Directorate of Education, GNCT of Delhi

Solution of Practice Paper

Term -II (2021-22)

Class – XII

Mathematics (Code: 041)

Q. No.	VALUE POINTS
	SECTION – A
1	Let I=$\int\left(\frac{1+x}{1+x^2}\right)dx=\int\left(\frac{1}{1+x^2}\right)dx+\int\left(\frac{x}{1+x^2}\right)dx$-------(1) $\int\left(\frac{x}{1+x^2}\right)dx=\tan^{-1}x$[formula] to evaluate $\int\left(\frac{x}{1+x^2}\right)dx$ put 1+x^2=t on differentiating both sides we get 2xdx=dt or xdx=$\frac{1}{2}dt$ $\int\left(\frac{x}{1+x^2}\right)dx=\int\left(\frac{1}{2}\frac{dt}{t}\right)=\frac{1}{2}\log\|t\|=\frac{1}{2}\log\|1+x^2$ $=\frac{1}{2}\log 1+x^2$ Putting these values in equation(1)we have I=$\tan^{-1}x+\frac{1}{2}\log 1+x^2$+c OR SOL-$\int\left(\frac{3x-6+6-1}{(x-2)^2}\right)dx=\int\left(\frac{3(x-2)+5}{(x-2)^2}\right)dx$=3log|x-2||+5$\int(x-2)^{-2}dx$ =3log|x-2|+5$\frac{(x-2)^{-1}}{-1}$+c=3log|x-2|-$\frac{5}{x-2}$+c
2	Given differential equation is $\frac{d}{dx}\{\left(\frac{dy}{dx}\right)^3\}=0$ $=>3\left(\frac{dy}{dx}\right)^{3-1}\frac{d}{dx}\left(\frac{dy}{dx}\right)=0$ $=>3\left(\frac{dy}{dx}\right)^2\frac{d^2y}{dx^2}=0$ clearly the highest order derivative occurring in the differential equation is $\frac{d^2y}{dx^2}$so its order is 2. Also it is a polynomial equation in derivative and the highest power raised to is $\frac{d^2y}{dx^2}$one so its degree is one . Hence the sum of the order and degree of the above given differential equation is 2+1=3

3

Sol-Let $\hat{a}$and $\hat{b}$be unit vectors such that $\hat{a}+\hat{b}$is also a unit vector

$|\hat{a}|=|\hat{b}|=|\hat{a}+\hat{b}|$----------------(1)

we know that $\left|\hat{a}+\hat{b}\right|^2=\left(\hat{a}+\hat{b}\right)^2=(\hat{a})^2+\left(\hat{b}\right)^2+2\,\hat{a}\,\hat{b}$-----------(2)

$\left|\hat{a}-\hat{b}\right|^2=\left(\hat{a}-\hat{b}\right)^2$=-$(\hat{a})^2+\left(\hat{b}\right)^2-2\,\hat{a}\,\hat{b}$-------(3)

adding equations (2) and (3) we have

$\left|\hat{a}+\hat{b}\right|^2+\left|\hat{a}-\hat{b}\right|^2=2(\hat{a})^2+2\left(\hat{b}\right)^2=|\hat{a}|^2+2\left|\hat{b}\right|^2$

putting the values of $|\hat{a}|\,\left|\hat{b}\right|\,\left|\hat{a}+\hat{b}\right|$(each=1)

$1^2+\left|\hat{a}-\hat{b}\right|^2=2|\hat{a}|^2+2\left|\hat{b}\right|^2=2(1)+2(1)$

$\left|\hat{a}-\hat{b}\right|^2=3 \Rightarrow \left|\hat{a}-\hat{b}\right|=\sqrt{3}$

4

$\frac{8}{\sqrt{89}},\frac{-4}{\sqrt{89}},\frac{3}{\sqrt{89}}$

5

Sol-

Three balls are drawn one by one without replacement from a bag containing 5 whit and 4 green balls .

