DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2022-2023)

Class : X

MATHEMATICS

Under the Guidance of

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Message

Remembering the words of John Dewey, "Education is not preparation for life, education is life itself", I highly commend the sincere efforts of the officials and subject experts from Directorate of Education involved in the development of Support Material for classes IX to XII for the session 2022-23.

The Support Material is a comprehensive, yet concise learning support tool to strengthen the subject competencies of the students. I am sure that this will help our students in performing to the best of their abilities.

I am sure that the Heads of Schools and teachers will motivate the students to utilise this material and the students will make optimum use of this Support Material to enrich themselves.

I would like to congratulate the team of the Examination Branch along with all the Subject Experts for their incessant and diligent efforts in making this material so useful for students.

I extend my Best Wishes to all the students for success in their future endeavours.

(Ashok Kumar)

HIMANSHU GUPTA, IAS
Director, Education & Sports

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MESSAGE

"A good education is a foundation for a better future."
- Elizabeth Warren

Believing in this quote, Directorate of Education, GNCT of Delhi tries to fulfill its objective of providing quality education to all its students.

Keeping this aim in mind, every year support material is developed for the students of classes IX to XII. Our expert faculty members undertake the responsibility to review and update the Support Material incorporating the latest changes made by CBSE. This helps the students become familiar with the new approaches and methods, enabling them to become good at problem solving and critical thinking. This year too, I am positive that it will help our students to excel in academics.

The support material is the outcome of persistent and sincere efforts of our dedicated team of subject experts from the Directorate of Education. This Support Material has been especially prepared for the students. I believe its thoughtful and intelligent use will definitely lead to learning enhancement.

Lastly, I would like to applaud the entire team for their valuable contribution in making this Support Material so beneficial and practical for our students.

Best wishes to all the students for a bright future.

(HIMANSHU GUPTA)

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सत्यमेव जयते

संदेश

शिक्षा निदेशालय, दिल्ली सरकार का महत्वपूर्ण लक्ष्य अपने विद्यार्थियों का सर्वांगीण विकास करना है। इस उद्देश्य को ध्यान में रखते हुए शिक्षा निदेशालय ने अपने विद्यार्थियों को उच्च कोटि के शैक्षणिक मानकों के अनुरूप विद्यार्थियों के स्तरानुकूल सहायक सामग्री उपलब्ध कराने का प्रयास किया है। कोरोना काल के कठिनतम समय में भी शिक्षण अधिगम की प्रक्रिया को निर्बाध रूप से संचालित करने के लिए संबंधित समस्त अकादमिक समूहों और क्रियान्वित करने वाले शिक्षकों को हार्दिक बधाई देती हूँ।

प्रत्येक वर्ष की भाँति इस वर्ष भी कक्षा 9वीं से कक्षा 12वीं तक की सहायक सामग्रियों में सी.बी.एस.ई. के नवीनतम दिशा-निर्देशों के अनुसार पाठ्यक्रम में आवश्यक संशोधन किए गए हैं। साथ ही साथ मूल्यांकन से संबंधित आवश्यक निर्देश भी दिए गए हैं। इन सहायक सामग्रियों में कठिन से कठिन पाठ्य सामग्री को भी सरलतम रूप में प्रस्तुत किया गया है ताकि शिक्षा निदेशालय के विद्यार्थियों को इसका भरपूर लाभ मिल सके।

मुझे आशा है कि इन सहायक सामग्रियों के गहन और निरंतर अध्ययन के फलस्वरूप विद्यार्थियों में गुणात्मक शैक्षणिक संवर्धन का विस्तार उनके प्रदर्शनो में भी परिलक्षित होगा। इस उत्कृष्ट सहायक सामग्री को तैयार करने में शामिल सभी अधिकारियों तथा शिक्षकों को हार्दिक बधाई देती हूँ तथा सभी विद्यार्थियों को उनके उज्ज्वल भविष्य की शुभकामनाएं देती हूँ।

रीता शर्मा

(रीता शर्मा)

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1][**संपूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य**] बनाने के लिए, तथा उसके समस्त नागरिकों को :

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की **समता**

प्राप्त कराने के लिए,

तथा उन **सब में**

व्यक्ति की गरिमा और [2][**राष्ट्र की एकता और अखंडता**] सुनिश्चित करने वाली **बंधुता**

बढ़ाने के लिए

दृढ़संकल्प होकर **अपनी इस संविधान सभा में** आज तारीख 26 नवंबर, 1949 ई. को **एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।**

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) "प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य" के स्थान पर प्रतिस्थापित।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) "राष्ट्र की एकता" के स्थान पर प्रतिस्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

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1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51 क

मूल कर्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की संप्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण बनाए रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभावों से परे हो, ऐसी प्रथाओं का त्याग करे जो महिलाओं के सम्मान के विरुद्ध हों;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की, जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणिमात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे, जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई ऊँचाइयों को छू सके; और

(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य को शिक्षा के अवसर प्रदान करे।

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2022-2023)

MATHEMATICS
Class : X

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

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SESSION-(2022-2023)
CLASS-X
Subject: Mathematics (Code: 041 & 241)

Course Structure

Units	Unit Name	Marks
I	Number Systems	06
II	Algebra	20
III	Coordinate Geometry	06
IV	Geometry	15
V	Trigonometry	12
VI	Mensuration	10
VII	Statistics and Probability	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

I. REAL NUMBER

Fundamental Theorem of Arithmetic - statement after reviewing work done earlier and after illustrating and motivating through examples. Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

UNIT IV : GEOMETRY

1. TRIANGLES

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY

Trigonometic ratios of an actue angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation /depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES

Area of sectors and segments of a circle. Problems based on areas and perimeter/circumference of the above said plane figures.

In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/ cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY

Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-Standard

QUESTION PAPER DESIGN
CLASS-X (2022-23)

Time: 3 Hours — ***Max. Marks: 80***

S.No.	Typology of Questions	Total Marks	% Weightage (approx)
1.	**Remembering :** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding :** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2.	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3.	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating :** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating :** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	18	22
	TOTAL	**80**	**100**

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5 + 5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS - BASIC

QUESTION PAPER DESIGN
CLASS-X (2022-23)

Time: 3 Hours ***Max. Marks: 80***

S.No.	Typology of Questions	Total Marks	% Weightage (approx)
1.	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	60	75
2.	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	12	15
3.	**Analysing:** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	8	10
	Total	**80**	**100**

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5 + 5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

Content

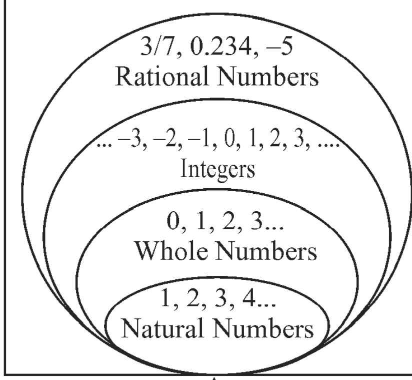
CHAPTER

1 Real Numbers

Decimal form of Real Numbers

RATIONAL NUMBERS

A number that can be expressed in the form of p/q where 'p' and 'q' are co-prime integers and $q \neq 0$, is called a rational number. Their decimal representation is either terminating or non-terminating and recurring (repeating).



IRRATIONAL NUMBERS

A number that cannot be expressed in the form p/q where 'p' and 'q' are co-prime integers, is called an irrational number. Their decimal representation is non-terminating and non-recurring (non-repeating).

(e.g. $\sqrt{3}, \sqrt{2}+\sqrt{5}, \pi, 0.102003102...$)

REAL NUMBERS

FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite number can be written as a product of primes in one and only one way, apart from the order in which the primes are written,

e.g. $286 = 2 \times 11 \times 13$

HCF USING PRIME FACTORIZATION

HCF is the product of the smallest power of each common prime factor of the given numbers,

e.g. HCF (240 and 360)

$240 = 2^4 \times 3 \times 5$

$360 = 2^3 \times 3^2 \times 5$

HCF = $\mathbf{2^3 \times 3 \times 5}$ = 120

LCM USING PRIME FACTORIZATION

LCM is the product of the greatest power of each prime factor of the given numbers.

e.g. LCM (240 and 360)

$240 = 2^4 \times 3 \times 5$

$360 = 2^3 \times 3^2 \times 5$

LCM = $\mathbf{2^4 \times 3^2 \times 5}$ = 720

RELATIONSHIP BETWEEN HCF AND LCM OF TWO NUMBERS

HCF is always a factor of the LCM of two numbers.

If 'a' and 'b' are two numbers, then

HCF (a, b) × LCM (a, b) = Product of 'a' and 'b'

e.g. For two numbers 24 and 36

Prime factorisation of 24 = $2^3 \times 3$ and 36 = $2^2 \times 3^2$

HCF (24, 36) = $2^2 \times 3 = 12$

LCM (24, 36) = $2^3 \times 3^2 = 72$

HCF × LCM = 12 × 72 = **864**

Product of 'a' and 'b' = 24 × 36 = **864**

Thus HCF **(a, b) ×** LCM **(a, b) = Product of 'a' and 'b'**

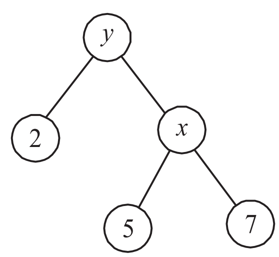
VERY SHORT ANSWER TYPE QUESTIONS

1. A number N when divided by 16 gives the remainder 5. _______ is the remainder when the same number is divided by 8.

2. HCF of $3^3 \times 5^4$ and $3^4 \times 5^2$ is ________ .

3. If $a = xy^2$ and $b = x^3y^5$ where x and y are prime numbers then LCM of (a, b) is ______ .

4. In the given factor tree find x and y



5. If n is a natural number, then $25^{2n} - 9^{2n}$ is always divisible by :

(i) 16 (ii) 34

(iii)both 16 or 34 (iv) None of these

6. Given HCF (2520, 6600) = 120 and LCM (2520, 6600) is 252k, then the value of 'k' is

(a) 165 (b) 550

(c) 990 (d) 1650

7. The product of HCF and LCM of the smallest prime number and the smallest composite number is

(a) 2 (b) 4

(c) 6 (d) 8

8. If the LCM of two numbers is 3600. then which of the following cannot be their HCF?

(a) 600 (b) 500

(c) 400 (d) 150

9. $p^n = (a \times 5)^n$ For p^n to end with the digit zero $a =$ _____ for natural number n.

(a) any natural number (b) even number

(b) odd number (d) none of these

10. HCF is always

(a) multiple of LCM (b) Factor of LCM

(c) divisible by LCM (d) (a) and (c) both

11. All decimal numbers are

(a) rational numbers (b) irrational numbers

(c) real numbers (d) integers

12. Which of these numbers always end with the digits 6.

(a) 4^n (b) 2^n (c) 6^n (d) 8^n

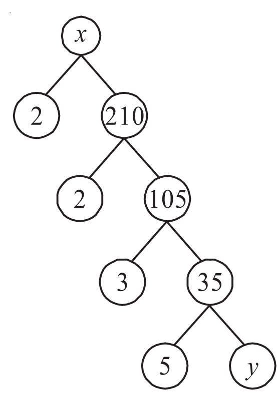
13. Write the prime factor of $2 \times 7 \times 11 \times 13 \times 17 + 21$

14. Write the form in which every odd integer can be written taking t as variable.

15. Find the least number which is divisible by all numbers from 1 to 10 (both inclusive).

16. The numbers 525 and 3000 are divisible by 3, 5, 15, 25 and 75. What is the HCF of 525 and 3000?

17. What is $x : y$ in the factor-tree?



SHORT ANSWER TYPE QUESTIONS-I

18. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

(NCERT Examplar)

19. What is the smallest number by which $\sqrt{5}-\sqrt{3}$ is to be multiplied to make it a rational number? Also find the number so obtained?

20. Find one rational number and one irrational number between $\sqrt{2}$ and $\sqrt{5}$.

21. If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of m.

(CBSE 2014)

22. Find the value of : $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$, where n is any positive odd integer. **(CBSE 2016)**

23. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times. **(CBSE 2016)**

SHORT ANSWER TYPE QUESTIONS-II

24. Express 2658 as a product of its prime factors.

25. If $7560 = 2^3 \times 3^p \times q \times 7$, find p and q.

26. Prove that $\sqrt{3}+\sqrt{5}$ is irrational number.

27. Prove that $5-\frac{3}{7}\sqrt{3}$ is an irrational number.

28. Prove that $\frac{1}{2-\sqrt{5}}$ is an irrational number.

29. Find HCF and LCM of 56 and 112 by prime factorization method.

30. Explain why:

(i) $7 \times 11 \times 13 \times 15 + 15$ is a composite number

(ii) $11 \times 13 \times 17 + 17$ is a composite number.

(iii) $1 \times 2 \times 3 \times 5 \times 7 + 3 \times 7$ is a composite number.

31. On a morning walk, three persons steps off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

(NCERT Exemplar)

32. During a sale, colour pencils were being sold in the pack of 24 each and crayons in the pack of 32 each. If you want full packs of both and the same number of pencils and crayons, how many packets of each would you need to buy? (CBSE : 2017)

33. Find the largest number that divides 31 and 99 leaving remainder 5 and 8 respectively.

34. The HCF of 65 and 117 is expressible in the form 65 m – 117. Find the value of m. Also find the LCM of 65 and 117 using prime factorisation method.

34. Find the largest number that divides 1251, 9377 and 15628 leaving remainder 1, 2 and 3 respectively. **(NCERT Exemplar)**

35. Find the largest number that divides 1251, 98733 and 15628 leaving remainder 1, 2 and 3 respectively.

36. Find the HCF of 180, 252 and 324.

37. Find the greatest number of six digits exactly divisible by 18, 24 and 36.

38. Three bells ring at intervals of 9, 12, 15 minutes respectively. If they start ringing together at a time, after how much time will they next ring together?

39. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm respectively. Find the length of the longest rod that can measure the three dimensions of the room exactly.

40. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of two given number. **(CBSE 2018)**

LONG ANSWER TYPE QUESTIONS

41. Find the HCF of 56, 96, 324 by prime factorization.

42. What will be the least possible number of the planks, if three pieces of timber 42 m. 49 m, and 63 m long have to be divided into planks of the same length?

43. Amit, Sunita and Sumit start preparing cards for all the persons in an old age home. In order to complete one card, they take 10, 16 and 20 minutes respectively. If they all started together, after what time will they begin preparing a new card together?

44. Aakriti decided to distribute milk in an orphanage on her birthday. The supplier brought two milk containers which contain 398 *l* and 436 *l* of milk. The milk is to be transferred to another containers so that 7 *l* and 11 *l* of milk is left in both the containers respectively. What will be the maximum capacity of the drum?

45. Find the smallest number, which when increased by 17, is exactly divisible by both 520 and 468.

46. A street shopkeeper prepares 396 Gulab jamuns and 342 ras-gullas. He packs them, in combination. Each containter consists of either gulab jamuns or ras-gulla but have equal number of pieces.
Find the number of pieces he should put in each box so that number of boxes are least. How many boxes will be packed in all. (CBSE 2016)

47. Find the number nearest to 110000 but greater than 1 lakh, which is exactly divisible by 8, 15, 21.

48. In a seminar, the no. of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of the them being of the the same subject. **(HOTS)**

49. State fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

50. Find the smallest number which when increased by 20 is exactly divisible by 90 and 144. Is LCM, a multiple of 144?

51. If the HCF of 1032 and 408 is expressible in the form $1032\,p - 408 \times 5$, find p.

52. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one of the number is 280. Find the other number.

ANSWERS AND HINTS

1. 5

2. $3^3 \times 5^2$

3. $x^3 \times y^5$

4. $x = 35, y = 70$

5. (*iii*) $25^{2n} - 9^{2n}$ is of the form $a^{2n} - b^{2n}$ which is divisible by both $a - b$ and $a + b$ so, by both $25 + 9 = 34$ and $25 - 9 = 16$.

6. (b) 550

7. (d) 8

8. (b) 500

9. (b) even number

10. (b) Factor of LCM

11. (c) real numbers

12. (c) 6^n

13. 7

14. $2t + 1$ or $2t - 1$

15. 2520

16. 75

17. 60 : 1

18. As 12 has factors 2, 2, 3. It doesnot has 5 as its factor so 12^n will never end with 0 or 5.

19. $\sqrt{5} + \sqrt{3}$, 2

21. HCF of 180 and 144 is 36.

$$13m - 3 = 36$$
$$13\ m = 39$$
$$m = 3$$

22. Given that n is a positive odd integer

$\Rightarrow$ $2n$ and $4n + 2$ are even positive integers and n and $2n + 1$ are odd positive integers.

$\therefore$ $(-1)^n = -1, (-1)^{2n} = +1, (-1)^{2n+1} = -1, (-1)^{2n+2} = +1$

$\therefore$ $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2} = -1 + 1 - 1 + 1 = 0$

23. HCF of 850 and 680 is $2 \times 5 \times 17 = 170$ litres.

24. $2658 = 2 \times 3 \times 443$

25. $p = 3$ and $q = 5$

26. Prove that $\sqrt{3}$ and $\sqrt{5}$ is irrational number separately.

27. 5 is rational no. and $\frac{3}{7}\sqrt{3}$ is an irrational number. Difference of a rational number and irrational number is an irrational number.

