CBSE Questions Bank

Year- 2025

Class – XII

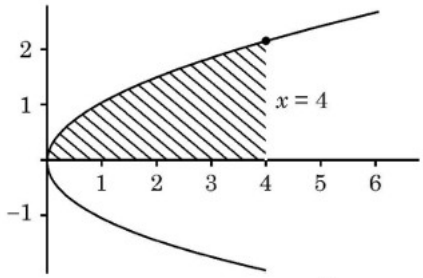
SUBJECT -Mathematics (Code : 041)

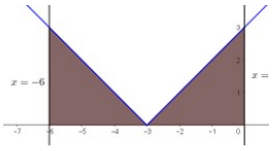
	CHAPTER Relation and Function/Continuity and Differentiability	Main/ Compartment.
	One Mark	
1	If $f(x) = \|x\| + \|x-1\|$, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	**Main 2025**
Ans	(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.	
2	**Assertion (A)** : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective. **Reason (R)** : A function is a bijective if it is both surjective and injective. **Assertion (A)** : $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$. **Reason (R)** : For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.	**CBSE MAIN 2025**
Ans	(D) Assertion (A) is false, but Reason (R) is true.	
3	Consider the function $f : R \to R$, defined as $f(x) = x^3$. *Assertion (A) :* f(x) is a one-one function. *Reason (R) :* f(x) is a one-one function. if co-domain = range.	**Compartment 2025**
Ans	(C) Assertion (A) is true, but Reason (R) is false.	
4	*Assertion (A) :* f(x) = [x], $x \in \mathbb{R}$, the greatest integer function is not differentiable at x = 2. *Reason (R) :* The greatest integer function is not continuous at any integral value.	**Compartment 2025**

Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	
	4 Marks **Case Study**	
1.	A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} : R_1 = {(2, 3), (3, 2)} R_2 = {(1, 2), (1, 3), (3, 2)} R_3 = {(1, 2), (2, 1), (1, 1)} R_4 = {(1, 1), (1, 2), (3, 3), (2, 2)} R_5 = {(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)} The students are asked to answer the following questions about the above relations : (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. **OR** (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?	**MAIN 2025**
Ans	$(i)\ R_4$ $(ii)\ R_5$ $(iii)(a)\ R_1$ and R_3 **OR** $(iii)(b)$ Required pairs to be added to make the relation R_2 as an equivalence relation are: $(1,1),(2,2),(3,3),(2,1),(3,1)$ and $(2,3)$	**1** **1** **1+1** **2**
	Chapter – 3 &4 **Matrices & Determinant**	
	1 Mark Question	**Main/ Compartment.**
1.	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is (A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**Main 2025**

Ans	(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
2	If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to : (A) 16 (B) 64 (C) 256 (D) 512	**CBSE Main 2025**
Sol	**(B) 64**	
3.	Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, C = [9 8 7], which of the following is defined ? (A) Only AB (B) Only AC (C) Only BA (D) All AB, AC and BA	
Ans.	**(A) Only AB**	
4	If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to (A) 0 (B) 1 (C) 7 (D) ± 7	**Delhi Main 2025**
Ans	**(B) 1**	
5	If $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$, then the value of $\lvert A \text{ adj } (A) \rvert$ is : (A) -1 (B) 1 (C) 2 (D) 3	**Delhi Main 2025**
Sol	**(A) -1**	
6	If A and B are invertible matrices, then which of the following is <u>not</u> correct ? (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$ (C) adj (A) = | A | A^{-1} (D) $\lvert A \rvert^{-1} = \lvert A^{-1} \rvert$	**Delhi Main 2025**

Ans	(A) $(A+B)^{-1} = B^{-1} + A^{-1}$	
7	Let A be a matrix of order m × n and B is a matrix such that A^TB and BA^T are defined. Then, the order of B is : (A) m × m (B) n × n (C) m × n (D) n × m	**CBSE MAIN 2025**
Sol	**(D) n X m**	
8	If A is a square matrix of order 3 such that det(A) = 9, then det(9 A^{-1}) is equal to (A) 9 (B) 9^2 (C) 9^3 (D) 9^4	**CBSE MAIN 2025**
Sol	(B) 9^2	
9	If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x-y) = ?$ (A) –7 (B) –3 (C) 3 (D) 7	**Delhi Main 2025**
Sol	**(B) -3**	**1**
10	For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of (4A) is : (A) 4B (B) B (C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$	**Delhi Main 2025**
Sol	(D) $\frac{1}{16}B$	**1**
11	If X, Y and XY are matrices of order 2 × 3, m × n and 2 × 5 respectively, then number of elements in matrix Y is : (A) 6 (B) 10 (C) 15 (D) 35	**Compartment 2024**
Sol	(C) 15	
	Chapter 7	

