x$ after solving we get $x>230$--------(ii)

Hence the number of litres of 3% solution of acid must be more tham 230 L and less than 920L

12	The given equation of circle is $4x^2+4y^2-12x-16y-21=0$ $\Rightarrow x^2+y^2-3x-4y-\frac{21}{4}=0$ $\Rightarrow\left(x^2-3x+\frac{9}{4}\right)+\left(y^2-4y+4\right)-\frac{9}{4}-4-\frac{21}{4}=0$ $\Rightarrow\left(x-\frac{3}{2}\right)^2+(y-2)^2-\frac{15}{2}-4=0$ $\Rightarrow\left(x-\frac{3}{2}\right)^2+(y-2)^2=\left(\sqrt{\frac{23}{2}}\right)^2$ therefore Centre=$(\frac{3}{2}$,2) and radius $r_1=\sqrt{\frac{23}{2}}$ Let 'r_2' be the radius of the concentric circle therefore its equation will be $\left(x-\frac{3}{2}\right)^2+(y-2)^2=r_2^2$ also $\frac{1}{2}\pi r_1^2=\pi r_2^2$ $\Rightarrow\pi r_1^2=2\pi r_2^2$ $\Rightarrow\frac{23}{2}=2\pi r_2^2$ $\Rightarrow r_2^2=\frac{23}{4}$ $\Rightarrow r_2=\frac{\sqrt{23}}{2}$ therefore $\left(x-\frac{3}{2}\right)^2+(y-2)^2=\frac{23}{4}$ $\Rightarrow x^2-3x+\frac{9}{4}+y^2-4y+4-\frac{7}{2}=\frac{23}{4}$ $\Rightarrow x^2+y^2-3x-4y+4-\frac{7}{2}=0$ $\Rightarrow 2x^2+2y^2-6x-8y+4+1=0$ is the required equation of the circle.
13	**let f(x)=tan** $\sqrt{x}$ f'(x)= $\lim_{h\to 0}\frac{\frac{\sin\sqrt{(x+h)}}{\cos\sqrt{(x+h)}}-\frac{\sin\sqrt{x}}{\cos\sqrt{x}}}{h}$ = $\lim_{h\to 0}\frac{\sin\sqrt{(x+h)}\cos\sqrt{x}-\sin\sqrt{x}\cos\sqrt{(x+h)}}{h\cos\sqrt{(x+h)}\cos\sqrt{x}}$ $\lim_{h\to 0}\frac{\sin\left(\sqrt{(x+h)}-\sqrt{x}\right)}{h\cos\sqrt{x}\cos\sqrt{(x+h)}}\times\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{\sqrt{x}}}$ = $\lim_{h\to 0}\frac{1}{\cos\sqrt{x}\cos\sqrt{(x+h)}}\times\lim_{h\to 0}\frac{\sin\left(\sqrt{(x+h)}-\sqrt{x}\right)}{\sqrt{x+h}-\sqrt{x}}\times\lim_{h\to 0}\frac{\left(\sqrt{(x+h)}-\sqrt{x}\right)}{h}$ $=sec^2\frac{\sqrt{x}}{2\sqrt{x}}$

14	**i) P(complex or very complex)** **=P(** E_1 **or** E_2 **)** **=P** $P(E_1 \cup E_2)$ = $P(E_1)$ + $P(E_2)$ **-** $P(E_1 \cap E_2)$ **=0.15+0.20-0=0.35** **(ii) P(neither very complex nor very simple)** = $P(E'_1 \cap' E_5)$ = $P(E_1 \cup E_5)'$ **=1-** $P(E_1 \cup E_5)$ **= 1-[** $P(E_1)$ + $P(E_5)$ **]** **=1-(0.15+0.08)** **=1-0.23** **=0.77**