29. HCF : 56, LCM : 112

30. (1) $15 \times (7 \times 11 \times 13 + 1)$ as it has more than two factors so it is composite no. Similarly for part (ii) and (iii)

31. LCM of 40, 42, 45 = 2520

Minimum distance each should walk 2520 cm.

32. LCM of 24 and 32 is 96

96 crayons or $\frac{96}{32}$ = 3 packs of crayons

96 pencils or $\frac{96}{24}$ = 4 packs of pencils.

33. Given number = 31 and 99

$31 - 5 = 26$ and $99 - 8 = 91$

Prime factors of $26 = 2 \times 13$

$91 = 7 \times 13$

HCF of (26, 91) = 13.

$\therefore$ 13 is the largest number which divides 31 and 99 leaving remainder 5 and 8 respectively.

34. HCF (117, 52) = 13.

Given that $65\,m - 117 = 13 \Rightarrow 65\,m = 130 \Rightarrow m = 2$.

LCM (65, 117) = $13 \times 3^2 \times 5 = 585$

35. $1251 - 1 = 1250$, $9377 - 2 = 9375$, $15628 - 3 = 15625$

HCF of (15625, 9375) = 3125

HCF of (3125, 1250) = 625

$\Rightarrow$ HCF of (1250, 9375, 15625) = 625

36. HCF (324, 252, 180) = 36

37. LCM of (18, 24, 36) = 72.

Greatest six digit number = 999999

$$\begin{array}{r|l}
72 & 999999 \quad (13888 \\
 & -72 \\ \hline
 & 279 \\
 & -216 \\ \hline
 & 639 \\
 & -576 \\ \hline
 & 639 \\
 & -576 \\ \hline
 & 639 \\
 & -576 \\ \hline
 & 63
\end{array}$$

Require six digit number

$$\begin{array}{r}
999999 \\
-\ 63 \\ \hline
999936
\end{array}$$

38. LCM of (9, 12, 15) = 180 minutes.

39. HCF of 8m 25 cm, 6m 75 cm and 4 m 50 cm = 75 cm

40. $$\begin{aligned} \text{HCF }(404, 96) &= 4 \\ \text{LCM }(404, 96) &= 9696 \\ \text{HCF} \times \text{LCM} &= 38,784 \\ \text{Also,} \quad 404 \times 96 &= 38,784 \end{aligned}$$

42. HCF of 42m, 49m and 63 m = 7 m

Number of planks = 42/7 + 63/7 = 6 + 7 + 9 = 22

43. LCM of 10, 16 and 20 minutes = 80 minutes

44. 17

45. 4663

LCM of (468, 520) = 4680

$\therefore$ Required no. = 4680 – 17 = 4663

46. HCF (396, 342) = 18

$$\text{No. of boxes} = \frac{396+342}{18} = \frac{738}{18} = 41$$

47. 109200

48. HCF of 60, 84 and 108 is $2^2 \times 3 = 12$ = No. of participants in each row.

$$\text{No. of rooms required} = \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12} = 21 \text{ rooms}$$

49. $$\text{HCF} = 24, \quad \text{LCM} = 540$$

$$\frac{\text{LCM}}{\text{HCF}} = \frac{540}{24} = 22.5, \text{ not an integer.}$$

Hence two numbers cannot have HCF and LCM as 24 and 540 respectively.

50. [The LCM of (90, 144) – 20] = Required No.

$\Rightarrow$ Required No. = 700

51. $p = 2$

52. HCF = 40, LCM = 560

$\therefore$ Other No. = 80.

PRACTICE-TEST

Real Number

Time : 45 Minutes *M.M. : 20*

SECTION A

1. Check whether $17 \times 19 \times 21 \times 23 + 7$ is a composite number. 1

2. What is the LCM of the smallest prime number and the smallest composite number? 1

3. HCF of x^4y^5 and x^8y^3 . 1

4. LCM of 14 and 122 . 1

SECTION B

5. Show that 9^n can never ends with unit digit zero. 2

6. Find the pairs of the natural numbers whose least common multiple is 78 and the greatest divisor is 13.

7. Find prime factors of 7650 using factor tree.

2

SECTION C

8. Prove that $\dfrac{1}{3-2\sqrt{5}}$ is an irrational number. 3

9. Find the HCF of 36, 96 and 120 by prime factorization. 3

SECTION D

10. Once a sports goods retailer organized a campaign "Run to remember" to spread awareness about benefits of walking. In that Soham and Baani participated. There was a circular path around a sports field. Soham took 12 minutes to drive one round of the field, while Baani took 18 minutes for the same. Suppose they started at the same point and at the same time and went in the same direction. After how many minutes have they met again at the starting point? 4

CHAPTER

2 Polynomials

Zeroes of cubic polynomial

If α, β and γ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d, a \neq 0$, then

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \alpha\gamma = c/a$

$\alpha\beta\gamma = -d/a$

Zeroes of quadratic polynomial

If α and β are zeroes of the quadratic polynomial $ax^2 + bx + c, a \neq 0$, then

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$

Quadratic polynomial P(x) with α and β as zeroes can be represented as $P(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Or $P(x) = (x - \alpha)(x - \beta)$

Zeroes of linear polynomial

If 'k' is a zero of $p(x) = ax + b$, then $p(k) = ak + b = 0$, i.e., $k = -b/a$ is the zero of the polynomial.

RELATIONSHIP BETWEEN ZEROES AND COEFFICIENT

Cubic polymomial

Polynomial of the degree 3 Can have at most 3 zeroes Standard form is $ax^3 + bx^2 + cx + d, a \neq 0$ e.g. $4x^3 - 5x^3 + 7$ etc.

Quadratic polynomial

Polynomial of the degree 2 Can have at most 2 zeroes Standard form is $ax^2 + bx + c, a \neq 0$ e.g. $4x^2 + 10x - 7$

Linear polynomial

Polynomial of the degree 1 Exactly 1 zero Standard form is $ax + b$

BASED ON DEGREE OF POLYNOMIAL

Trinomial

A polynomial with three terms e.g. $4x^3 + 5x^2 - 7$

Binomial

A polynomial with two terms e.g. $5x^2 - 7$, $3x/5+7$

Monomial

A polynomial with one term, e.g. $10x^2$, -7 etc.

BASED ON NUMBER OF TERMS

ZERO OF POLYNOMIAL

A real number 'a' is a zero of the polynomial P(x), if P(a) = 0. It means (x – a) is a factor of P(x).

TYPES OF POLYNOMIALS

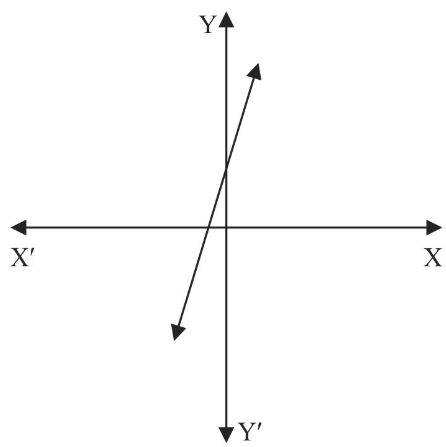
POLYNOMIALS

If 'x' is a variable, 'n' is a natural number, a_0, a_1, a_2, a_3, as... are real numbers then, $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots\ldots\ldots a_1 x + a_0 (n \neq 0)$ is called a polynomial in 'x'.

GRAPH OF A POLYNOMIAL

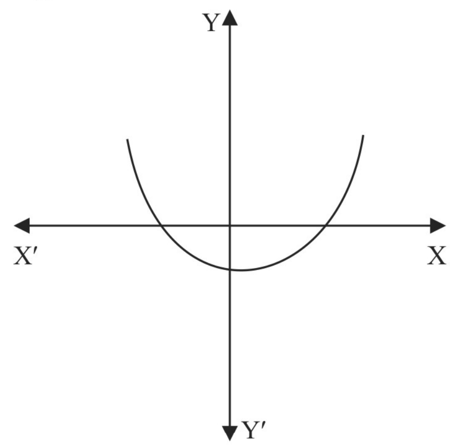
GEOMETRICAL REPRESENTATION OF A LINEAR POLYNOMIAL

Graph of a linear polynomial $P(x) = ax + b, a \neq 0$ is a straight line cutting x - axis exactly at one point.

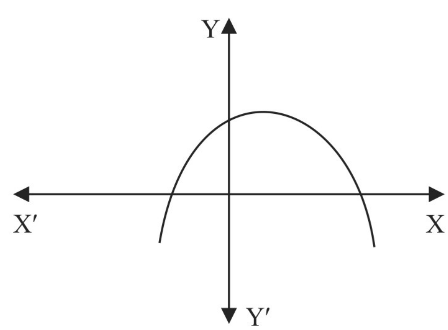


GEOMETRICAL REPRESENTATION OF A QUADRATIC POLYNOMIAL

Graph of a quadratic polynomial $P(x) = ax^2 + bx + c, a \neq 0$, is a parabola open upwards, if $a > 0$. e.g. $5x^2 + 4x + l$



Graph of a quadratic polynomial $P(x) = -ax^2 + bx + c, a \neq 0$, is a parabola open downwards, if $a < 0$. e.g. $-x^2 + 7x + 1$



In general a polynomial P(x) of degree 'n' crosses the x-axis at most 'n' points.

VERY SHORT ANSWER TYPE QUESTIONS

1. If one zero of the polynomial $P(x) = 5x^2 + 13x + k$ is reciprocal of the other, then value of k is

(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

2. If α and β are the zeroes of the polynomial $p(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, the c = ________ .

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) – 10 (c) 5 (d) – 5

4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

5. What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial:

(a) 1 (b) 2 (c) 4 (d) 5

6. If α and β are the zeroes of the polynomial

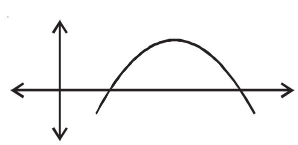
$$f(x) = x^2 + x + 1, \quad \text{then} \quad \frac{1}{\alpha} + \frac{1}{\beta} =$$

7. If a quadratic polynomial $f(x)$ is not factorizable into linear factors, then it has no real zero. (True/False)

8. If a quadratic polynomial $f(x)$ is a square of a linear polynomial, then its two zeroes are coincident. (True/False).

9. If $p(x) = x^3 - 2x^2 - x + 2 = (x + 1)(x - 2)(x - d)$ then what is the value of d?

10. The quadratic polynomial $ax^2 + bx + c,\ a \neq 0$ is represented by this graph then a is



(a) Natural no. (b) Whole no. (c) Negative Integer (d) Irrational no.

11. What will be the number of zeroes of a linear polynomial $p(x)$ if its graph (*i*) passes through the origin. (*ii*) doesn't intersect or touch x-axis at any point?

12. Find the quadratic polynomial whose zeroes are

$\left(5+2\sqrt{3}\right)$ and $\left(5-2\sqrt{3}\right)$

13. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)x - 9$ is negative of the other, find values of k.

14. What number should be subtracted to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.

15. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

16. What will be the number of real zeroes of the polynomial $x^2 + 1$?

17. If α and β are zeroes of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeroes are 2α and 2β (CBSE)

18. If α and $\frac{1}{\alpha}$ are zeroes of $4x^2 - 17x + k - 4$, find the value of k.

19. What will be the number of zeroes of the polynomials whose graphs are parallel to (*i*) y-axis (ii) x-axis?

20. What will be the number of zeroes of the polynomials whose graphs are either touching or intersecting the axis only at the points:

    (*i*) (–3, 0), (0, 2) & (3, 0) (*ii*) (0, 4), (0, 0) and (0, –4)

SHORT ANSWER TYPE (I) QUESTIONS

21. For what value of k, $x^2 - 4x + k$ touches x-axis.

22. If the product of zeroes of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeroes.

23. If zeroes of $x^2 - kx + 6$ are in the ratio 3 : 2, find k.

24. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k.

25. If α and β are the zeroes of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m. (CBSE)

26. If the sum of squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, find the value of k.

27. If α and β are zeroes of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. What should be added to the polynomial $x^3 - 3x^2 + 6x - 15$, so that it is completely divisible by $x - 3$? (CBSE 2016)

29. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

(CBSE, 2012)

30. Find a quadratic polynomial whose zeroes are $\frac{3+\sqrt{5}}{5}$ and $\frac{3-\sqrt{5}}{5}$.

(CBSE, 2013)

SHORT ANSWER TYPE (II) QUESTIONS

31. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find the values of k and a.
32. Obtain zeroes of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.
33. Form a quadratic polynomial, whose one zero is 8 and the product of zeroes is -56.
34. -5 is one of the zeroes of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k.
35. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeroes as half of their product.
36. If zeroes of the polynomial $ax^2 + bx - c$, $a \neq 0$ are additive inverse of each other then what is the value of b?
37. If α and β are zeroes of $x^2 - x - 2$, find a polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$
38. If α, β are zeroes of the quadratic polynomial $2x^2 + 5 + \text{k}$, then find the value of 'k' such that $(\alpha + \beta)^2 - \alpha\beta = 24$.
39. If one zero of the polynomial $2x^2 - 3x + p$ is 3, find the other zero and the value of 'p'.
40. Find a quadratic polynomial, whose zeroes are in the ratio 2 : 3 and their sum is 15.

LONG ANSWER TYPE QUESTIONS

41. If $(x + a)$ is a factor of two quadratic polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = (n - q)/(m - p)$

42. If one zero of the quadratic polynomial $4x^2 - 8kx + 8x - 9$ is the negative of the other, then find the zeroes of $kx^2 + 3kx^2 + 2$

43. If α, β are zeroes of the quadratic polynomial $x^2 - 5x - 3$, then form a polynomial whose zeroes are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$.

44. If one zero of the polynomial $(k + 1) x^2 - 5x + 5$ is multiplicative inverse of the other, then find the zeroes of $kx^2 - 3kx + 9$.

45. If the product of the zeroes of the quadratic polynomial $kx^2 + 11x + 42$ is 7, then find the zeroes of the polynomial $(k - 4)x^2 + (k + 1)x + 5$.

46. If α and β are zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1+\dfrac{\beta}{\alpha}$ and $1+\dfrac{\alpha}{\beta}$. **(CBSE)**

47. Form a quadratic polynomial one of whose zero is $2 + \sqrt{5}$ and sum of the zeroes is 4.

48. Form a polynomial whose zeroes are the reciprocal of the zeroes of $p(x) = ax^2 + bx + c$, $a \neq 0$.

49. If $(x + 2)$ is a factor of $x^2 + px + 2q$ and $p + q = 4$ then what are the values of p and q?

50. What should be subtracted from $x^3 - 3x^2 + 6x - 15$, so that it is completely divisible by $(x - 3)$?

51. If sum of the zeroes of $5x^2 + (p + q + r)x + pqr$ is zero, then find $p^3 + q^3 + r^3$.

52. If the zeroes of $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$ find p and q.

ANSWERS AND HINTS

1. (b) 5

2. 1

3. (b) –10

4. (d) a = 0, b = –6

5. (b) 2

6. – 1

7. True

8. True

9. 1

10. (c) Negative Integer

11. (i) 1 (ii) 0

12. $x^2 - 10x + 13$

13. $k = 0, 5$

14. (– 2)

15. (i) 2 (ii) 0

16. 0

17. $[3x^2 - 7x - 6]\ k$

18. $k = 8$

19. (i) 1 (ii) 0

20. (i) 2 (ii) 1

21. 4

22. $a = -\frac{3}{2}$, sum of zeroes = – 4

23. – 5, 5

24. 5

25. 6

26. 12

27. $4t^2 + t - 1$

28. On dividing $x^3 - 3x^2 + 6x - 15$ by x – 3, remainder is + 3, hence – 3 must be added to $x^3 - 3x^2 + 6x - 15$.

29. $$\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn} = \frac{\left(-\frac{11}{3}\right)^2 - 2\left(-\frac{4}{3}\right)}{-\frac{4}{3}} = -\frac{145}{12}$$

30. $\alpha + \beta = \frac{6}{5}, \quad \alpha\beta = \frac{4}{25}$,

$25x^2 - 30x + 4$

31. $k = -3, 5$ and $a = -27, 125$

32. $-\frac{2}{\sqrt{3}}, \quad \frac{\sqrt{3}}{4}$

33. $\alpha\beta = -56$ and $\beta = -7$

so, $\alpha = 8$, Now $\alpha + \beta = 1$

Required polynomial is $x^2 - x - 56$

34. $\frac{7}{4}$

35. 1

36. $b = 0$

37. $x^2 - 4x - 5$

38. $(\alpha + \beta) = -5/2$ and $\alpha\beta = k/2$

Substituting the above values in $(\alpha + \beta)^2 - \alpha\beta = 24$. Solve to get 'k' = $\frac{-71}{2}$.