	Integral	
	One Mark Questions	
1	**O** $\int_{-1}^{1} \frac{\lvert x \rvert}{x}\, dx,\ x \neq 0$ is equal to (A) -1 (B) 0 (C) 1 (D) 2	**Main CBSE Delhi 2025**
Ans	(B) 0	
2	If $\int \frac{2^{\frac{1}{x}}}{x^2}\, dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to (A) $\frac{-1}{\log 2}$ (B) $-\log 2$ (C) -1 (D) $\frac{1}{2}$	**Main CBSE Delhi 2025**
Ans	(A) $\frac{-1}{\log 2}$	
3	$\int \frac{1-2\sin x}{\cos^2 x}\, dx$ is equal to : (A) $\tan x - 2\sec x + C$ (B) $-\tan x + 2\sec x + C$ (C) $-\tan x - 2\sec x + C$ (D) $\tan x + 2\sec x + C$	**Main CBSE Delhi 2025**
Ans	(A) $\tan x - 2\sec x + C$	
4	The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by  (A) $\int_0^4 x\, dx$ (B) $\int_0^2 y^2\, dy$ (C) $2\int_0^4 \sqrt{x}\, dx$ (D) $\int_0^4 \sqrt{x}\, dx$	**Main CBSE Delhi 2025**

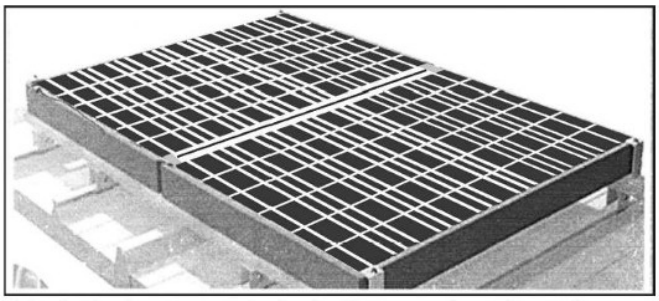
Ans	(D) $\int_0^4 \sqrt{x}\ dx$	
	3 marks Questions	
1	(a) Find : $\int \frac{\cos 2x}{(\sin x + \cos x)^2}\ dx$ **OR** (b) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\ dx$	**Main CBSE Delhi 2025**
Ans(a)	$\int \frac{\cos 2x}{(\sin x+\cos x)^2} dx = \int \frac{\cos^2 x - \sin^2 x}{(\sin x+\cos x)^2} dx$ $= \int \frac{\cos x - \sin x}{\sin x + \cos x} dx$ $= \log\lvert \sin x + \cos x \rvert + C$	**1** **1** **1**
	OR	
Ans(b)	$I = \int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x} dx$ -- (i) $I = \int_0^{\pi/2} \frac{5\sin\left(\frac{\pi}{2}-x\right)+3\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx = \int_0^{\pi/2} \frac{5\cos x + 3\sin x}{\cos x + \sin x} dx$ -- (ii) Adding (i) and (ii), we get $2I = \int_0^{\pi/2} 8dx \Rightarrow I = 4x\Big]_0^{\frac{\pi}{2}} = 2\pi$	**1½** **1½**
2	Sketch the graph of y = | x + 3 | and find the area of the region enclosed by the curve, x-axis, between x = –6 and x = 0, using integration.	
Ans	 **Required Area** $= \int_{-6}^{0} y\,dx$ $= 2\int_{-3}^{0} (x+3)\,dx$ $= 2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$ $= 9$	**For correct graph: 1 mark** **½** **½** **½** **½**