Let X denote number of green balls (out of three green balls drawn)

=> X=0,1,2 and 3 only(and not upto 4 , the number of green balls in the bag

P(X=0)= Probability of getting green no green balls in the three draws i.e., all the three white balls =P(WWW)=$\frac{5}{9}x\frac{4}{8}x\frac{3}{7}=\frac{60}{504}=\frac{5}{42}$

P(X=1)= Probability of getting one green balls (and hence two white balls) in three draws

=P(GWW)+P(WGW)+P(WWG)=$\frac{4}{9}x\frac{5}{8}x\frac{4}{7}+\frac{5}{9}x\frac{4}{8}x\frac{4}{7}+\frac{5}{9}x\frac{4}{8}x\frac{4}{7}$

$=\frac{240}{504}=\frac{20}{42}$

similarly P(X=2)= Probability of getting two green balls (and hence one white ball) in three draws=$\frac{15}{42}$

and similarly P(X=3)= Probability of getting all the three green balls =

P(GGG)=$\frac{4}{9}x\frac{3}{8}x\frac{2}{7}=\frac{2}{42}$

Hence probability distribution of X is

X	0	1	2	3
P(X)	$\frac{5}{42}$	$\frac{20}{42}$	$\frac{15}{42}$	$\frac{15}{42}$

6

Solution- There are 26 red cards and 26 black cards in a pack of 52 playing cards .

Let R and B denote the event of drawing red card and black card respectively

Now required probability=P(first card is red and second one is black)+P(first card is black and second one is red)

=P(R)+$P\left(\frac{B}{R}\right)$+P(B)$P\left(\frac{R}{B}\right)$

$=\frac{26}{52}\text{x}\frac{26}{51}+\frac{26}{52}\text{x}\frac{26}{51}$

$=\frac{26}{51}$

SECTION – B

7	Let I=$\int\left(\frac{dx}{1+x+x^2+x^3}\right)=\int\left(\frac{dx}{(1+x)+x^2(1+x)}\right)=\int\left(\frac{dx}{(1+x)(1+x^2)}\right)$ let$\left(\frac{dx}{(1+x)(1+x^2)}\right)=\frac{A}{1+X}+\frac{BX+C}{1+x^2}$--------------(1) Multiply both sides by L.C.M $(1+x)(1+x^2)$we get 1= A$(1+x^2)$+(BX+C) $(1+x)$ OR 1=A+Ax^2+Bx+B x^2+C+Cx Equating coefficients of x^2,x and constant term ,we get A+B=0 B+C=0 Putting values A+C=1 Solving A,B,C we get A=$\frac{1}{2}$,C=$\frac{1}{2}$,B=-A=$\frac{-1}{2}$ Putting values of A,B,C in (1) we get $\left(\frac{1}{(1+x)(1+x^2)}\right)=\frac{\frac{1}{2}}{1+X}+\frac{\left(\frac{-1}{2}x+\frac{1}{2}\right)}{1+X^2}$ $=\frac{1}{2}\frac{1}{1+X}-\frac{1}{2}\frac{x}{1+x^2}+\frac{1}{2}\frac{1}{1+x^2}$ Hence I=$\frac{1}{2}\log\lvert(1+x)\rvert-\frac{1}{4}\log(1+x^2)+\frac{1}{2}\tan^{-1}x$+c
8	The given differential equation is $xd\,y$ −ydx= $\sqrt{x^2+y^2}$dx dividing by dx x$\frac{dy}{dx}$-y= =>x$\frac{dy}{dx}$=y+$\sqrt{x^2+y^2}$=>$\frac{dy}{dx}-\frac{y}{x}=\frac{\sqrt{x^2+y^2}}{x}$ Put y=vx=>$\frac{dy}{dx}$=v+x$\frac{dv}{dx}$ Therefore v+x$\frac{dv}{dx}-\frac{vx}{x}=\frac{\sqrt{x^2+v^2x^2}}{x}$=>v+x$\frac{dv}{dx}$-v=$\sqrt{1+v^2}$ =>$\int\left(\frac{dv}{\sqrt{1+v^2}}\right)=\int\left(\frac{dx}{x}\right)$=>log |v+$\sqrt{1+v^2}$|=log|x|+log|c|=>loglog$\left\lvert\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right\rvert$=log|Cx|=>$\frac{y}{x}+\sqrt{\left(\frac{x^2+y^2x^2}{x^2}\right)}$ =Cx=>y+$\sqrt{x^2+y^2}$=Cx^2 **OR** The given differential equation is $\frac{dY}{dx}$-3ycotx=sin2x given that y=2 when x=$\frac{\pi}{2}$ Compairing with $\frac{dY}{dx}$+Py=Q we have P=3cotx and Q=sin2x