39. 3 is a zero, so $2(3)^2 - 3 \times 3 + p = 0$

$p = 9$, Now $\alpha\beta = \frac{c}{a}$, solve to get the other zero $\frac{-3}{2}$.

40. $\alpha : \beta = 2:3$. So $\alpha = 2\beta/3$

Using $(\alpha + \beta) = 15$, solve to get α and β as 9 and 6 respectively.

Required polynomial is $x^2 - 15x + 54$

41. Since $(x + 2)$ is a factor of $x^2 + px + q$

$(-a)^2 - ap + q = 0$

$(-a)^2 = ap - q$ ....(1)

Similarly from $x^2 + mx + n$

$(a)^2 = am - n$ .... (2)

Comparing equatin (1) and (2)

$a = (n - q)/(m - p)$

42. $f(x) = 4x^2 + (8 - 8k)x - 9$

$(\alpha + \beta) = -(8 - 8k)/4$

$k = 1$

Substitute $k = 1$ in $kx^2 + 3kx^2 + 2$ and solve for $x = -2$ and -1

43. For given polynomial, $(\alpha + \beta) = 5$, $\alpha\beta = -3$

For Required polynomial

Sum of zeroes $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$= 5(a + p)$

$= 25$

Product of zeroes $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6\ [(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta$

$= 147$

Required polynomial is $x^2 - 25x + 147$

44. $f(x) = (k + 1)x^2 - 5x + 5$

$(\alpha\beta) = 1$

$5/(k + 1) = 1$

$k = 4$

Substituting $k = 4$ in $kx^2 - 3kx + 9$ solve to get zeroes $x = 3/2$ and $3/2$

45. $f(x) = kx^2 + 11x + 42$

$(\alpha\beta) = 7$

k = 6

Substituting k = 6 in $(k-4)x^2 + (k+1)x + 5$, solve to get zeroes $x = -1$ and $x = -5/2$

46. $x^2 - \frac{16}{3}x + \frac{16}{3}$ or $\frac{1}{3}(3x^2 - 16x + 16)$

47. $\alpha + \beta = 4$

$(2+\sqrt{5}) + \beta = 4$

$\beta = 2 - \sqrt{5}$

$\alpha\beta = -1 \quad \therefore$ Polynomial $= k[x^2 - 4x - 1]$

48. $k\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right]$

49. $p = 3$, $q = 1$

50. 3

51. Product of the zeroes = 3 pqr

52. $p = -5$ and $q = -6$

PRACTICE-TEST

Polynomials

Time : 45 Minutes ***M.M. : 20***

SECTION-A

1. If α and β are zeroes of a quadratic polynomial $p(x)$, then factorize $p(x)$. **1**

2. If α and β are zeroes of $x^2 - x - 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. **1**

3. If one of the zeroes of quadratic polynomial $(K-1)x^2 + kx + 1$ is -3 then the value of K is, **1**

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

4. A quadratic polynomial, whose zeroes are -3 and 4, is **1**

(*a*) $x^2 - x + 12$ (b) $x^2 + x + 12$

(*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

SECTION-B

5. If α and β are zeroes of $x^2 - (k + 6)x + 2(2k - 1)$. find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **2**

6. Find a quadratic polynomial one of whose zeroes is $(3+\sqrt{2})$ and the sum of its zeroes is 6. **2**

7. If zeroes of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$ then find the value of a. **2**

SECTION-C

8. If α and β are zeroes of the polynomial p(s) = 3s2 – 6s + 4, then find the value of $\alpha/\beta + \beta/\alpha + 2(1/\alpha + 1/\beta) + 3\alpha\beta$ **3**

9. If truth and lie are zeroes of the polynomial $px^2 + qx + r$, $(p \neq 0)$ and zeroes are reciprocal to each other, Find the relation between p and r. **3**

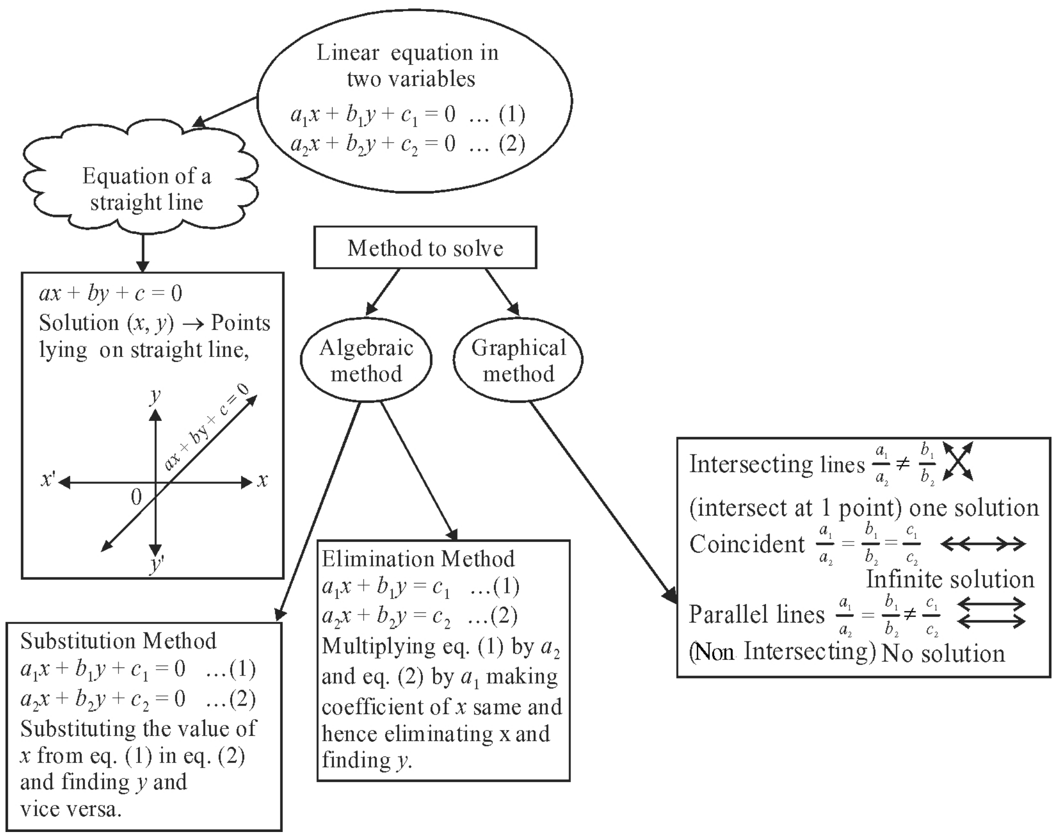
SECTION-D

10. Find the zeroes of the polynomial $\sqrt{3x^2} + 10x + 7\sqrt{3}$. Also verify the relationship between the zeroes and their coefficients.

CHAPTER

3 Pair of Linear Equations in Two Variables

KEY POINTS



VERY SHORT ANSWER TYPE QUESTIONS

1. If the lines given by $3x + 2ky = 2$ and $2x + 5y = 1$ are parallel, then the value of k is _______ .

2. If $x = a$ and $y = b$ is the solution of the equation $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively _______ .

3. A pair of linear equations which has a unique solution $x = 2$ and $y = -3$ is
   (a) $x + y = 1$ and $2x - 3y = -5$
   (b) $2x + 5y = -11$ and $2x - 3y = -22$
   (c) $2x + 5y = -11$ and $4x + 10y = -22$
   (d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$
4. The area of the triangle formed by the lines $x = 3$, $y = 4$ and $x = y$ is _____ .
5. The value of k for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has a non-zero solutions is ____ .
6. If a pair of linear equations in two variables is consistent, then the lines represented by two equations are:
   (a) Intersecting (b) Parallel
   (c) always coincident (d) intersecting or coincident
7. For $2x + 3y = 4$, y can be written in terms of x as _______ .
8. One of the common solution of $ax + by = c$ and y axis is
   (a) $\left(0, \frac{c}{b}\right)$ (b) $\left(0, \frac{b}{c}\right)$
   (c) $\left(\frac{c}{b}, 0\right)$ (d) $\left(0, -\frac{c}{b}\right)$
9. If $ax + by = c$ and $lx + my = n$ has unique solution then the relation between the coefficient will be:
   (a) $am \neq lb$ (b) $am = lb$ (c) $ab = lm$ (d) $ab \neq lm$
10. In ΔABC, $\angle C = 3\angle B$, $\angle C = 2(\angle A + \angle B)$ then, $\angle A$, $\angle B$, $\angle C$ are respectively.
    (a) 30°, 60°, 90° (b) 20°, 40°, 120°
    (c) 45°, 45°, 90° (d) 110°, 40°, 50°
11. If $x = 3m - 1$ and $y = 4$ is a solution of the equation $x + y = 6$, then find the value of m.
12. What is the point of intersection of the line represented by $3x - 2y = 6$ and the y-axis?
13. For what value of p, system of equations $2x + py = 8$ and $x + y = 6$ have no solution.
14. A motor cyclist is moving along the line $x - y = 2$ and another motor cyclist is moving along the line $x - y = 4$ find out their moving direction.
15. Find the value of k for which pair of linear equations $3x + 2y = -5$ and $x - ky = 2$ has a unique solution.

16. Write the solution of $y = x$ and $y = -x$.

17. If $2x + 5y = 4$, write another linear equation, so that lines represented by the pair are coincident.

18. Check whether the graph of the pair of linear equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ is intersecting lines or parallel lines.

19. What is the value of p, for which the pair of linear equations $x + y = 3$ and $3x + py = 9$ is inconsistent.

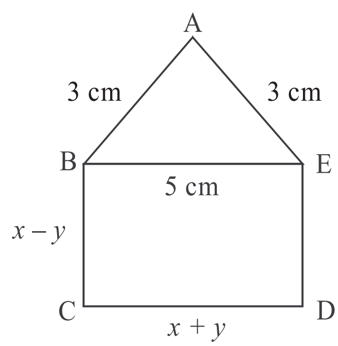
20. If we draw lines of $x = 2$ and $y = 3$ what kind of lines do we get?

SHORT ANSWER TYPE (I) QUESTIONS (2 MARKS QUESTIONS)

21. Form a pair of linear equations for: The sum of the numerator and denominator of the fraction is 3 less than twice the denominator. If the numerator and denominator both are decreased by 1, the numerator becomes half the denominator.

22. For what value of p the pair of linear equations $(p + 2)x - (2p + 1)y = 3(2p - 1)$ and $2x - 3y = 7$ has a unique solution.

23. ABCDE is a pentagon with BE || CD and BC || DE, BC is perpendicular to CD If the perimeter of ABCDE is 21 cm, find x and y.



24. Solve for x and y

$$x - \frac{y}{2} = 3 \text{ and } \frac{x}{2} - \frac{2y}{3} = \frac{2}{3}$$

25. Solve for x and y

$3x + 2y = 11$ and $2x + 3y = 4$

Also find p if $p = 8x + 5y$

26. Solve the pair of linear equations by substitution method $x - 7y + 42 = 0$ and $x - 3y - 6 = 0$

27. Ram is walking along the line joining (1, 4) and (0, 6)
Rahim is walking along the line Joining (3, 4) and (1, 0)
Represent on graph and find the point where both of them cross each other

28. Given the linear equation $2x + 3y - 12 = 0$, write another linear equation in these variables, such that. geometrical representation of the pair so formed is
(*i*) Parallel Lines (*ii*) Coincident Lines

29. The difference of two numbers is 66. If one number is four times the other, find the numbers.

30. For what value of k, the following system of equations will be inconsistent
$kx + 3y = k - 3$
$12x + ky = k$

SHORT ANSWERS TYPE (II) QUESTIONS

31. Solve graphically the pair of linear equations $5x - y = 5$ and $3x - 2y = -4$
Also find the co-ordinates of the points where these lines intersect y-axis

32. Solve (CBSE)

$$\frac{x}{a} + \frac{y}{b} = a + b$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

33. For what values of a and b the following pair of linear equations have infinite number of solutions? (CBSE)

$$2x + 3y = 7$$

$$a(x + y) - b(x - y) = 3a + b - 2$$

34. Find the value of k for no solutions

$$(3k + 1)x + 3y - 2 = 0$$

$$(k^2 + 1)x + (k - 2)y - 5 = 0$$

35. Solve the pair of linear equations

$$152x - 378y = -74$$

$$-378x + 152y = -604$$

36. Pinky scored 40 marks in a test getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks were deducted for each wrong answer, then pinky again would have scored 40 marks. How many questions were there in the test?

37. Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of the father.

38. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gain ₹ 2000. But if he sells the T.V. at 10% gain and fridge at 5% loss, he gains ₹ 1500 on the transaction. Find the actual price of the T.V. and the fridge

39. Sunita has some ₹ 50 and ₹ 100 notes amounting to a total of ₹ 15,500. If the total number of notes is 200, then find how many notes of ₹ 50 and ₹ 100 each, she has.

LONG ANSWER TYPE QUESTIONS

40. Solve graphically the pair of linear equations $3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$

Find the co-ordinates of vertices of triangular region formed by these lines and x-axis. Also calculate the area of this triangle.

41. A and B are two points 150 km apart on a highway. Two cars start with different speeds from A and B at same time. If they move in same direction, they meet in 15 hours. If they move in opposite direction, they meet in one hour. Find their speeds

42. The ratio of incomes of two persons A and B is 3 : 4 and the ratio of their expenditures is 5 : 7. If their savings are ₹ 15,000 annually find their annual incomes.

43. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

(HOTS, Exampler)

44. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket costs ₹ 2530. One reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket. (Exemplar)

45. Determine graphically, the vertices of the triangle formed by the lines $y = x$, $3y = x$ and $x + y = 8$. **(NCERT Exemplar)**

46. Draw the graphs of the equations x = 3, x = 5 and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x-axis.

(NCERT Exemplar, HOTS)

47. A nirudh takes 3 hours more than Nishi to walk 30 km. But if Anirudh doubles his speed, he is ahead of Nishi by 1½ hours. Find their speed of walking.

48. In a two digit number, the ten's place digit is 3 times the unit's place digit. When the number is decreased by 54, digits get reversed. Find the original number.

49. A two-digit number is 3 more than 4 times the sum of the digits. If 18 is added to the number, digits reversed. Find the number.

50. Find the values of a and b for infinite solutions

(i) $2x - (a - 4)y = 2b + 1$

$4x - (a - 1)y = 5b - 1$

(ii) $2x + 3y = 7$

$2ax + ay = 28 - by$

ANSWERS AND HINTS

1. $k = \frac{15}{4}$

2. $a = 3$ and $b = 1$

3. (c) $2x + 5y = -11$ and $4x + 10y = -22$

4. $\frac{1}{2}$ sq. unit

5. $k \neq 6$

6. (d) intersecting or coincident

7. $y = \frac{4 - 2x}{3}$

8. (a) $\left(0, \frac{c}{b}\right)$

9. (a) am ≠ lb

10. (b) 20°, 40°, 120°

11. $m = 1$

12. (0, –3)

13. $p = 2$

14. move parallel

15. $k \neq \frac{-2}{3}$

16. (0, 0)

17. $4x + 10y = 8$

18. Parallel lines

19. $p = 3$

20. Intersecting lines

21. $x - y = -3$, $2x - y = 1$

22. $p \neq 4$

23. $x = 5$, $y = 0$

24. 4, 2

25. $x = 5$, $y = -2$, $p = 30$

26. 42, 12

27. (2, 2)

28. (*i*) $4x + 6y + 10 = 0$

(*ii*) $4x + 6y - 24 = 0$

29. 88, 22

30. $k = -6$

31. (2, 5) (0, – 5) and (0, 2)

32. $x = a^2$, $y = b^2$

33. $a = 5$, $b = 1$

34. $k = -1$

35. 2, 1

36. 40 questions

37. 45 years

38. T.V. = ₹ 20,000 Fridge = ₹ 10,000

39. ₹ 50 notes = 90, ₹ 100 notes = 110

40. Solution (3, 3), Vertices (– 1, 0) (7, 0) and (3, 3), Area = 12 square units

41. 80 km/hr , 70 km/hr

42. ₹ 90,000, ₹ 1,20,000

43. Let the no. of bananas in lots A be x and in lots B be y

Case I : $\frac{2}{3}x + y = 400 \quad \Rightarrow \quad 2x + 3y = 1200$

Case 2 : $x + \frac{4}{5}y = 460 \quad \Rightarrow \quad 5x + 4y = 2300$

$x = 300$, $y = 200$, Total bananas = 500.

44. Let the cost of full and half ticket be ₹ x & ₹ $\frac{x}{2}$ and reservation charge by ₹ y per ticket.

Case I : $x + y = 2530$

Case 2 : $x + y + \frac{x}{2} + y = 3810$

$x = 2500$, $y = 30$

Full first class fare is ₹ 2500 and reservation charge is ₹ 30.