3	(a) Find : $\int \frac{x+\sin x}{1+\cos x}\,dx$ **OR** (b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$	**Main CBSE Delhi 2025**
Ans (a)	$\int \frac{x+\sin x}{1+\cos x}dx$ $= \int \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$ $= \int x\left(\frac{1}{2}\sec^2\frac{x}{2}\right)dx + \int \tan\frac{x}{2}dx$ $= x\tan\frac{x}{2} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx$ $= x\tan\frac{x}{2} + C$	**1** **½** **1** **½**
	OR	
Ans(b)	$\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$ $= \frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^4 x\sqrt{\tan x}}$ $= \frac{1}{2}\int_0^{\pi/4} \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}dx$ Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$ $\therefore I = \frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}}dt$ $= \frac{1}{2}\int_0^1 \left(\frac{1}{\sqrt{t}} + t^{3/2}\right)dt$ $= \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_0^1$ $= \frac{6}{5}$	**½** **½** **½** **1** **½**

	Chapter 9 **DIFFERENTIAL EQUATION**	
1	Which of the following is <u>not</u> a homogeneous function of x and y ? (A) $y^2 - xy$ (B) $x - 3y$ (C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$	**CBSE MAIN 2025**
Ans	**tan x- secy**	
2	The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is (A) xe^x (B) $\frac{e^x}{x}$ (C) $\frac{x}{e^x}$ (D) $xe^{\frac{1}{x}}$	**Main CBSE Delhi 2025**
Ans	(B) $\frac{e^x}{x}$	
3	The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is (A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$ (C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$	**Main CBSE Delhi 2025**
Ans	(B) $\frac{1}{\sqrt{y}}$	
	Probability	
	One Mark Question	
1.	If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to : (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$	

Ans	(C) $\frac{2}{3}$	
	3 marks Questions	
1	(a) The probability distribution for the number of students being absent in a class on a Saturday is as follows : X: 0, 2, 4, 5 P(X): p, 2p, 3p, p Where X is the number of students absent. (i) Calculate p. (ii) Calculate the mean of the number of absent students on Saturday. **OR** (b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.	
Ans (a)	$(i)\ \text{Since}\sum P(X)=1\Rightarrow p+2p+3p+p=1$	½
	$\Rightarrow p=\frac{1}{7}$	½
	$(ii)\ \text{Mean}=\sum X.P(X)=0(p)+2(2p)+4(3p)+5(p)$	1
	$=21p=21\left(\frac{1}{7}\right)=3$	1
	OR	
Ans (b)	Let E_1 : The applicant is a male E_2 : The applicant is a female A : The candidate chosen will have distinction in the written test.	½
	$P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A\mid E_1)=0.4, P(A\mid E_2)=0.35$	1
	$\therefore P(A)=P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)$	
	$=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$	1
	$=\frac{11}{30}$	½

	Four Marks Question	
1	 ɔank offers loan to its customers on different types of interest namely, ɛd rate, floating rate and variable rate. From the past data with the ık, it is known that a customer avails loan on fixed rate, floating rate variable rate with probabilities 10%, 20% and 70% respectively. A ɪtomer after availing loan can pay the loan or default on loan ɔayment. The bank data suggests that the probability that a person ʼaults on loan after availing it at fixed rate, floating rate and variable e is 5%, 3% and 1% respectively. sed on the above information, answer the following : What is the probability that a customer after availing the loan will default on the loan repayment ? A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ?	
Ans	E_1 **:customer avails loan on fixed rate** E_2 **:customer avails loan on floating rate** E_3 **:customer avails loan on variable rate** **A: the person defaults on the loan** $P(E_1)=\frac{1}{10}, P(E_2)=\frac{2}{10}, P(E_3)=\frac{7}{10}$ $P(A\mid E_1)=\frac{5}{100}, P(A\mid E_2)=\frac{3}{100}, P(A\mid E_3)=\frac{1}{100}$ **1** $(i)\ P(A)=P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)$ **1** $=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$ $=\frac{18}{1000}$ **or** $\frac{9}{500}$ $(ii)\ P(E_3\mid A)=\frac{P(E_3).P(A\mid E_3)}{P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)}$ **1** $=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$ **1** $=\frac{7}{18}$	
	Differentiation and its Application	
	Two Mark	
1	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **OR** (b) If $\tan^{-1}(x^2+y^2)=a^2$, then find $\frac{dy}{dx}$.	