	$\int Pdx$ $=-3\int cotxdx=-3\log sinx=\log(sinx)^{-3}$ IF$==\int e^{pdx}=e^{\log(sinx)^{-3}}=(sinx)^{-3}=\frac{1}{\sin^3 x}$ The general solution is y(IF.)=$\int Q(IF)$dx+C or $y\frac{1}{\sin^3 x}=\int \sin 2x\frac{1}{\sin^3 x}$dx+C $\frac{y}{\sin^3 x}=\int\frac{(2\,sinxcosx)}{\sin^3 x}$dx+C $=2\int\frac{cosx}{\sin^2 x}$dx+C$=2\int\frac{cosx}{sinxcosx}$dx+C$=\int 2\,cosecx\,cotx$dx=-2cosecx+C or $\frac{y}{\sin^3 x}=\frac{-2}{sinx}$+C $y=\sin^3 x$ multiplying every term by LCM=$\sin^3 x$ $y=-2\sin^2 x+c\sin^3 x$To find C putting y=2 when x=$\frac{\pi}{2}$(given in (1) $2=-2\sin^2\frac{\pi}{2}+c\sin^3\frac{\pi}{2}$ or 2=-2+c or c=4 puttung c=4 the required particular solution is $y=-2\sin^2 x+4\sin^3 x$
9	$\vec{a}X\vec{b}=\vec{c}$=>$\vec{c}\perp\vec{a}$and$\vec{c}\perp\vec{b}$ (By def of cross product) ----------(1) similarly $\vec{b}\,x\,\vec{c}=\vec{a}$=>$\vec{a}\perp\vec{b}$and $\vec{a}\perp\vec{c}$-----------(2) from (1) and (2) we have$\vec{a},\vec{b},\vec{c}$are mutually at right angles. Now $\vec{a}\,x\,\vec{b}=\vec{c}$(given)=>$\|\vec{a}\,x\,\vec{b}\|=\|\vec{c}\|$ Using (2)$\|\vec{a}\|\,\|\vec{b}\|\sin 90^0=\|\vec{c}\|$ $\|\vec{a}\|\,\|\vec{b}\|=\|\vec{c}\|$-------(3) similarly$\vec{b}\,x\,\vec{c}=\vec{a}$=>$\|\vec{b}\,x\,\vec{c}\|=\|\vec{a}\|$ =>$\|\vec{b}\|\,\|\vec{c}\|=\|\vec{a}\|$----(4) Dividing (3) by(4) (to eliminate $\|\vec{b}\|$) we have $\frac{\|\vec{a}\|}{\|\vec{c}\|}=\frac{\|\vec{c}\|}{\|\vec{a}\|}$=>$\|\vec{a}\|^2=\|\vec{c}\|^2$therefore $\|\vec{a}\|=\|\vec{c}\|$------(5) dividing (3) by (5) $\|\vec{b}\|$=1 putting it in (4) we have $\|\vec{c}\|=\|\vec{a}\|$
10	Solution-Here $\vec{b_1}=\hat{i}+2\hat{j}-2\hat{k}$and $\vec{b_2}=2\hat{i}+4\hat{j}-4\hat{k}=2(\hat{i}+2\hat{j}-2\hat{k})$ $=2\vec{b_1}$Therefore , the lines are parallel now using the formula for distance between parallel lines = $\frac{\|(\vec{a_2}-\vec{a_1})\times\vec{b}\|}{\|\vec{b}\|}$ Shortest distance is 2√5 **OR** Sol- Given point is $\vec{a_1}$=(1 ,2 ,-4) =$\hat{i}+2\hat{j}-4\hat{k}$ we know that vector along the line $\vec{r}$= $\hat{i}+2\hat{j}-4\hat{k}=\lambda(2\hat{i}+3\hat{j}+6\hat{k})$is $\vec{b_1}=(2\hat{i}+3\hat{j}+6\hat{k})$and a vector along the line $\vec{b_2}=(\hat{i}+\hat{j}-\hat{k})$ we know vector equation of the plane passing through point $\vec{a}$and parallel to the two given lines is ($\vec{r}$ -$\vec{a_1}$).$\vec{n}$=0 where $\vec{n}=\vec{b_1}$x$\vec{b_2}$------------------(1) now $\vec{n}=\vec{b_1}$x$\vec{b_2}$