45. Vertices of the triangle are (0, 0) (4, 4) (6, 2).

46. Area of quadrilateral ABCD where,

$A(3, 0)$, $B(5, 0)$

C(5, 6), $D(3, 2)$

$$= \frac{1}{2} \times AB \times (AD + BC)$$

$$= \frac{1}{2} \times 2 \times (6 + 2) = 8 \text{ sq. units.}$$

47. $\frac{10}{3}$ km/hr, 5 km/hr

48. 93

49. 35

50. (i) 7, 3

(ii) 4, 8

PRACTICE-TEST

Pair of Linear Equations In Two Variables

Time : 45 Minutes ***M.M. : 20***

SECTION-A

1. For what value of k system of equations
$x + 2y = 3$ and $5x + ky + 7 = 0$ has a unique solution. **1**

2. Does the point (2, 3) lie on line represented by the graph of $3x - 2y = 5$. **1**

3. The pair of equations $x = a$ and $y = b$ graphically representes lines which are: **1**

(a) Parallel (b) Intersecting at (b, a)

(c) Coincident (d) Intersecting at (a, b)

4. For what value of K, the equations $3x - y + 8 = 0$ and $6x - Ky = -16$ represent coincident lines? **1**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) –2

SECTION-B

5. For what value of a and b the pair of linear equations have infinite number of solutions

$$\begin{aligned} 2x - 3y &= 7 \\ ax + 3y &= b \end{aligned}$$ **2**

6. Solve for x and y

$$\begin{aligned} 0.4x + 0.3y &= 1.7 \\ 0.7x - 0.2y &= 0.8 \end{aligned}$$ 2

7. If the system of equations $6x + 2y = 3$ and $kx + y = 2$ has a unique solution, find the value of k. **2**

SECTION-C

8. Solve for x and y

$$\begin{aligned} x + y &= a + b \\ ax - by &= a^2 - b^2 \end{aligned}$$ **3**

9. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their ages. **3**

SECTION-D

10. Solve the following pair of equations graphically.
$3x + 5y = 12$ and $3x - 5y = -18$. **4**

Also shade the region enclosed by these two lines and x-axis.

CHAPTER

4 Quadratic Equations

Quadratic Polynomial

A polynomial of the form $ax^2 + bx + c$, $a \neq 0$ is called a Quadratic polynomial, Degree of this polynomla is 2

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a Quadratic equation, in one variable x, where a, b, c are real numbers.

Roots of a Quadratic Equations

A real number α is called root of $ax^2 + bx + c = 0$, when it satisfies this equation i.e., $a\alpha^2 + b\alpha + c = 0$. A quadratic equation have at most 2 real roots.

Methods for solving Quadratic Equations

By factorisation

(a) By using identities
(b) By splitting the middle term

By Using Quadratic Formula (Given by Sridhar Acharya)

Quadratic equation $ax^2 + bx + c = 0$ has two roots α and β given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $D = b^2 - 4ac$ is called discriminant

Nature of roots

Case I : $D > 0$, $b^2 - 4ac > 0$ Roots are real and distinct

Case II : $D = 0$, $b^2 - 4ac = 0$ Roots are real and equal

Case III : $D < 0$, $b^2 - 4ac < 0$ Roots are not real

Real roots can be rational or irrational

NOTES:

1. Real and distinct roots are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. Real and equal roots are $\dfrac{-b}{2a}, \dfrac{-b}{2a}$
3. There are quadratic equation which donot have any real roots e.g. $x^2 + 1 = 0$

VERY SHORT ANSWER TYPE QUESTIONS

Multiple Choice Questions:

1. Which of the following is not a Quadratic Equation?

(*a*) $2(x - 1)^2 = 4x^2 - 2x + 1$ (*b*) $3x - x^2 = x^2 + 6$

(*c*) $\left(\sqrt{3}x + \sqrt{2}\right)^2 = 2x^2 - 5x$ (*d*) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

2. Which of the following equation has 2 as a root

(*a*) $x^2 + 4 = 0$ (*b*) $x^2 - 4 = 0$

(*c*) $x^2 + 3x - 12 = 0$ (*d*) $3x^2 - 6x - 2 = 0$

3. If $\dfrac{1}{2}$ is a root of $x^2 + px - \dfrac{5}{4} = 0$ then value of p is

(*a*) 2 (*b*) –2

(*c*) $\dfrac{1}{4}$ (*d*) $\dfrac{1}{2}$

4. Every Quadratic Equation can have at most

(*a*) Three roots (*b*) One root

(*c*) Two roots (*d*) Any number of roots

5. Roots of Quadratic equation $x^2 - 7x = 0$ will be

(*a*) 7 (*b*) 0, –7

(*c*) 0, 5 (*d*) 0, 7

6. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

(*a*) 4 (*b*) ± 4

(*c*) – 4 (*d*) 0 **(CBSE 2020)**

7. **Fill in the blanks:**

(*a*) If $px^2 + qx + r = 0$ has equal roots then value of r will be _______ .

(*b*) The quadratic equation $x^2 - 5x - 6 = 0$ if expressed as $(x + p)(x + q) = 0$ then value of p and q respectively are _______ and ________ .

(*c*) The value of k for which the roots of qaudratic equations $x^2 + 4x + k = 0$ are real is _______ .

(*d*) If roots of $4x^2 - 2x + c = 0$ are reciprocal of each other then the value of c is _________ .

(*e*) If in a quadratic equation $ax^2 + bx + c = 0$, value of a is zero then it become a ______ equation.

8. **Write whether the following statements are true or false. Justify your answers.**

(*a*) Every quadratic equation has atleast one real roots.

(*b*) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

(*c*) 0.3 is a root of $x^2 - 0.9 = 0$.

(*d*) The graph of a quadratic polynomial is a straight line.

(*e*) The discriminant of $(x - 2)^2 = 0$ is positive.

9. **Match the following :**

(*i*) Roots of $3x^2 - 27 = 0$	(*a*) 169/9
(*ii*) D of $2x^2 + \frac{5}{3}x - 2 = 0$	(*b*) 0
(*iii*) Sum of roots of $8x^2 + 2x - 3 = 0$	(*c*) $x^2 - (a + b)x + ab = 0$
(*iv*) A quadratic equation with roots a and b	(*d*) 3, – 3
(*v*) The product of roots of $x^2 + 8x = 0$	(*e*) $\frac{-1}{4}$

SHORT ANSWER TYPE QUESTIONS-I

10. If the Quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$ $(p \neq 0)$ has two equal roots then find the value of p.

11. **Solve for *x* by factorisation**

(*a*) $8x^2 - 22x - 21 = 0$

(*b*) $3\sqrt{5}x^2 + 25x + 10\sqrt{5} = 0$

(*c*) $3x^2 - 2\sqrt{6}\,x + 2 = 0$ **(CBSE 2010)** (*d*) $2x^2 + ax - a^2 = 0$ **(CBSE 2014)**

(*e*) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

(*f*) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(*g*) $(x - 1)^2 - 5(x - 1) - 6 = 0$

12. For what vlaue of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots?

(CBSE 2020)

13. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots find the value of k.

(CBSE 2014, 2016)

14. If $x = \dfrac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$. Find the value of a and b. **(CBSE 2016)**

15. Find value of p for which the product of roots of the quadratic equation $px^2 + 6x + 4p = 0$ is equal to the sum of the roots.

16. The sides of two squares are x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 Find the sides of these two squares.

17. Find K if the difference of roots of the quadratic equation$x^2 - 5x + (3k - 3) = 0$ is 11.

SHORT ANSWER TYPE QUESTIONS-II

18. Find the positive value of k for which the quadratic equation $x^2 + kx + 64 = 0$ and the quadratic equation $x^2 - 8x + k = 0$ both will have real roots.

19. Solve for x

(*a*) $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$ $\quad a + b + x \neq 0,$ **(CBSE 2005)**

$a, b, x \neq 0$

(*b*) $\dfrac{1}{2a+b+2x} = \dfrac{1}{2a} + \dfrac{1}{b} + \dfrac{1}{2x}$ $\quad 2a + b + 2x \neq 0,$

$a, b, x \neq 0$

(*c*) $\dfrac{2x}{x-3} + \dfrac{1}{2x+3} + \dfrac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, \dfrac{-3}{2}$

(*d*) $\dfrac{1}{x-1} - \dfrac{1}{x+5} = \dfrac{6}{7}, x \neq 1, 5$ **(CBSE 2010)**

(*e*) $4x^2 + 4bx - (a^2 - b^2) = 0$

(*f*) $4x^2 - 2(a^2 + b^2)\,x + a^2b^2 = 0$

(g) $\dfrac{2}{x+1}+\dfrac{3}{2(x-2)} = \dfrac{23}{5x}, x \neq 0, -1, 2$

(h) $\left(\dfrac{2x}{x-5}\right)^2 + \dfrac{10x}{(x-5)} - 24 = 0, x \neq 5$

(i) $4x^2 - 4a^2x + a^4 - b^4 = 0$

(j) $2a^2x^2 + b(6a^2 + 1)x + 3b^2 = 0$

(k) $3\left(\dfrac{7x+1}{5x-3}\right) - 4\left(\dfrac{5x-3}{7x+1}\right) = 11, x \neq \dfrac{3}{5}, \dfrac{-1}{7}$

(l) $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30}, x \neq -4, 7$ **(NCERT)**

(m) $\dfrac{x-4}{x-5} + \dfrac{x-6}{x-7} = \dfrac{10}{3}, x \neq 5, 7$ **(CBSE 2014)**

(n) $\dfrac{1}{x+1}+\dfrac{2}{x+2}=\dfrac{4}{x+4},$ $\quad x \neq -1, -2, -4$

(o) $\dfrac{1}{2x-3}+\dfrac{1}{x-5}=1,$ $\quad x \neq \dfrac{3}{2}, 5$

(p) $x^2 + 5\sqrt{5}x - 70 = 0$

(q) $\dfrac{16}{x}-1=\dfrac{15}{x+1}, x \neq 0, -1$ **(CBSE 2014)**

20. Solve by using quadratic formula $abx^2 + (b^2 - ac)x - bc = 0$. **(CBSE 2005)**

21. If the roots of the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$ are equal find p and then find the roots of this quadratic equation.

22. Find the nature of roots of the quadratic equation $3x^2 - 4\sqrt{3}x + 4 = 0$

If the roots are real, find them. **(CBSE 2020)**

23. Solve $9x^2 - 6a^2x + a^4 - b^4 = 0$ using quadratic formula. **(CBSE 2020)**

LONG ANSWER TYPE QUESTIONS

24. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

25. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

26. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policemen will catch the thief?

27. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **(CBSE 2020)**

28. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed, so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

29. A farmer wishes to grow a 100 m^2 recangular garden. Since he has only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of this house act as the fourth side fence. Find the dimensions of his garden.

30. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of a pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal at what distance from the hole is the snake caught?

31. If the price of a book is reduced by ₹ 5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

32. ₹ 6500 were divided equally among a certain number of persons. If there been 15 more persons, each would have got ₹ 30 less. Find the original number of persons.

33. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight. **(CBSE 2020, Outside Delhi)**

34. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than the fast train, find the speed of the two trains.

(CBSE 2020, Outside Delhi)

35. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the orignal point in 4 hrs 30 minutes. Find the speed of the stream.

36. Sum of areas of two squares is 400 cm^2. If the difference of their perimeter is 16 cm. Find the side of each square.

37. The area of an isosceles triangle is 60 cm^2. The length of equal sides is 13 cm find length of its base.

38. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$. Find the fraction.

39. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

40. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

(CBSE 2006)

41. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers. **(CBSE 2010)**

42. A piece of cloth costs ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, then the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

43. A motor boat whose speed is 24 km/hr in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream

(CBSE 2016)

44. If the roots of the quadratic equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, prove $2b = a + c$.

45. If the equation $(1 + m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

46. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train. **(CBSE 2020)**

47. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park.

(CBSE 2020)

ANSWERS AND HINTS

1. (***d***) [$x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^2 \Rightarrow 4x^3 = 3 \Rightarrow$ degree = 3]

2. (***b***) [Check by substituting $x = 2$ in the equation.]

3. (***a***) [Substitute $x = \frac{1}{2}$ in $x^2 + px - \frac{5}{4} = 0$.]

4. (***c***) [$\because$ A quadratic polynomial is of degree 2 and it has atmost two zeroes.]

5. (***d***) [$x(x-7) = 0 \Rightarrow x = 0, x = 7$.]

6. (***b***) ± 4 ($D = 0, k^2 - 16 = 0$)

7. (*a*) [$r = \frac{q^2}{4p}$ ($D = 0 \Rightarrow q^2 - 4pr = 0$)]

(*b*) $p = -6, q = 1$ [$x^2 - 5x - 6 = 0 \Rightarrow (x-6)(x+1) = 0$]

(*c*) $K \leq 4$ [$D \geq 0 \Rightarrow 16 - 4K \geq 0 \Rightarrow 16 \geq 4K \Rightarrow 4 \geq$ K]

(*d*) $c = 4$ ($\because$ product $= 1 \Rightarrow \frac{c}{a} = 1 \Rightarrow \frac{c}{4} = 1$)

(*e*) Linear equation ($x = 0 \Rightarrow ax^2 + bx + c = 0$ reduces to $bx + c = 0$)

8. (*a*) False (A quadratic equation has atmost two real root).

(*b*) True (Coefficient of $x^2 = a$, Constant $= -c$, $D = b^2 - 4ac = b^2 - 4(a)(-c) = b^2 + 4ac > 0$)

(*c*) False ($(0.3)^2 - 0.9 = 0.09 - 0.9 \neq 0$)

(*d*) False (Degree of quadratic polynomial is 2 not 1 $\because$ Not a straight line)

(*e*) True, D = 0

9. (*i*) $\rightarrow$ *d*

(*ii*) $\rightarrow$ *a*

(*iii*) $\rightarrow$ *e*

(*iv*) $\rightarrow$ *c*

(*v*) $\rightarrow$ *b*

10. $D = 0$

$$20p^2 - 60p = 0, p \neq 0$$
$$20p(p-3) = 0$$
$$p = 3$$

11. (*a*) $x = \frac{7}{2}, x = -\frac{3}{4}$ (*b*) $x = -\sqrt{5}, x = \frac{-2\sqrt{5}}{3}$

(*c*) $x = \sqrt{\frac{2}{3}}, x = \sqrt{\frac{2}{3}}$ (*d*) $x = \frac{a}{2}, x = -a$

(*e*) $x = -\sqrt{3}, x = \frac{-7\sqrt{3}}{3}$ (*f*) $x = -\sqrt{2}, x = \frac{-5\sqrt{2}}{2}$

(*g*) Take $(x - 1) = y$

$y^2 - 5y - 6 = 0 \Rightarrow (y + 1)(y - 6) = 0$

$y = -1, y = 6$

$x - 1 = -1, x - 1 = 6$

$x = 0, x = 7$

12. $D < 0, (-6)^2 - 4(3a)(1) < 0, 12a > 36 \Rightarrow a > 3$

13. $2(-5)^2 + p(-5) - 15 = 0 \Rightarrow p = 7$

$\therefore\ 7x^2 + 7x + k = 0, \quad D = 49 - 28k = 0$

$\Rightarrow k = \frac{49}{28} = \frac{7}{4}$

14. Substituting, $x = \frac{2}{3}$ we get $4a + 9b = -42$ ...(1)

Substituting, $x = -3$ we get $9a + b = 21$ ...(2)

Solve (1) and (2) to get $a = 3, b = -6$.

15. Product $= \frac{c}{a} = \frac{4p}{p} = 4,$ sum $= \frac{-b}{a} = \frac{-6}{p}$

ATQ $= \frac{-6}{p} = 4 \quad \Rightarrow \quad p = \frac{-6}{4} = \frac{-3}{2}$

16. $x^2 + (x + 4)^2 = 656$

$x^2 + 4x - 320 = 0$

$D = 1296 \quad x = \frac{-4 \pm \sqrt{1296}}{2} = \frac{-4+36}{2}, \frac{-4-36}{2}$

$x = \frac{32}{2} = 16$, (rejecting –ve value)

Sides are 16 cm, 20 cm

17. ATQ $\alpha - \beta = 11$

Solve to get $\alpha = 8, \beta = -3$

Sum of roots $\alpha + \beta = \frac{-b}{a} = 5$

$\alpha = 8, \beta = -3$

Product of roots $= \frac{c}{a}$

$-24 = 3k - 3$

$-21 = 3k \quad \Rightarrow \quad k = -7$ Ans.