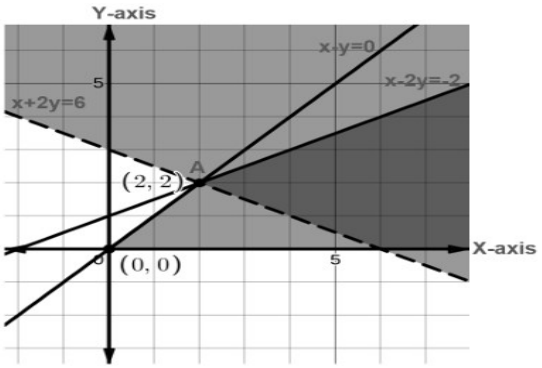
Ans (a)	**Let** $u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}(-2\cos x \sin x)\log 2$	**1**
	Let $v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$	½
	Now $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$	½
	OR	
Ans (b)	$\tan^{-1}(x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$	½
	Differentiate both sides wrt x,	
	$2x + 2y\frac{dy}{dx} = 0$	**1**
	$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	½
	Four Marks / Case Based	
1	 A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres. Based on this information, answer the following questions : (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. (ii) Write the area of the solar panel as a function of x. (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **OR** (iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.	

Ans	$(i)\ 2x+3y=300$	
	$(ii)\ A=xy=\frac{x}{3}(300-2x)$	1
	$(iii)(a)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	1
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$	½
	Also, $\frac{d^2A}{dx^2}=-\frac{4}{3}<0$. So, A is maximum at $x=75$	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\,m^2$	½
	OR	
	$(iii)(b)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$	½
	As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x=75$ from left to right, which means $x=75$ is the point of maximum.	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\,m^2$	½
	Note : Full credit to be given if the student takes equation as $2x+2y=300$ **or** $2x+4y=300$ **or** $4x+4y=300$ **or** $4x+3y=300$ **The solutions of sub-parts will differ and marks may be given accordingly.**	
	5 marks Question	
1	(a) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.	
	OR	
	(b) If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=\sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.	
Ans (a)	Let $x=\sin A,\ y=\sin B \Rightarrow A=\sin^{-1}x,\ B=\sin^{-1}y$	1
	$\therefore \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$	
	$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$	
	$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	1
	$\Rightarrow \cot\left(\frac{A-B}{2}\right)=a \Rightarrow A-B=2\cot^{-1}a$	1
	$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$	
	differentiate both sides wrt x,	½
	$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$	
	$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$	1½
	OR	

Ans (b)	$f(x)=2x^3-15x^2+36x+1$ $\Rightarrow f'(x)=6(x^2-5x+6)=6(x-2)(x-3)$ $f'(x)=0\Rightarrow x=2,3\in[1,5]$ Now $f(1)=24,\ f(2)=29,\ f(3)=28,\ f(5)=56$ Hence, the absolute maximum value is 56 and the absolute minimum value is 24.	
2	(a) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$. **OR** (b) If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=\sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.	
Ans (a)	Let $x=\sin A,\ y=\sin B\Rightarrow A=\sin^{-1}x,\ B=\sin^{-1}y$ $\therefore\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ $\Rightarrow\cos A+\cos B=a(\sin A-\sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow\cot\left(\frac{A-B}{2}\right)=a\Rightarrow A-B=2\cot^{-1}a$ $\Rightarrow\sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$ differentiate both sides wrt x, $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$ $\Rightarrow\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$	1 1 1 ½ 1½
Ans(b)	$x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ $\Rightarrow\frac{dx}{d\theta}=a\left(-\sin\theta+\frac{1}{\tan\frac{\theta}{2}}\times\sec^2\frac{\theta}{2}\times\frac{1}{2}\right)$ $=a\left(-\sin\theta+\frac{1}{\sin\theta}\right)=a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$ $\frac{dx}{d\theta}=a\cot\theta\cos\theta$ Also, $y=\sin\theta\Rightarrow\frac{dy}{d\theta}=\cos\theta$ $\therefore\frac{dy}{dx}=\frac{\tan\theta}{a}$ Differetiating wrt x, $\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$ $=\frac{\sec^3\theta\tan\theta}{a^2}$ $\left.\frac{d^2y}{dx^2}\right]_{\text{at }\theta=\frac{\pi}{4}}=\frac{2\sqrt{2}}{a^2}$	½ ½ ½ ½ 1 1 1