$$=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$$

$=\hat{i}(-3-6)-\hat{j}(-2-6)+\hat{k}(2-3)=-9\hat{i}+8\hat{j}-\hat{k}$

from equation (1)$\vec{r}.\vec{n}-\vec{a}_1.\vec{n}=0 \Rightarrow \vec{r}.\vec{n}=\vec{a}_1.\vec{n}$-------(2)

putting values of $\vec{a}_1$ and $\vec{n}$ in (2) equatoin of required plane is

i.e.,$\vec{r}.(-9\hat{i}+8\hat{j}-\hat{k})=(\hat{i}+2\hat{j}-4\hat{k}).(-9\hat{i}+8\hat{j}-\hat{k})=-9+16+4$

or $\vec{r}.(-9\hat{i}+8\hat{j}-\hat{k})=11$ **(Vector form of the plane)**

or$(x\hat{i}+y\hat{j}+z\hat{k}).(-9\hat{i}+8\hat{j}-\hat{k})=11$

or 9x+8y-z=11

or 9x+8y-z-11=0 **(Cartesian form of the plane)**

SECTION – C

11 Let $I=\int_{-1}^{2}|x^3-x|dx$

Again let f(x) $=x^3-x=x(x^2-1)=x(x-1)(x+1)$

Now break the limit at x=0,1 (because on putting f(x)=0 we get x=0,1,-1)

It is clear that $x^3-x\geqslant 0$ on[-1,0]

$x^3-x\leqslant 0$ on[0,1]

$x^3-x\geqslant 0$ on[1,2]

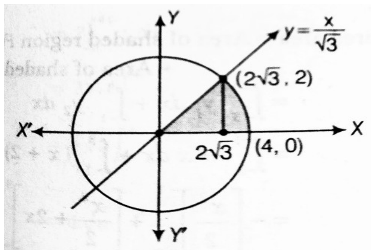
Hence the interval of the integral can be subdivided as

$$\int_{-1}^{2}|x^3-x|dx=\int_{-1}^{0}(x^3-x)dx-\int_{0}^{1}(x^3-x)dx+\int_{1}^{2}(x^3-x)dx$$

$$==\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x-x^3)dx+\int_{1}^{2}(x^3-x)dx=\frac{11}{4}$$

12 Given equation of the circle is $x^2+y^2=16$ and $\sqrt{3}y$=x, represents a line through origin.

The line y=$\frac{1}{\sqrt{3}}x$ intersect the circle .