18. $x^2 + kx + 64 = 0 \rightarrow D_1 = k^2 - 256 \geq 0, \quad k^2 \geq 256$

$\Rightarrow \quad k \geq 16 \quad ...(1)$

$k \leq -16$

$x^2 - 8x + k = 0 \rightarrow D_2 = 64 - 4k \geq 0, 64 \geq 4k$

$\Rightarrow \quad k \leq 16 \quad ...(2)$

(1) and (2) gives $k = 16$

19. (*a*) $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{x-a-b-x}{(a+b+x)x} = \frac{a+b}{ab}$

$-(a+b)\,ab = (a+b)\,(a+b+x)\,x$

$x^2 + xa + bx + ab = 0$

$(x+a)\,(x+b) = 0, x = -a, x = -b$

(*b*) $\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\frac{2x-2a-b-2x}{(2a+b+2x)2x} = \frac{2a+b}{2ab}$

$-(2a+b)2ab = (2a+b)\,(2a+b+2x)2x$

$2x^2 + 2xa + bx + ab = 0$

$(x + a)(2x + b) = 0$, $x = -a$, $x = -\frac{b}{2}$

(c) Take LCM to get $2x^2 + 5x + 3 = 0$, $x = -1$, $x \neq \frac{-3}{2}$. (given)

(d) Take LCM to get $x^2 + 4x - 12 = 0$

Ans. $x = 2, -6$

(e) $(4x^2 + 4bx + b^2) - a^2 = 0$

$(2x + b)^2 - a^2 = 0$ apply $A^2 - B^2 = (A + B)(A - B)$

Ans. $x = -\frac{(a+b)}{2}$, $x = \frac{a-b}{2}$

(f) $4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$2x(2x - a^2) - b^2(2x - a^2) = 0 \Rightarrow (2x - b^2)(2x - a^2) = 0$

$x = \frac{b^2}{2}, \frac{a^2}{2}$

(g) Take LCM to get $11x^2 - 21x - 92 = 0$

$11x^2 - 44x + 23x - 92 = 0$. Solve and get

$x = 4$, $x = \frac{-23}{11}$

(h) $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y$ $\therefore$ $y^2 + 5y - 24 = 0$. Solve to get $y = 3$, $y = -8$

Sub, $\frac{2x}{x-5} = 3$, $\frac{2x}{x-5} = -8$

Ans. $x = 15$, $x = 4$

(i) $4x^2 - 4a^2x + a^4 - b^4 = 0$

$(2x - a^2)^2 - (b^2)^2 = 0$

$(2x - a^2 - b^2)(2x - a^2 + b^2) = 0$

$x = \frac{a^2 + b^2}{2}$, $x = \frac{a^2 - b^2}{2}$

(j) Find $D = b^2 (6a^2 - 1)^2$

Use $x = \dfrac{-B \pm \sqrt{D}}{2A}$ to get answer

Ans. $x = \dfrac{-b}{2a^2}, -3b$

(k) Let $\dfrac{7x+1}{5x-3} = y$

$\therefore\ 3y - \dfrac{4}{y} = 11 \Rightarrow 3y^2 - 11y - 4 = 0$. Solve to get

$y = -\dfrac{1}{3}, y = 4$

Substitute y and get $x = 0, 1$

(l) Take LCM to get $x^2 - 3x + 2 = 0$

Solve to get $x = 1, x = 2$

(m) Take LCM to get $2x^2 - 27x + 88 = 0$

$x = 8, \dfrac{11}{2}$

(n) Take LCM to get $x^2 - 4x - 8 = 0$ (Use quadratic formula)

Ans. $x = 2 \pm 2\sqrt{3}$

(o) Take LCM to get $2x^2 - 16x + 23 = 0$

Solve using Quadratic formula

Ans. $x = \dfrac{-8 \pm 3\sqrt{2}}{2}$

(p) $x^2 + 7\sqrt{5}x - 2\sqrt{5}x - 70 = 0$

$\left(x + 7\sqrt{5}\right)\left(x - 2\sqrt{5}\right) = 0$

$x = 2\sqrt{5}, -7\sqrt{5}$

(q) $\dfrac{16-x}{x} = \dfrac{15}{x+1}$

$x^2 - 16 = 0$

$x = \pm 4$

20. $abx^2 + b^2x - acx - bc = 0$

$(bx - c)(ax + b) = 0$

$x = -\frac{b}{a}, \frac{c}{b}$

21. $D = 0$

$\therefore p^2 - 2p - 3 = 0$; $p = -1, 3$

rejecting $p = -1$,

Ans. $p = 3$.

22. Find D, $D = \left(-4\sqrt{3}\right)^2 - 4(3)(4) = 0$

$\therefore$ Roots are equal and real

Roots are $\frac{-b}{2a}, \frac{-b}{2a} = \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

23. $D = (-6a^2)^2 - 4(9)(a^4 - b^4)$

$= 36b^4$

$$x = \frac{-(-6a^2) \pm \sqrt{36b^4}}{2 \times 9} = \frac{a^2 \pm b^2}{3}$$

24. Equation $\frac{54}{x} + \frac{63}{x+6} = 3$, $x \to$ speed of train at first, $x + 6 \to$ Increased speed.

Ans. $x = 36$, $x \neq -3$.

25. Let the natural number be x.

ATQ, $x + 12 = \frac{160}{x}$ to get $\quad x^2 + 12x - 160 = 0$

$$(x + 20)(x - 8) = 0$$

$$x = 8, \quad x = -20 \text{ (rejected)}$$

26. Let time taken by thief be n minutes.

Policeman will catch the thief in $(n - 1)$ minutes.

Total distance covered by thief = $(100\,n)$ metres...(1)

(as distance covered in 1 min = 100 min)

Distance covered by policemen

$100 + 110 + 120 + ... +$ to $(n - 1)$ 10 ...(2)

(1) and (2) $\Rightarrow 100\, n = \frac{(n-1)}{2}\ [2 \times 100 + (n-2)\ 10]$

Solve and get $n^2 - 3n - 18 = 0$

$n = 6, \quad n \neq -3$

Policeman will catch the thief in 5 minutes.

27. Time taken by top of smaller diameter = x hrs

Time taken by larger tap = $(x-9)$ hrs

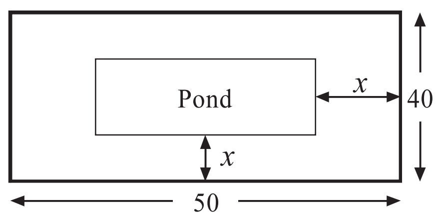
ATQ $\frac{1}{x} + \frac{1}{x-9} = \frac{1}{6}$ and get $x^2 - 21x + 54 = 0$

Ans. $x = 3, x = 18$

$x = 3$ rejected as $x - 9 = -6 < 0$

$\therefore\ x = 18$ hrs $x - 9 = 18 - 9 = 9$ hrs

28.



Length of rectangular lawn = 50 m

Breadth of rectangular lawn = 40 m

Length of pond = $50 - 2x$

Breadth of pond = $40 - 2x$

Area of lawn – Area of pond = area of grass

$50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

get $x^2 - 45x + 296 = 0$

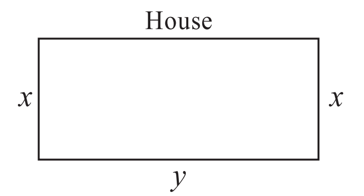
$x = 37, x = 8$

$x = 37$ rejected $\because\ 40 - 2x = 40 - 2(37) < 0$

Ans. Length of pond = 34 m

Breadth of pond = 24 m

29. $x + y + x = 30, xy = 100$

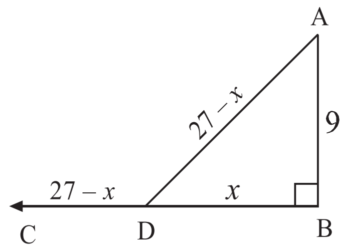


Solve $x = 5\text{m}, 10\text{ m},$

$y = 20\text{ m}, 10\text{ m}$

$\therefore$ dim. are 5 m × 20 m or 10 m × 10 m

30.



In ΔABD, pythagoras theorem $9^2 + x^2 = (27 - x)^2$. Solve it to get $x = 12$ m.

31. Let original list price = ₹ x

ATQ $\frac{300}{x-5} - \frac{300}{x} = 5$

Solve and get $x = 20, x = -15 \rightarrow$ rejected

Ans. ₹ 20

32. Let original number of persons be x

ATQ $\frac{6500}{x} - \frac{6500}{x+15} = 30$

Solve and get $x = 50, x = -65$ (rejected).

33. ATQ $\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$ [Speed of aircraft = x km/hr]

Solve to get $x = 600, x \neq -400$

Duration of flight $\frac{600}{600} = 1\text{hr.}$

34. ATQ $\dfrac{600}{x} - \dfrac{600}{x+10} = 3$ (Speed of slow train x km/hr)

Solve to get $x = 40$, $x = -50$ (rejected).

Ans. 40 km/hr, 50 km/hr.

35. ATQ $\dfrac{30}{15-x} + \dfrac{30}{15+x} = \dfrac{9}{2}$ (Speed of stream x km/hr)

Solve to get x = 5, $x = -5$ (rejected)

Ans. 5 km/hr

36. $x^2 + y^2 = 400$ ...(1)

$4x - 4y = 16 \Rightarrow x - y = 4$ ...(2)

$y - x = 4$ ...(3)

Solve (1) and (2) to get $x = 16$, $x = -12$ (rejected)

Solve (1) and (3) to get $x = 12$, $x = -16$ (rejected)

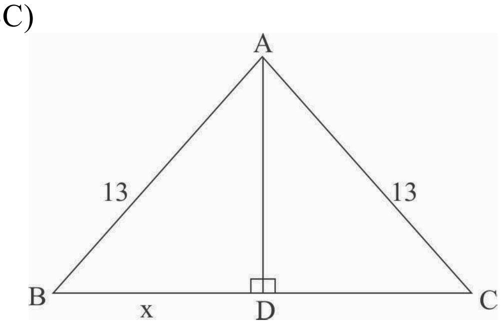
Ans. $x = 16$ m, $y = 12$ m from (1) and (2)

$x = 12$ m, $y = 16$ m from (1) and (3)

37. BC = 2x, BD = x (Draw a $\perp$ from A on BC)

Use pythagoreas to get

AD = $\sqrt{169 - x^2} = 60$



A = $\dfrac{1}{2} \times 2x \times \sqrt{169 - x^2} = 60$

Solve to get $x^2 = 144$, $x^2 = 25$

$x = 12$ or x = 5

$x = -12, -5$ (rejected)

base $2x = 24$ cm or 10 cm

38. Fraction is $\dfrac{x}{2x+1}$

ATQ $\dfrac{x}{2x+1} + \dfrac{2x+1}{x} = 2\dfrac{16}{21} = \dfrac{58}{21}$

Solve to get $x = 3$, $x = \dfrac{-7}{11}$ (rejected)

Ans. Fraction = $\dfrac{3}{7}$.

39. Age of sister = x years

Age of girl = $2x$

ATQ $(x + 4)(2x + 4) = 160$

Solve to get $x^2 + 6x - 72 = 0$

Ans. $x = 6$ years, $x = -12$ (rejected)

$2x = 12$ years

40. Let tens place digit = x, then units digits = $\dfrac{18}{x}$.

No, $10x + \dfrac{18}{x}$

ATQ $\left(10x + \dfrac{18}{x}\right) - \left(\dfrac{10 \times 18}{x} + x\right) = 63$

Solve to get $x = 9, x \neq -2$ (rejected).

Ans. Number is 92

41. Let no. be $x, x + 1, x + 2$ (rejected).

ATQ $(x)^2 + (x + 1)(x + 2) = 46$

To get $2x^2 + 3x - 44 = 0$

Use quadratic formula to get $x = 4, x = -\dfrac{11}{2}$ (rejected)

$\therefore$ Numbers are 4, 5, 6.

42. Let length of piece be x metre.

ATQ $\dfrac{200}{x} - \dfrac{200}{x+5} = 2$

Solve to get $x^2 + 5x - 500 = 0$

Solve to get $x = 20, x = -25$ (rejected)

Rate per meter = $\dfrac{200}{x} = \dfrac{200}{20} = ₹\ 10$

43. Let speed of boat = x

ATQ $\dfrac{32}{24-x} - \dfrac{32}{24+x} = 1$

$x^2 + 64x - 576 = 0$

$(x + 72)(x - 8) = 0$

$x = 8$ km/hr

$x = -72$ km/hr (rejected)

44. Find D and let $D = 0$

$(c - a)^2 - 4(b - c)(a - b) = 0$

Solve to get $(a + c - 2b)^2 = 0$

$\therefore\ a + c = 2b$

45. $D = 0$

$(2\text{ mnc})^2 - 4(1 + m^2)\, n^2 (c^2 - a^2) = 0$

to get $4n^2c^2 = 4n^2a^2(1 + m^2)$

$\therefore\ c^2 = a^2(1 + m^2)$

46. Let the speed of the train = x km/hr

ATQ, $\dfrac{480}{x-8} - \dfrac{480}{x} = 3$

$x^2 - 8x - 1280 = 0$

$x = 40, -32$ (rejected)

$x = 40$ km/hr

47. Let Lm be the length of the rectangular park

Breadth = (L – 3) m

Altitude of the isosceles triangle = 12 m

ATQ $L(L - 3) = \dfrac{1}{2}(12)(L - 3) + 4$

$L^2 - 9L + 14 = 0$

$(L - 7)(L - 2) = 0$

$\Rightarrow\ L = 7, 2$

So, L = 7m ($L = 2$ rejected $\because L - 3 = -1$)

$\because$ Length = 7 m, Breadth = 4 m

Practice Test

Quadratic Equations

Time: 45 Minutes **M.M : 20**

SECTION-A

1. The value of k is .... if $x = 3$ is one root of $x^2 - 2kx - 6 = 0$. **1**
2. If the discriminant of $3x^2 + 2x + \alpha = 0$ is double the discriminant of $x^2 - 4x + 2 = 0$ then value of α is **1**
3. If discriminant of $6x^2 - bx + 2 = 0$ is 1 then value of b is **1**
4. $(x - 1)^3 = x^3 + 1$ is quadratic equation. (T/F) **1**

SECTION-B

5. If roots of $x^2 + kx + 12 = 0$ are in the ratio 1 : 3 find k. **2**
6. Solve for x : $21x^2 - 2x + \dfrac{1}{21} = 0$ **2**
7. Find k if the quadratic equation has equal roots : $kx(x - 2) + 6 = 0$. **2**

SECTION-C

8. Solve using quadratic formula **3**

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

9. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$ is a perfect square. **3**

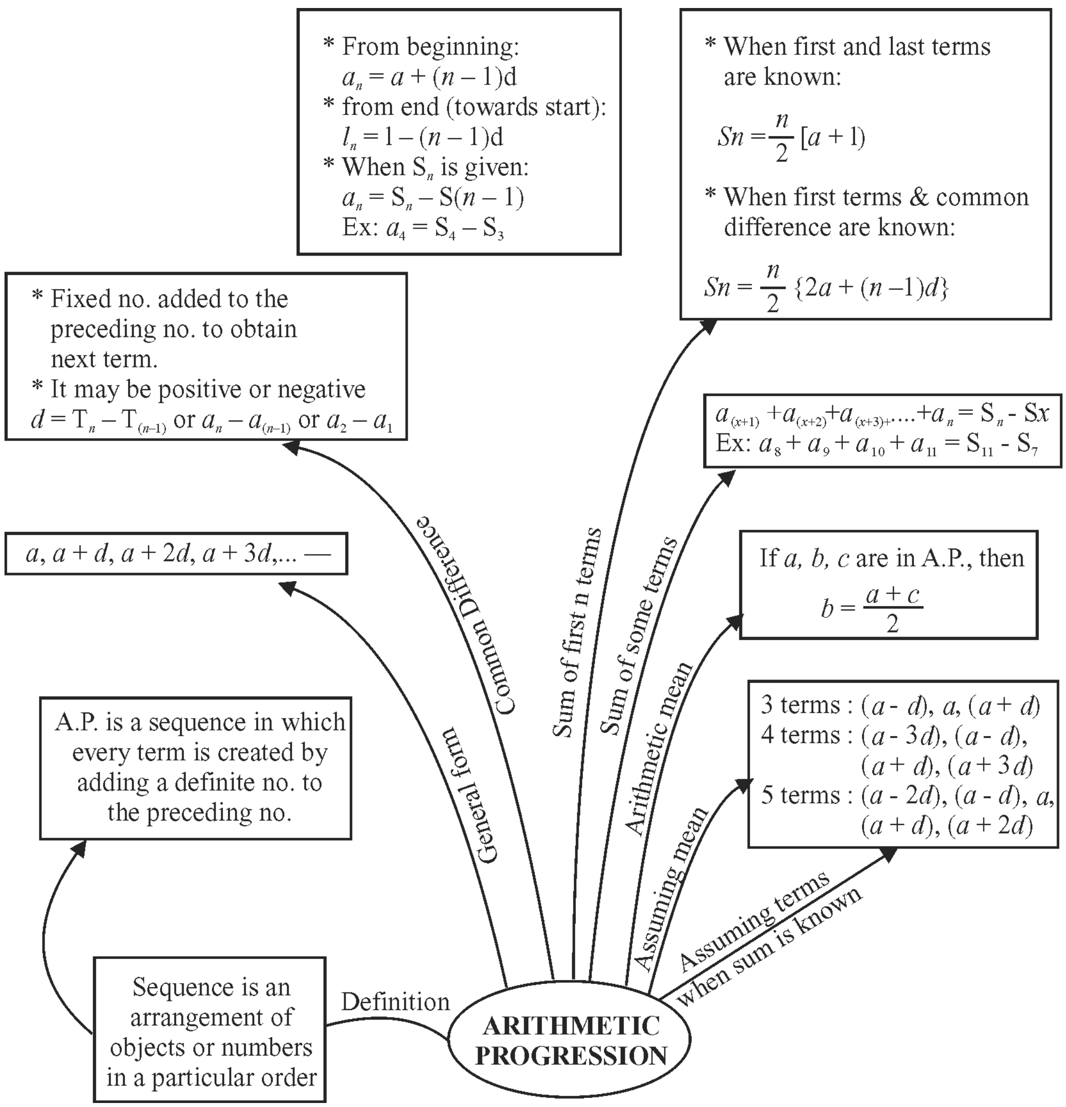
SECTION-D

10. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately. **(CBSE 2018)**

4

CHAPTER

5 Arithmetic Progression



* $a \rightarrow$ first term, $d \rightarrow$ common difference; $a_n \rightarrow n^{th}$ term; S_n Sum of first n terms; $l \rightarrow$ last term

VERY SHORT ANSWER TYPE QUESTIONS

1. Find 5^{th} term of an A.P. whose n^{th} term is $3n - 5$

2. Find the sum of first 10 even numbers.

3. Write the n^{th} term of odd numbers.

4. Write the sum of first n natural numbers.

5. Write the sum of first n even numbers.

6. Find the n^{th} term of the A.P. – 10, – 15, – 20, – 25, ...

7. Find the common difference of A.P. $4\frac{1}{9}, 4\frac{2}{9}, 4\frac{1}{3}$, ...

8. Write the common difference of an A.P. whose n^{th} term is $a_n = 3n + 7$

9. What will be the value of $a_8 - a_4$ for the following A.P.

4, 9, 14, ..., 254

10. What is value of a_{16} for the A.P. – 10, – 12, – 14, – 16, ...

11. 3, $k - 2$, 5 are in A.P. find k.