	Chapter Linear programming	
	One Mark Question	
1	The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If Z = 18x + 9y be the objective function, then (A) Z is maximum at (2, 72), minimum at (15, 20) (B) Z is maximum at (15, 20) minimum at (40, 15) (C) Z is maximum at (40, 15), minimum at (15, 20) (D) Z is maximum at (40, 15), minimum at (2, 72)	CBSE MAIN 2025
Ans	(C) Z is maximum at (40, 15), minimum at (15, 20)	
2	If the feasible region of a linear programming problem with objective function Z = ax + by, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.	CBSE MAIN 2025
Ans	(C) It will have both maximum and minimum values.	
3	In an LPP, corner points of the feasible region determined by the system of linear constraints are (1, 1), (3, 0) and (0, 3). If Z = ax + by, where a, b > 0 is to be minimized, the condition on a and b, so that the minimum of Z occurs at (3, 0) and (1, 1), will be : (A) a = 2b (B) $a = \frac{b}{2}$ (C) a = 3b (D) a = b	
Ans	(B) $a = \frac{b}{2}$	
	3 Marks Question	
1.	Solve the following linear programming problem graphically : Maximise Z = x + 2y Subject to the constraints : $x - y \geq 0$ $x - 2y \geq -2$ $x \geq 0, y \geq 0$	

Ans



Corner Point	Value of $Z = x + 2y$
$O(0,0)$	0
$A(2,2)$	6

Since feasible region is unbounded. Plot x + 2y > 6 which has common region with feasible region, thus Z has no maximum value.

For correct graph and shading $1^1/2$

For correct table 1

½

2.

Main Set

Solve the following linear programming problem graphically :

Maximise Z = 20x + 30y

Subject to the constraints :

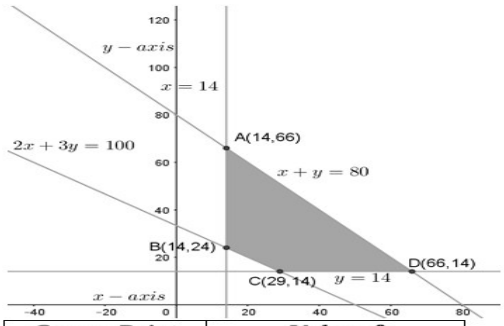
x + y ≤ 80

2x + 3y ≥ 100

x ≥ 14

y ≥ 14

Sol



Corner Point	**Value of $Z = 20x + 30y$**
$A(14,66)$	**2260**
$B(14,24)$	**1000**
$C(66,14)$	**1740**
$D(29,14)$	**1000**

Max(Z) = 2260

For correct graph and shading 1½

For correct table 1

½

3

Compartment 2025

Solve the following LPP graphically :

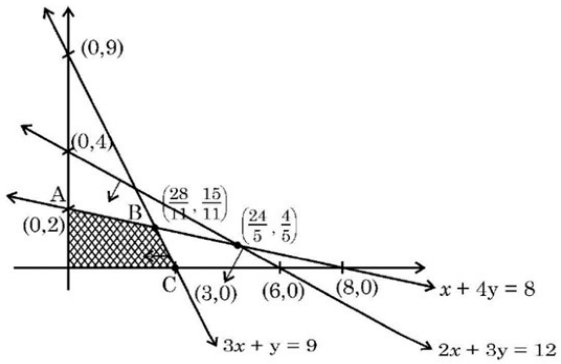
Maximize Z = 2x + 3y

subject to the constraints x + 4y ≤ 8

2x + 3y ≤ 12

3x + y ≤ 9

x > 0, y > 0

Sol		
	Chapter Vector and 3D	
	One Mark Question	
1	If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ? (A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$ (C) $\lvert \vec{b} \rvert > \lvert \vec{a} \rvert$ (D) $\lvert \vec{a} \rvert = \lvert \vec{b} \rvert$	**Delhi Mains 2025**
Ans	(B) $\vec{a} \perp \vec{b}$	
2	If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to : (A) -18 (B) 18 (C) $\frac{-34}{9}$ (D) $\frac{34}{9}$	**Delhi Main CBSE 2025**
Ans	(B) 18	
3.	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\lvert \vec{a} \rvert = \sqrt{37}$, $\lvert \vec{b} \rvert = 3$ and $\lvert \vec{c} \rvert = 4$, then angle between $\vec{b}$ and $\vec{c}$ is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	

Ans	(C) $\frac{\pi}{3}$	
4	Let A = $[a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = \ddot{j} - 2\ddot{i}$. Then which of the following is true ? (A) $a_{12} > 0$ (B) all $a_{ij} < 0$ (C) $a_{13} + a_{31} = -6$ (D) $a_{23} > a_{32}$	CBSE Main 2025
Ans	(D) $a_{23} > a_{32}$	
	Two Mark Question	
1	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	**Delhi Main CBSE 2025**
Ans	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ **Area of parallelogram** $=\frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$ $=\frac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\frac{\sqrt{62}}{2}$	**1** **1**
2	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	
Ans	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ **Area of parallelogram** $=\frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$ $=\frac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\frac{\sqrt{62}}{2}$	**1** **1**