Therefore $x^2+\frac{x^2}{3}=16$

$\frac{3x^2+x^2}{3}=16 \Rightarrow 4x^2=48$

$\Rightarrow x^2=12 \Rightarrow$ x=$\pm 2\sqrt{3}$

When x=$2\sqrt{3}$ then y=$\frac{2\sqrt{3}}{\sqrt{3}}$=2

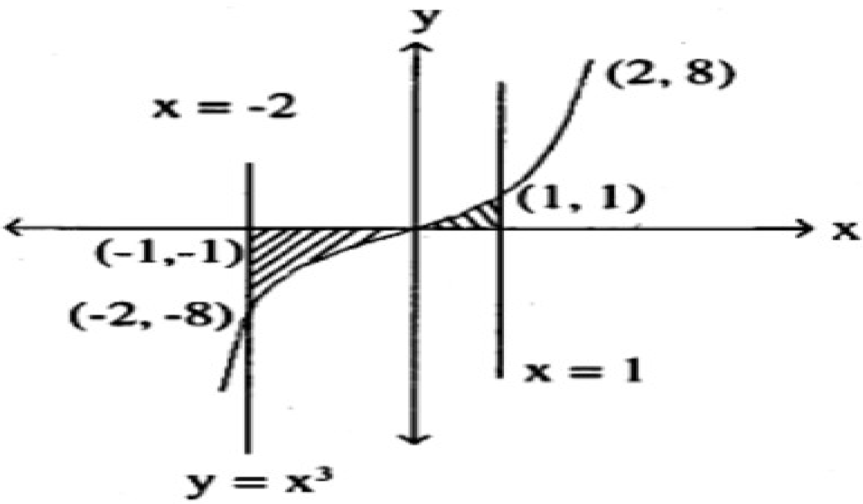
Required area shaded in first quadrant =(Area under the line y=$\frac{1}{\sqrt{3}}x$ from x=0 to $2\sqrt{3}$) +

(Area under the circle from x=$2\sqrt{3}$to x=4

$$=\int_{0}^{2\sqrt{3}}\frac{1}{\sqrt{3}}x\,dx+\int_{2\sqrt{3}}^{4}\sqrt{16-x^2}\,dx$$

after solving it we get required area =$\frac{4\pi}{3}$sq units

OR



Required area (shaded)
= Area under curve y = x^3 with respect to 'x' axis from x = -2 to x = 1

$$= \int_{-2}^{1}|x^3|\, dx = \left|\int_{2}^{0}|x^3|dx\right| + \int_{0}^{1}|x^3|dx$$

$$= \left|\int_{-2}^{0}-x^3dx\right| + \int_{0}^{1}x^3dx$$

As cube a value below 0 is negative

$$= \left.\frac{-x^4}{4}\right]_{-2}^{0} + \left.\frac{x^4}{4}\right]_{0}^{1} = -\left[0-\frac{(-2)^4}{4}\right]+\left[\frac{1}{4}-\frac{0}{4}\right]$$

$$= -\left[\frac{-16}{4}\right]+\frac{1}{4} = \frac{16}{4}+\frac{1}{4} = \frac{17}{4} \text{ sq.units}$$

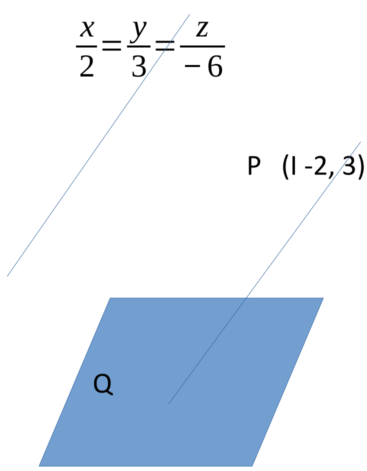
13

We know that d.r.'s of the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ are is denominators 2 ,3 ,-6

therefore d.r.'s of any line parallel to it are also 2 ,3 ,-6 therefore equation of the line through P (1 ,-2 ,3) and parallel to the given line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ are

$\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ $\qquad$ $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}(=\lambda)$--------(1)

let this line meet the given plane x-y+z=5 in the point Q say) from equation (1) x-1=2λ y+2=3 λ,z-3=-6λ=>x=2λ+1 ,y=3λ-2
z=-6λ+3
therefore point Q is Q(2λ+1 , 3λ-2 , -6λ+3) for some real λ
But this point Q lies on the plane x-y+z=5 therefore
(2λ+1)-(3λ-2)(-6λ+3)=
2λ+1-3λ+2-6λ+3 =5=>-7λ=-1=>$\lambda=\frac{1}{7}$



putting $\lambda=\frac{1}{7}$ in (3) coordinates of point Q are ($\frac{9}{7}$, $\frac{-11}{7}$ $\frac{15}{7}$)

Required distance is PQ

$$\sqrt{\left(\frac{9}{7}-1\right)^2+\left(\frac{-11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2}=\mathbf{1}$$

$$\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{-6}{7}\right)^2}$$

	$\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=\sqrt{\frac{49}{49}}$ =**1**
14	(a)0.9 2. (b)0.083