12. For what value of p, the following terms are three consecutive terms of an A.P.

$\frac{4}{5}$, p, 2.

13. Determine the 36^{th} term of the A.P. whose first two terms are –3 and 4 respectively.

14. Multiple Choice Questions:

(*a*) 30^{th} term of the A.P. 10, 7, 4 .... is

(A) 97 (B) 77

(C) –77 (D) –87

(*b*) 11^{th} term of an A.P. – 3, $-\frac{1}{2}$, , ... is

(A) 28 (B) 22

(C) –38 (D) $-48\frac{1}{2}$

(*c*) In an A.P. if $d = -4$, $n = 7$, $a_n = 4$, then a is

(A) 6 (B) 7

(C) 120 (D) 28

(*d*) The first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$ then y equals: **(CBSE 2014)**

(A) -3 (B) 4

(C) 5 (D) 2

(*e*) The list of numbers $-10, -6, -2, 2, \ldots$ is

(A) An A.P. with $d = -16$ (B) An A.P. with $d = 4$

(C) An A.P. with $d = -4$ (D) Not an A.P.

(*f*) The 11^{th} term from the last term of an A.P. $10, 7, 4, \ldots, -62$ is **(NCERT)**

(A) 25 (B) -32

(C) 16 (D) 0

(*g*) The famous mathematician associated with finding the sum of the first 100 natural numbers is

(A) Pythagoras (B) Newton

(C) Gauss (D) Euclid

(*h*) What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?

(A) 8 (B) -8

(C) -4 (D) 4

(*i*) The nth term of the A.P. $\left(1+\sqrt{3}\right), \left(1+2\sqrt{3}\right), \left(1+3\sqrt{3}\right), \ldots$ is

(A) $1 + n\sqrt{3}$ (B) $n + \sqrt{3}$

(C) $n\left(1+\sqrt{3}\right)$ (D) $n\sqrt{3}$

(*j*) The common difference of the A.P. $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2} \ldots\ldots$ is

(A) $\sqrt{2}$ (B) 1

(C) $2\sqrt{2}$ (D) $-\sqrt{2}$

(*k*) The first term of an A.P. is p and the common difference is q, then its 10^{th} term is

(A) $a + 9p$ (B) $p - 9q$

(C) $p + 9q$ (D) $2p + 9q$

15. Match the following :

Column A	Column B
(*a*) $a = -18$, $n = 10$, $d = 2$ then a_n of A.P.	(*a*) $\frac{a+c}{2}$
(*b*) a, b and c are in A.P. then their Arithmetic mean is	(*b*) 0
(*c*) If 2, 4, 6, are in A.P. then 4, 8, 12 will also be an	(*c*) -41
(*d*) If $a_n = 9 - 5n$ of an A.P. then a_{10} will be	(*d*) 8
(*e*) If $d = -2$, $n = 5$ and $a_n = 0$ in A.P. then a is	(*e*) A.P.

16. State True/False and justify

(*a*) 301 is a term of an A.P. 5, 11, 17, 23 .... (NCERT)

(*b*) Difference of m^{th} and n^{th} term of an A.P. = $(m - n)\,d$.

(*c*) 2, 5, 9, 14, ... is an A.P.

(*d*) Sum of first 20 natural numbers is 410.

(*e*) n^{th} term of an A.P. 5, 10, 15, 20 .... n terms and n^{th} term of A.P. 15, 30, 45, 60, ... n terms are same.

SHORT ANSWER TYPE QUESTIONS-I

17. Is 144 a term of the A.P. 3, 7, 11, ... ? Justify your answer.

18. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b^2)$ are in A.P.

19. Which term of the A.P. 5, 15, 25, ... will be 130 more than its 31^{st} term?

20. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively, Find the sum of all terms of this A.P.

21. Find the sum of first 15 multiples of 8.

22. Is the sequence formed in the following situations an A.P.

(*i*) Number of students left in the school auditorium from the total strength of 1000 students when they leave the auditorium in batches of 25.

(*ii*) The amount of money in the account every year when Rs. 100 are deposit annually to accumulate at compound interest at 4% per annum.

23. Find the sum of even positive integers between 1 and 200.

24. If $4m + 8$, $2m^2 + 3m + 6$, $3m^2 + 4m + 4$ are three consecutive terms of an A.P. find m.

25. How many terms of the A.P. 22, 20, 18, ... should be taken so that their sum is zero.

26. If 10 times of 10^{th} term is equal to 20 times of 20^{th} term of an A.P. Find its 30^{th} term.

27. Solve $1 + 4 + 7 + 10 + ... + x = 287$ **(CBSE 2020)**

28. In an A.P., the sum of first n terms is $\frac{3n^2}{2}+\frac{5n}{2}$. Find its 25th terms. **(NCERT)**

29. Find how many two digit numbers are divisible by 6? **(CBSE 2011)**

30. If $\frac{1}{x+2}$, $\frac{1}{x+3}$ and $\frac{1}{x+5}$ are in A.P. find x. **(CBSE 2011)**

31. Find the middle term of an A.P. $-6, -2, 2, ... 58$. **(CBSE 2011)**

32. In an A.P. find S_n, where $a_n = 5n - 1$. Hence find the sum of the first 20 terms. **(CBSE 2011)**

33. Which term of A.P. 3, 7, 11, 15 .... is 79? Also find the sum $3 + 7 + 11 + ... + 79$. **(CBSE 2011C)**

34. Which term of the A.P. : 121, 117, 113 ... is the first negative terms ? **(NCERT)**

35. Find the 15^{th} term from the last term of the A.P. 3, 8, 13, ... 253. **(CBSE 2022)**

SHORT ANSWER TYPE QUESTIONS-II

36. Find the middle terms of the A.P. 7, 13, 19, ..., 241.

37. Find the sum of integers between 10 and 500 which are divisible by 7.

38. The sum of 5^{th} and 9^{th} terms of an A.P. is 72 and the sum of 7^{th} and 12^{th} term is 97. Find the A.P.

39. If the m^{th} term of an A.P. be $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$, show that its $(mn)^{th}$ is 1.

40. If the m^{th} term of *an* A.P. is $\frac{1}{n}$ and the n^{th} terms is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}(mn+1)$.

41. If the p^{th} term A.P. is q and the q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.

42. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

43. The sum of 5^{th} and 9^{th} terms of an A.P. is 30. If its 25^{th} term is three times its 8^{th} term, find the A.P.

44. If m times the m^{th} terms of an A.P. is equal to n times of n^{th} term and $m \neq n$, show that its $(m+n)^{th}$ term is zero. **(CBSE 2014)**

45. Which term of the A.P. 3, 15, 27, 39 .... will be 120 more than its 21st term?

(CBSE 2018)

46. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, find the n^{th} term.

(CBSE 2018)

47. The sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$. Find the A.P.

(CBSE 2022)

48. In an A.P., the first term is 12 and the common difference is 6. If the last term of the A.P. is 252, then find its middle term. **(CBSE 2022)**

49. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, then find the n^{th} term of the A.P. **(CBSE 2020)**

(NCERT)

50. If the sum of the first 14 terms of an A.P. is 1050 and its fourth term is 40, find its 20th term. **(CBSE 2020)**

51. Find the number of terms in the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots$ of which the sum is 300, explain the double answer. **(NCERT)**

52. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. **(NCERT)**

53. Find the sum of n terms of the series: $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots$

(CBSE 2017)

LONG ANSWER TYPE QUESTIONS

54. The sum of third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first 16 terms of the A.P.

55. Determine the A.P. whose 4th term is 18 and the difference of 9th term from the 15th term is 30.

56. The sum of first 9 terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1:2. Find the first and fifteenth terms of the A.P.

57. The sum of the first 9 terms of an A.P. is 171 and the sum of its first 24 terms is 996. Find the first term and common difference of the A.P. **(CBSE 2020)**

58. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 term is 161. Find the 28^{th} term of this A.P.

59. The sum of first 20 terms of an A.P. is one third of the sum of next 20 term. If first term is 1, find the sum of first 30 terms of this A.P.

60. If the sum of the first four terms of an AP is 40 and the sum of the first fourteen terms of an AP is 280. Find the sum of first n terms of the A.P. **(CBSE 2018)**

61. Ramkali required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly savings by ₹ 20 every week. Find wheather she will be able to send her daughter to school after 12 weeks.

(CBSE 2015)

62. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of last 15 terms is 2565. Find the A.P. **(CBSE 2014)**

63. The sum of first n terms of an A.P. is $5n^2 + 3n$. If the m^{th} term is 168, find the value of m. Also find the 20^{th} term of the A.P. **(CBSE 2013)**

64. If the sum of the first seven terms of an A.P. is 49 and the sum of its first 17 terms is 289. Find the sum of first n terms of an A.P. **(CBSE 2016)**

65. If the 4^{th} term of an A.P. is zero, prove that the 25^{th} term of the A.P. is three times its 11^{th} term. **(CBSE 2016)**

66. In an A.P. if $S_5 + S_7 = 167$ and $S_{10} = 235$. Find the A.P., where S_n denotes the sum of its first n terms. **(CBSE 2015)**

67. In an AP prove $S_{12} = 3\,(S_8 - S_4)$ where S_n represent the sum of first n terms of an A.P. **(CBSE 2015)**

68. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

69. Find the sum of first 16 terms of an Arithmetic Progression whose 4^{th} and 9^{th} terms are -15 and -30 respectively. **(CBSE 2020)**

70. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P.

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. $a_n = 3n - 5 \qquad a_5 = 10$

2. $S_n = \dfrac{10}{2}[2 \times 2 + 9 \times 2] = 110$

3. 1, 3, 5, ...

$a_n = 1 + (n - 1)2 = 2n - 1.$

4. $1 + 2 + ... + n = \dfrac{n}{2}[1 + n]$

5. $2 + 4 + 6 + ... + 2n = \dfrac{n}{2}[2 + 2n] = n(n + 1)$

6. $a_n = a + (n - 1)d = -5(n + 1)$

7. $d = a_2 - a_1 = \dfrac{1}{9}$

8. $a_1 = 3 + 7 = 10,\ a_2 = 6 + 7 = 13,\ d = 3$

9. $(a + 7d) - (a + 3d) = 4d = 20$

10. $a_{16} = a + 15d = -40$

11. 3, $k - 2$, 5 are in A.P.

$\therefore \quad k - 2 = \dfrac{3+5}{2} = 4 \qquad k = 6$

12. $p = \dfrac{7}{5}$ (same as Q.11)

13. $a = -3; a_2 = 4; d = 7$

$a_n = a + (n-1)d$

$a_{36} = -3 + 35 \times 7$

$a_{36} = 242$

14. (*a*) C (*b*) B

(*c*) D (*d*) C

(*e*) B (*f*) B

(*g*) C (*h*) A

(*i*) A (*j*) A

(*k*) C

15. (*a*) $\rightarrow$ (*b*) (*b*) $\rightarrow$ (*a*)

(*c*) $\rightarrow$ (*e*) (*d*) $\rightarrow$ (*c*)

(*e*) $\rightarrow$ (*d*)

16. (*a*) False, $301 = 5 + (n - 1)\, 6$

Solving we get $n = \dfrac{151}{3}$ which is not a natural number.

$\therefore$ 301 is not a term of this A.P.

(*b*) True $[a + (m - 1)\, d] - [a + (n - 1)\, d] = (m - n)\, d$

(*c*) False $\because\ a_2 - a_1 = 5 - 2 = 3$

$\because\ a_3 - a_2 = 9 - 5 = 4$

(*d*) False $\because\ S_n = \dfrac{n(n+1)}{2} = \dfrac{20 \times 21}{2} = 210$

(*e*) False (If a, b, c, d ... are in AP then ka, kb, kc, kd ..... are in AP)

$k \neq 0$, n^{th} term $= k$ times n^{th} term of original A.P. of new A.P.

17. $144 = 3 + (n - 1)\, 4$

$\dfrac{141}{4} + 1 = n$ which is not possible

18. $a_1 = (a - b)^2 \quad a_2 = a^2 + b^2 \quad a_3 = (a + b)^2$

$$\begin{aligned} a_2 - a_1 &= a^2 + b^2 - (a - b)^2 \\ &= 2ab \\ a_3 - a_2 &= (a + b)^2 - (a^2 + b^2) \\ &= 2ab \\ a_2 - a_1 &= a_3 - a_2 \end{aligned}$$

$\therefore$ in A.P.

19. Let $a_n = 130 + a_{31}$

Solve to get $n = 44$

Ans. 44th term

20. $a = 12,\ d = 6,\ a_n = 252 \quad \Rightarrow \quad n = 41$

Find $S_{41} = 5412$, use $S_n = \dfrac{n}{2}\, [2a + (n - 1)\, d]$

21. $S_{15} = \frac{15}{2}[2a + 14d]$

where $a = 8$, $d = 8$

Ans. 960

22. (*i*) Yes $\rightarrow$ (1000, 975, 950, 925 .....)

(*ii*) No $\rightarrow$ (104, 108.16, 112.48 .....)

23. $2 + 4 + 6 + ... + 198$

$a = 2$, $d = 2$, $a_n = 198 \Rightarrow n = 99$

$S_n = \frac{n}{2}[a + l] = 9900$

24. $b = \frac{a+c}{2}$

$\therefore\ 2m^2 + 3m + 6 = \frac{4m+8+3m^2+4m+4}{2}$

Solve to get $m^2 - 2m = 0$

$m = 0, 2$

25. $S_n = 0 \Rightarrow \frac{n}{2}\ [44 + (n - 1)(-2)] = 0.$

Solve $n = 23$

26. ATQ $10\ a_{10} = 20\ a_{20}$

$\Rightarrow a_{10} = 2a_{20}$

$a + 9d = 2a + 38d$

$a = -29d$ ...(1)

$a_{30} = a + 29d$

Substitute a from (1)

Ans. $a_{30} = 0$

27. a = 1, $d = 3$, $a_n = x$

$S_n = 287$

$287 = \frac{n}{2}[2\times1+(n-1)3]$

$\Rightarrow 3n^2 - n - 574 = 0$

$n = 14, \dfrac{-41}{3}$ (rejected)

$\therefore\ n = 14$

$\therefore\ x = a_{14} = 40$

28. $S_n = \dfrac{3n^2}{2} + \dfrac{5n}{2}$

$a_n = S_n - S_{(n-1)}$

$a_{25} = S_{25} - S_{24}$

$= \left[\dfrac{3(25)^2}{2} + \dfrac{5(25)}{2}\right] - \left[\dfrac{3(24)^2}{2} - \dfrac{5(24)}{2}\right]$

$= \dfrac{3}{2}\left[(25)^2 - (24)^2\right] + \dfrac{5}{2}(25 - 24)$

$= \dfrac{3}{2} \times 49 + \dfrac{5}{2} = 76$

$= \dfrac{3}{2} \times 49 + \dfrac{5}{2} = 76$

$a_{25} = 76$

29. Two digit numbers divisible by 6 are 12, 18, 24, ... 96.

$a_2 - a_1 = a_3 - a_2 = 6$

$\therefore$ A.P., $a_n = 96 \Rightarrow n = 15$

30. $\dfrac{2}{x+3} = \dfrac{1}{x+2} + \dfrac{1}{x+5}$ $(2b = a + c)$

Solve to get $x = 1$.

31. $a_n = a + (n - 1)\, d$

$58 = -6 + (n - 1)\, 4$

find $n = 17$

Find Middle term using concept of median

$= \left(\frac{n+1}{2}\right)^{\text{th}}$ term = 9th term

$a_9 = -6 + 8(4) = 26$

32. $a_n = 5n - 1$

Find A.P. $a_1 = 4$, $a_2 = 9$, $a_3 = 14$

4, 9, 14, ...