1	(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. **OR** (b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.	
Ans (a)	**Let the required angle between the kite strings be θ.** **Then, $\cos\theta = \frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$** **$\Rightarrow \cos\theta = = \frac{(3\hat{i}+\hat{j}+2\hat{k})(2\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}$** **$\Rightarrow \theta = \cos^{-1}\left(\frac{12}{\sqrt{336}}\right)$ or $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$**	**1½** **½**
	OR	
Ans (b)	$\overrightarrow{BA} = -6\hat{i}+2\hat{j}+3\hat{k}$ **Required unit vector of magnitude 21** $= 21 \times \left(\frac{-6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{36+4+9}}\right)$ $= 3\left(-6\hat{i}+2\hat{j}+3\hat{k}\right)$ or $-18\hat{i}+6\hat{j}+9\hat{k}$	**1** **½** **½**
	3 Mark Question	
1	(a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. **OR** (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.	

Ans (a)

Rewriting the lines, we get

$\vec{r}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+\hat{j}-2\hat{k}\right)$ **and** $\vec{r}=\left(\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}-2\hat{k}\right)$ ½

Let $\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{a}_2=\hat{i}-\hat{j}-\hat{k}$, $\vec{b}_1=-\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$

Note that the dr'sof given lines are not proportional so, they are not parallel lines.

The lines will be skew if they do not intersect each other also.

Here $\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k}$, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&-2\\1&2&-2\end{vmatrix}=2\hat{i}-4\hat{j}-3\hat{k}$ ½ + ½

Consider $\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)$

$=\left(\hat{j}-4\hat{k}\right).\left(2\hat{i}-4\hat{j}-3\hat{k}\right)=8\neq0$ ½

Hence lines will not intersect. So the lines are skew.

Shortest Distance $=\dfrac{\left|\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ 1

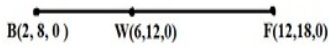
$=\dfrac{8}{\sqrt{4+16+9}}=\dfrac{8}{\sqrt{29}}$

OR

Ans (b)

Let the wicket keeper divides the line segment in ratio $k:1$

$\therefore\vec{W}=\dfrac{k\vec{F}+1.\vec{B}}{k+1}$ 1



$\Rightarrow 6\hat{i}+12\hat{j}=\left(\dfrac{12k+2}{k+1}\right)\hat{i}+\left(\dfrac{18k+8}{k+1}\right)\hat{j}$ 1

$\Rightarrow k=\dfrac{2}{3}$ 1

Hence, the required ratio is $2:3$

5 Mark question

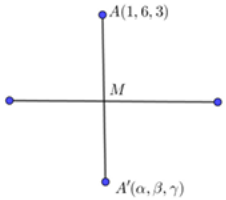
1

(a) Find the image A' of the point A(1, 6, 3) in the line $\dfrac{x}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{3}$. Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\dfrac{x+5}{1}=\dfrac{y+3}{4}=\dfrac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.

Ans(a)

	The equation of given line is $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$ Any arbitrary point on the line is $M(\lambda,2\lambda+1,3\lambda+2)$ dr'sof AM are $<\lambda-1,2\lambda-5,3\lambda-1>$ Here $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ $\Rightarrow\lambda=1$ $\therefore M(1,3,5)$ is the foot perpendicular of the point A to the given line. Let image of point A in the line be $A'(\alpha,\beta,\gamma)$ Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2},\frac{6+\beta}{2},\frac{3+\gamma}{2}\right)=M(1,3,5)$ $\Rightarrow A'(1,0,7)$ is the image of A. Also, Equation of AA' is $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$ 	1 1 ½ ½ 1 1
	OR	
Ans(b)	The given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ and $Q(2,4,-1)$ Any random point on the line will be given by $P(\lambda-5,4\lambda-3,-9\lambda+6)$ Since $PQ=7\Rightarrow\sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$ $\Rightarrow 98(\lambda^2-2\lambda+1)=0\Rightarrow\lambda=1$ Hence, the required point is $P(-4,1,-3)$ The equation of line PQ is $\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$ or $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$	1 1 1 1 1