$a_2 - a_1 = 5 = a_3 - a_2$

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[8 + (n-1)\,5]$

$= \frac{n}{2}[5n + 3]$

$S_{20} = \frac{20}{2}\,[100 + 3] = 10 \times 103 = 1030$

33. $79 = 3 + (n-1)\,4$

$n = 20$

$S_{20} = \frac{20}{2}\,[3 + 79] = 10[82]$

$S_{20} = 820$

34. Let $a_n < 0$

$121 + (n-1)(-4) < 0$

$121 - 4n + 4 < 0$

$125 < 4n$

$n > \frac{125}{4}$

$\therefore\ n = 32$

32nd term will be first negative term.

35. 15th term from end using $[l - (n-1)\,d]$

$= 253 - 14 \times 5$

$= 253 - 70 = 183$

SHORT ANSWER TYPE QUESTIONS-II

36. Same as Q.27, $n = 40$ Middle terms are a_{20}, a_{21}

Ans. 121, 127

37. Numbers between 10 and 500 which are divisible by 7, 14, 21, 28 ..., 497

Find n, using $a_n = a + (n-1)\,d$, then use $S_n = \frac{n}{2}\,[2a + (n-1)\,d]$

Ans. $S_n = 17885$. $(n = 70)$

38. $a_5 + a_9 = 72$

$a_7 + a_{12} = 97$

Solve these equations to get a and d, $a = 6$, $d = 5$

$\therefore$ A.P., 6, 11, 16, 21, 26, ...

39. $a_m = \frac{1}{n} \quad \Rightarrow a + (m-1)d = \frac{1}{n}$

$a_n = \frac{1}{m} \quad \Rightarrow a + (n-1)d = \frac{1}{m}$

$$(m-n)\,d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$\therefore d = \frac{1}{mn}$, find $a = \frac{1}{mn}$

$a_{mn} = a + (mn-1)\,d$

$= \frac{1}{mn} + (mn-1)\frac{1}{mn}$

$a_{mn} = 1.$

40. $a_m = a + (m-1)d = \frac{1}{n}$ ...(1)

$a_m = a + (n-1)d = \frac{1}{m}$ ...(2)

Subtracting equation 2 from equation 1, we get

$d = \frac{1}{mn}$

$$a = \frac{1}{mn}$$

$$S_{mn} = \frac{mn}{2}\{2a + (mn - 1)d\}$$

$$S_{mn} = \frac{1}{2}(mn + 1)$$

41. $a_p = q, \quad a_q = p$

Soved to get a and d, $a = q + p - 1$, $d = -1$

$a_n = p + q - n$

42. Numbers divisible by both 2 and 5

$\Rightarrow$ Numbers divisible by 10.

Numbers between 101 and 999 divisible by 2 and 5 both 110, 120, 130, 140, ..., 990.

Use $a_n = 990$ to get $n = 89$.

43. ATQ $a_5 + a_9 = 30$

$a_{25} = 3\ a_8$

Solve to get $a = 3$, $d = 2$

A.P. 3, 5, 7, 9, ...

44. $m \times a_m = n \times a_n$

$$a(m - n) = d[(m - n) - (m^2 - n^2)]$$

$$(m - n)\{a + (m + n - 1)d\} = 0$$

$$(m - n)a_{(m+n)} = 0$$

$$a_{(m+n)} = 0$$

45. Let $a_n = 120 + a_{21}$

$3 + (n - 1)d = 120 + [3 + 20d]$

$3 + (n - 1)12 = 120 + [3 + 20 \times 12]$

$= 120 + 243$

$(n-1)12 = 363 - 3 = 360$

$n = 31$

46. $S_n = 3n^2 - 4n$

$a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - [3(n-1)^2 - 4(n-1)]$

$= (3n^2 - 4n) - [3n^2 + 3 - 6n - 4n + 4]$

$= -[7 - 6n]$

$a_n = 6n - 7$

47. $S_n = 3n^2 + 2n$

$S_1 = 5;\ S_2 = 16; S_3 = 33$

$a_n = S_n - S_{(n-1)}$

$a = S_1 = 5$

$a_2 = S_2 - S_1 = 16 - 5 = 11$

$a_3 = S_3 - S_2 = 33 - 16 = 17$

A.P. : 5, 11, 17, ...

48. $a = 12; d = 6; a_n = 252$

$a_n = a + (n-1)d$

Substitute the values and find$n$

$n = 41$

Middle terms $= \dfrac{41+1}{2} = 21^{\text{st}}$ term

$a_{21} = 132$

Middle term of A.P. is 132

49. ATQ,

$a_{17} = 5 + 2 \times a_8$

$a + 16d = 5 + 2a + 14\,d$

$a - 2d = -5$ ...(1)

$a_{11} = a + 10d = 43$ ....(2)

Solving (1) & (2), we get

$a = 3,\ d = 4$

$\therefore\ a_n = 4n - 1$

50. $S_{14} = 1050,\ a_4 = 40$

$S_{14} = \dfrac{14}{2}\ [2 \times a + 13d]$

$\dfrac{1050}{7} = 2a + 13d$

Solve $2a + 13d = 150$ and $a + 3d = 40$ to get $a = 10,\ d = 10$

$a_{20} = a + 19d = 10 + 190 = 200$

51. $a = 20;\ d = {}^{-2}\!/_{3}$

$S_n = 300$

$S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

Substitute the values and find n

$n = 25$ or 36

Sum of 26^{th} to 36^{th} term is 0.

52. $a = 5;\ l = 45;\ Sn = 400$

$S_n = \dfrac{n}{2}(a + l)$

$n = 16$

$l = a + (n - 1)d$

$d = \dfrac{8}{3}$

53. $\left(4 - \dfrac{1}{n}\right) + \left(4 - \dfrac{2}{n}\right) + \left(4 - \dfrac{3}{n}\right) \ldots$

$= (4 + 4 + 4 + \ldots) - \dfrac{1}{n}(1 + 2 + 3 + \ldots)$

$= 4n - \dfrac{1}{n} \times \dfrac{n(n+1)}{2}$

$= \dfrac{7n-1}{2}$ **(NCERT)**

LONG ANSWER TYPE QUESTIONS

54. $a_3 + a_7 = 6, \quad a_3 \times a_7 = 8$

On Solving

$a = 1, \quad d = \frac{1}{2} \quad S_n = 16$

$a = 5, \quad d = \frac{-1}{2} \quad S_n = 20$

Ans. 76, 20

55. **ATQ** $a_4 = 18$ ...(1), $a_{15} - a_9 = 30$ ...(2)

equation (2) will give $d = 5$

Substitute $d = 5$ in (1) to get $a = 3$

A.P. 3, 8, 13, ...

56. ATQ $S_9 = 162 \Rightarrow \frac{9}{2}[2a + 8d] = 162$ ...(1)

ATQ $\frac{a_6}{a_{13}} = \frac{1}{2}$ solve and get $a = 2d$

Sub $a = 2d$ in (1) to get $d = 3$, $a = 6$

$a_{15} = a + 14d$

Ans. $a_{15} = 48$, $a = 6$

57. $S_9 = 171, \quad S_{24} = 996$

$a + 4d = 19, \quad 2a + 23d = 83$

Solve to get,

$d = 3$, $a = 7$

58. ATQ $S_7 = 63$, ...(1)

Sum of next 7 terms $= S_{14} - S_7 = 161$ ...(2)

Use $S_n = \frac{n}{2}[2a + (n - 1)d]$

Solve (1) and (2) to get a and d then find a_{28} using $a_n = a + (n - 1)d$.

$a = 3$, $d = 2$

Ans. $a_{28} = 57$

59. ATQ $S_{20} = \frac{1}{3}(S_{40} - S_{20})$, $a = 1$

Use $S_n = \frac{n}{2}[2a + (n-1)d]$ and $a = 1$ to find d, $d = 2$

then find S_{30}.

Ans. 900

60. $S_4 = 40 \Rightarrow \frac{4}{2}[2a + 3d] = 40$

$S_{14} = 280 \Rightarrow \frac{14}{2}[2a + 13d] = 280$

Solve to get $a = 7$, $d = 2$

Ans. $S_n = n^2 + 6n$ (using $S_n = \frac{n}{2}[2a + (n-1)d]$

61. $a = 100$, $d = 20$, $n = 12$

$S_{12} = \frac{12}{2}[200 + 220] = 6 \times 420$

$= 2520 > 2500$

$\therefore$ Ram kali will be able to send her daughter to school after 12 weeks.

62. $S_{10} = 210 \Rightarrow 5[2a + 9d] = 210$

$2a + 9d = 42$ ...(1)

$S_{50} - S_{35} = 2565 \Rightarrow \frac{50}{2}[2a + 49d] - \frac{35}{2}[2a + 34d] = 2565$

$\frac{15}{2}(2a) + d[25 \times 49 - 35 \times 17] = 2565$

$15a + d[1225 - 595] = 2565$

or $15a + 630d = 2565$

or $3a + 126d = 513$ ...(2)

Solve (1) and (2) $d = 4$, $a = 3$.

63. $S_n = 5n^2 + 3n$

$S_1 = a_1 = 8$

$S_2 = a_1 + a_2$

$26 = 8 + a_2 \Rightarrow a_2 = 18$

$d = 18 - 8 = 10$

$a_m = 168 \quad \Rightarrow \quad a + (m-1)d = 168$

$\qquad 8 + (m-1)10 = 168 \Rightarrow m = 17$

$a_{20} = a + 19d = 8 + 190 = 198$

64. $S_7 = 49$, $S_{17} = 289$ (Solve just like Q 54.)

65. $a_4 = 0 \quad \Rightarrow \quad a + 3d = 0 \quad \Rightarrow \quad a = -3d$

$a_{25} = a + 24d = -3d + 24d = 21d$

$\therefore\ a_{25} = 3a_{11}$

$a_{11} = a + 10d = -3d + 10d = 7d$

66. Use $S_n = \frac{n}{2}\,[2a + (n-1)\,d]$

$S_5 + S_7 = 167 \qquad S_{10} = 235$

Solve to get $a = 1$, $d = 5$

A.P. = 1, 6, 11, 16, 21, ...

67. L.H.S. = $S_{12} = \frac{12}{2}\,[2a + 11d] = 6\,[2a + 11d]$

R.H.S. = $3\left[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)\right] = 3[4a + 22d] = 6[2a + 11d]$

$\therefore$ L.H.S. = R.H.S.

68. Four consecutive terms are :

$a - 3d, a - d, a + d, a + 3d$

$d = 8$

$$\frac{\text{Product of Extremes}}{\text{Product of means}} = \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

Put $a = 8$ and solve to get

$\Rightarrow \quad d^2 = 4$

$\qquad d = \pm 2$

$\therefore \quad$ for $a = 8$, $d = 2$ terms are 2, 6, 10, 14

$\qquad$ for $a = 8$, $d = -2$ terms are 14, 10, 6, 2

69. $a_4 = -15$, $a_9 = -30$

$a + 3d = -15$, $a + 8d = -30$

Solve to get $a = -6$, $d = -3$

$S_{16} = -456$ $\left[S_n = \dfrac{n}{2}\{2a + (n-1)\,d\}\right]$

70. $a, a_2, a_3, ..., a_{36}, a_{37}$

3 middle most terms – a_{18}, a_{19}, a_{20}

$a_{18} + a_{19} + a_{20} = 225 \Rightarrow a + 18d = 75$ ...(1)

$a_{35} + a_{36} + a_{37} = 429 \Rightarrow a + 35d = 143$ ...(2)

Solving (1) and (2)

$a = 3$; $d = 4$

A.P. $\rightarrow$ 3, 7, 11, ..., 147

Practice Test

Arithmetic Progression

Time: 45 Minutes **M.M. : 20**

Section-A

1. Find the sum of first 10 natural numbers. **1**

2. What is the common difference of an A.P. $8\frac{1}{8}, 8\frac{2}{8}, 8\frac{3}{8}, \ldots\ldots\ldots\ldots$ **1**

3. If k, $2k - 1$ and $2k + 1$ are in A.P. them value of k is **1**

4. The 10th term from the end of the AP 8, 10, 12, ..., 126 is **1**

Section-B

5. How many 2 digit number are there in between 6 and 102 which are divisible by 6. **2**

6. The sum of n terms of an A.P. is $n^2 + 3n$. Find its 20th term. **2**

7. Find the sum $(-5) + (-8) + (-11) + \ldots + (-230)$ **2**

Section-C

8. Find the five terms of an A.P. whose sum is $12\frac{1}{2}$ and first and last term ratio is 2 : 3. **3**

9. Find the middle term of an A.P. 20,16,12,.......,– 176. **3**

Section-D

10. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers. **4**

CHAPTER

6 Triangles

TRIANGLES

SIMILAR

(a) Corresponding angles are equal

or

(b) Corresponding sides are proportional

other Theorems

Thales/Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

SIMILARITY CRITERIA

(a) AAA criterion: If in two triangles, corresponding angles are equal, then their corresponding sides are proportional and hence the two triangles are similar.

(b) SSS criterion: If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

(c) SAS criterion: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Converse of Thales/B.P.T.

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Key Points:

1. **Similar Triangles:** Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are proportional.

2. **Criteria for Similarity:**

   In ΔABC and ΔDEF

   (*i*) **AAA Similarity:** $\Delta ABC \sim \Delta DEF$ when $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

   (*ii*) **SAS Similarity :**

   $\Delta ABC \sim \Delta DEF$ when $\dfrac{AB}{DE} = \dfrac{BC}{EF}$ and $\angle B = \angle E$

   (*iii*) **SSS Similarity :** $\Delta ABC \sim \Delta DEF$, $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

3. (***a***) **(Theorems with Proof)**

   Basic Proportionality Theorem : If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

   (***b***) **(Theorems without Proof)**

   Converse of BPT Theorem : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (without proof).

VERY SHORT ANSWER TYPE QUESTIONS

1. **Fill in the blanks :**

   (*i*) All equilateral triangles are __________ .

   (*ii*) If $\Delta ABC \sim \Delta FED$, then $\dfrac{AB}{\_\_\_} = \dfrac{\_\_\_}{ED}$.

   (*iii*) Circles with equal radii are _________ .

   (*iv*) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the _________ ratio.

(v) If two triangles are similar, their corresponding sides are __________.

(CBSE 2020)

(vi) In ΔABC, AB = $6\sqrt{3}$, AC = 12 cm and BC = 6 cm, then $\angle$B = _______ .

2. State True or False :

(i) All the similar figures are always congruent.

(ii) The Basic Proportionality Theorem was given by Pythagoras.

(iii) The mid-point theorem can be proved by Basic Proprotionality Theorem.

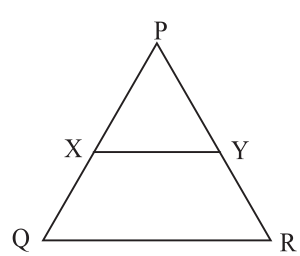
(iv) Two squares are similar figures.

(v) If all the sides of a triangle are proportional to the sides of other triangle, then the two triangles are cngruent.

3. Match the following :

Column I	**Column II**
(a) If corresponding angles are equal in two triangles, then the two triangles are similar.	(i) SAS similarity criterion
(b) If sides of one triangle are proportional to the sides of the other triangle, then the two triangles are similar.	(ii) ASA similarity criterion
(c) If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.	(iii) AAA similarity criterion
	(iv) SSS similarity criterion

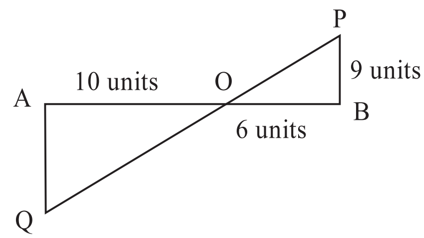
4. In the following figure, XY $\parallel$ QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



(*a*) XY = QR (*b*) $XY = \frac{1}{3} QR$

(*c*) $XY^2 = QR^2$ (*d*) $XY = \frac{1}{2} QR$

5. In the following figure, $QA \perp AB$ and $PB \perp AB$, then AQ is



(*a*) 15 units (*b*) 8 units

(*c*) 5 units (*d*) 9 units

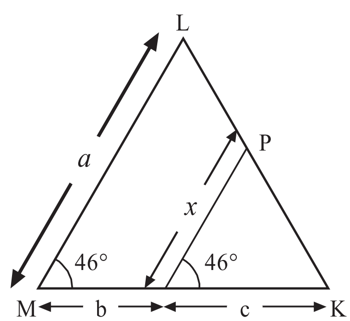
6. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true? **(NCERT Exempler)**

(*a*) BC.EF = AC.FD (*b*) AB.EF = AC.DE

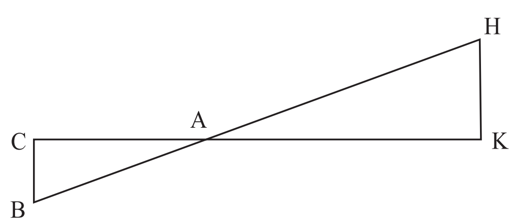
(*c*) BC.DE = AB.EF (*d*) BC.DE = AB.FD

7. Write the Statement of Basic Proportionality Theorem.

8. In the given Fig., $\angle M = \angle N = 46°$, Express x in terms of a, b and c.



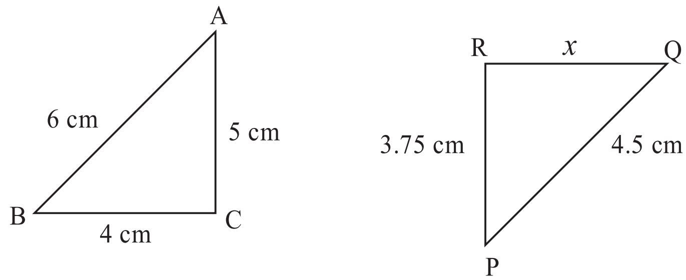
9. In the given Fig. $\Delta AHK \sim \Delta ABC$. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. **(CBSE 2010)**



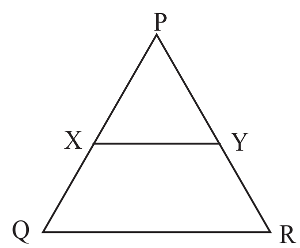
10. It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$?

11. If the corresponding Medians of two similar triangles are in the ratio 5 : 7. Then find the ratio of their sides.

12. In the given figure, if $\Delta ABC \sim \Delta PQR$, find the value of x?

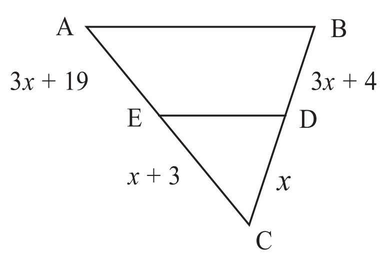


13. In the given figure, $XY \parallel QR$ and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, find XY : QR.



14. In the given figure, find the value of x which will make $DE \parallel AB$?

(NCERT Exempler)

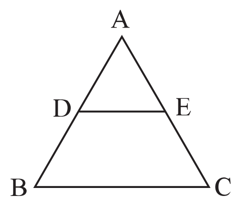


15. If ΔABC and ΔDEF are similar triangles such that $\angle A = 45°$ and $\angle F = 56°$, then find the value of $\angle C$.

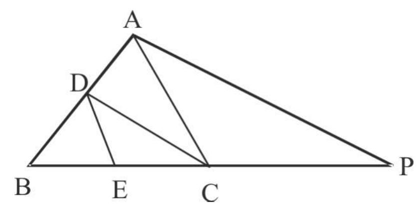
16. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then find the ratio of their corresponding attitudes.

SHORT ANSWER TYPE QUESTIONS-I

17. In the given fig. $\frac{BD}{AB} = \frac{CE}{AC}$, then prove that DE || BC

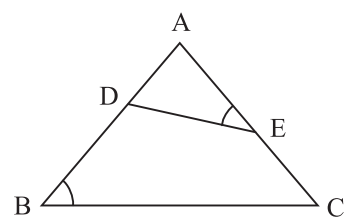


18. In the given Fig., DE || AC and DC || AP Prove that $\frac{BE}{EC} = \frac{BC}{CP}$ **(CBSE 2020)**

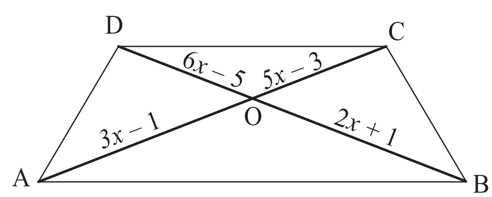


19. In ΔPQR, MN || QR, using B.P.T. prove that $\frac{PM}{PQ} = \frac{PN}{PR}$.

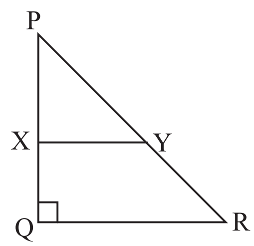
20. In the given Fig., D and E are points on sides AB and CA of ΔABC such that ΔB = $\angle$AED. Show that ΔABC ~ ΔAED.



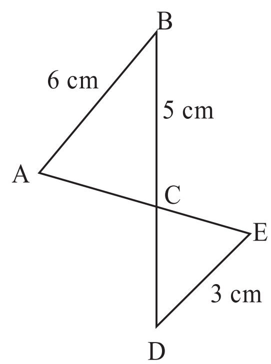
21. In the given fig., AB || DC and diagonals AC and BD intersects at O. If OA = $3x - 1$ and OB = $2x + 1$, OC = $5x - 3$ and OD = $6x - 5$, find the value of x.



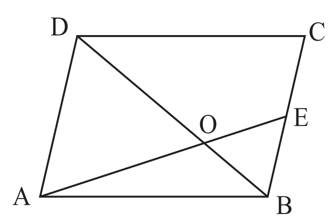
22. In the given Fig. PQR is a triangle, right angled at Q. If XY || QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.



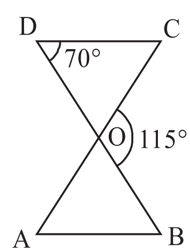
23. In the given figure, AB || DE. Find the length of CD.



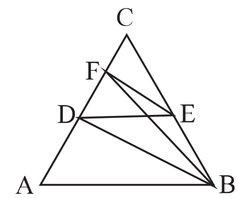
24. In the given figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm find BC.



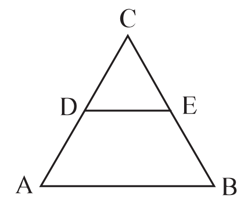
25. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 70°$. Find, (*i*) $\angle DOC$, (*ii*) $\angle DCO$, (*iii*) $\angle OAB$, (*iv*) $\angle OBA$.



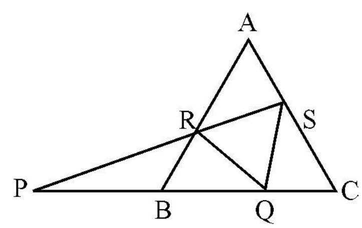
26. In the given fig., AB || DE and BD || EF prove that $DC^2 = CF \times AC$



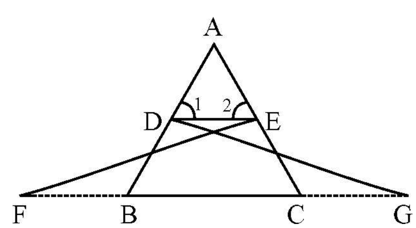
27. In the given fig. $\frac{AD}{DC} = \frac{BE}{EC}$ and $\angle CDE = \angle CED$, prove that ΔCAB is isoceles.



28. In the given fig., QS || BA, QR || CA and PQ = 10 cm. Find PB × PC.



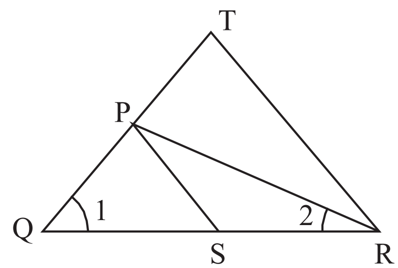
29. In the given fig., $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$ prve that $\Delta ADE \sim \Delta ABC$.



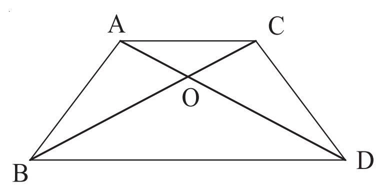
SHORT ANSWER TYPE QUESTIONS-II

30. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ then prove that $\Delta PQS \sim \Delta TQR$.

(NCERT)



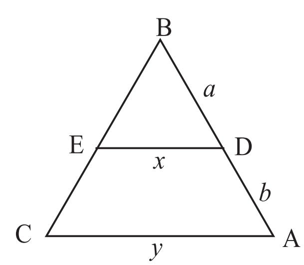
31. In the adjoining figure ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta DBC)} = \frac{AO}{DO}$. **(CBSE 2020)**



32. If AD and PS are medians of ΔABC and ΔPQR respectively where $\Delta ABC \sim \Delta PQR$, Prove that $\frac{AB}{PQ} = \frac{AD}{PS}$.

33. In the given figure, DE $\|$ AC. Which of the following is correct?

$x = \frac{a+b}{ay}$ or $x = \frac{ay}{a+b}$



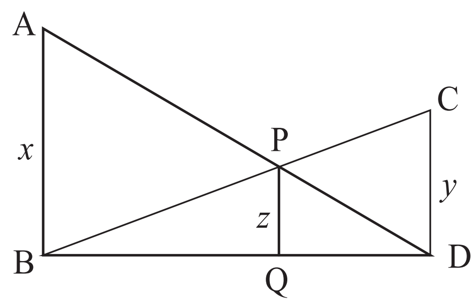
34. If three parallel lines are intersected by two transversals,them prove that the intercepts made by them on the transversals are proportional.

35. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how far she is away from the base of the pole.

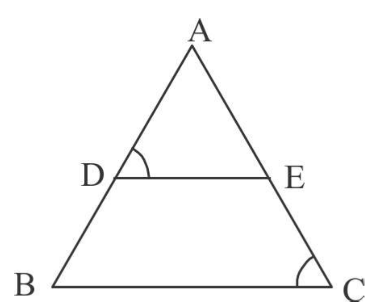
(NCERT Exempler)

36. Two poles of height a metrs and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

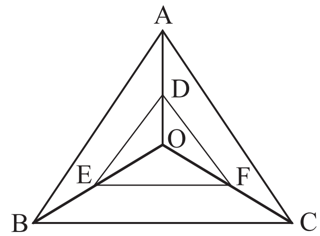
37. In the given figure AB || PQ || CD, AB = x, CD = y and PQ = z. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



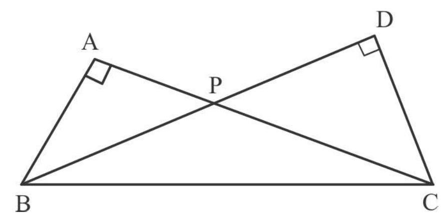
38. In the given figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that ΔBAC is an isoscles triangle. **(CBSE 2020)**



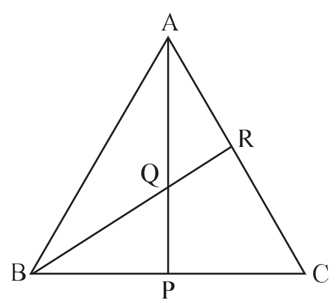
39. In the figure, a point O inside ΔABC is joined to its vertices. From a point D on AO, DE is drawn parallel to AB and from a point E on BO, EF is drawn parallel to BC. Prove that DF $\parallel$ AC.



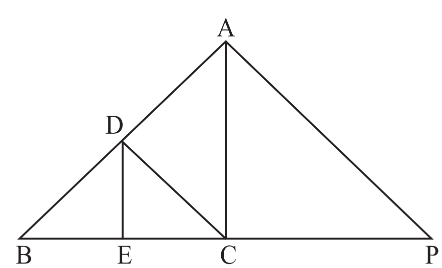
40. Two triangles ΔBAC and ΔBDC, right angled at A and D respectively are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P. PRove that AP × PC = DP × PB. **(CBSE 2019)**



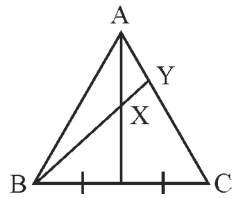
41. In the given fig., P is the mid point of BC and Q is the mid point of AP. If BQ when produced meets AC at R, prove that RA = 1/3 CA. **(CBSE)**



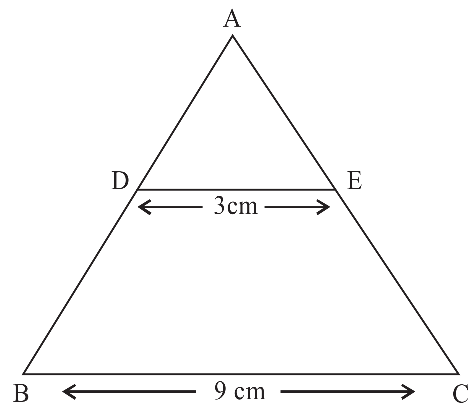
42. In the given figure DE $\parallel$ AC and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that DC $\parallel$ AP.



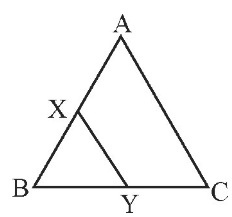
43. In ΔABC, AD is a median, X is a point on AD such that AX : XD = 2 : 3. Ray BX intersects AC in Y. Prove that BX = 4XY.



44. In the given figure, DE || BC, DE = 3 cm, BC = 9 cm and ar (ΔADE) = 30 cm^2. Find ar (BCED).



45. In the given figure, the line segment XY is Parallel to AC ofΔABC and it divides the triangle into two parts of equal areas. Prove that $\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$.



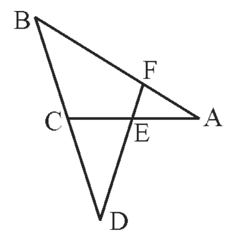
46. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that $\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$.

47. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

(CBSE 2019, 2020)

48. Through the mid point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL.

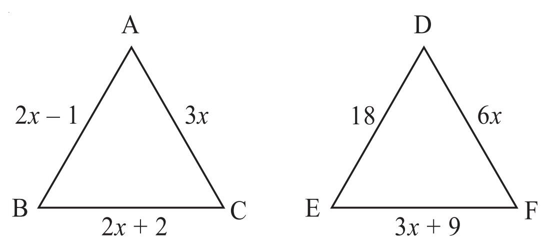
49. In the given figure, $\angle AEF = \angle AFE$ and E is the mid-point of CA. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$.



50. Sides AB and AC and median AD of ΔABC are respectively proportional to sides PQ and PR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$.

(CBSE 2020)

51. In figure if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle. **(CBSE 2020)**



52. The Perimeters of two similar triangles are 30 cm and 20 cm repsectively. If one side of the first triangle is 9 cm long. Find the length of the corresponding side of the second triangle. **(CBSE 2020)**

53. If in ΔABC, D be a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$, then show that AD is bisector of $\angle A$.

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. (*i*) Similar (*ii*) $\frac{AB}{FE} = \frac{BC}{ED}$ (*iii*) Congruent

(*iv*) Same (*v*) Proportional (*vi*) 90°

2. (*i*) False (*ii*) False (*iii*) True

(*iv*) True (*v*) False

3. (*a*) (*iii*) AAA similarity criterion.

(*b*) (*iv*) SSS similarity criterion.

(*c*) (*i*) SAS similarity criterion.

4. (B) $XY = \frac{1}{3}QR$

5. (A) 15 units

6. (C) BC.DE = AB.EF

7. See point 3(*i*) of Key Points.

8. $\Delta KPN \sim \Delta KLM$

$$\frac{x}{a} = \frac{c}{b+c}$$

$$x = \frac{ac}{b+c}$$

9. $\frac{AK}{AC} = \frac{HK}{BC} \Rightarrow \frac{10}{AC} = \frac{7}{3.5} \Rightarrow AC = 5 \text{ cm}$

10. $\angle D = \angle R$ (True)

$\angle F = \angle P$ (False)

11. 5 : 7

12. $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{6}{4.5} = \frac{4}{x} \Rightarrow x = 3\text{cm}$

13. $\Delta PXY \sim \Delta PQR$

$$\frac{PX}{PQ} = \frac{XY}{QR} = \frac{1}{3}$$

$\therefore$ XY : QR = 1 : 3

14. $\dfrac{x+3}{3x+19} = \dfrac{x}{3x+4}$ (By B.P.T.)

$x = 2$

15. $\angle F = \angle C = 56°$

16. 2 : 3

17. $\dfrac{BD}{AB} = \dfrac{CE}{AC}$

Substracting 1 from reciprocal

$$\frac{AB}{BD} - 1 = \frac{AC}{CE} - 1$$

$$\frac{AD}{BD} = \frac{AE}{CE} - 1$$

$\Rightarrow$ DE $\parallel$ BC

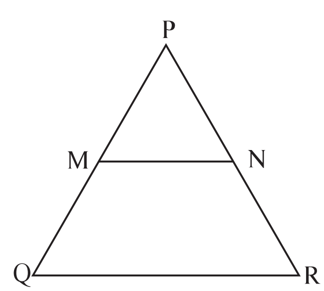
18. DE $\parallel$ AC, $\dfrac{AD}{DB} = \dfrac{EC}{BE}$ ...(1) [$\because$ BPT]

DC $\parallel$ AP, $\dfrac{AD}{DB} = \dfrac{CP}{BC}$ ...(2) [$\because$ BPT]

From (1) and (2), we get

$$\frac{BE}{EC} = \frac{BC}{CP}$$

19. In ΔPQR, MN $\parallel$ QR



$$\frac{MQ}{PM} = \frac{NR}{PN}$$

Adding 1 to both sides and we get

$\frac{PQ}{PM} = \frac{PR}{PN}$

Apply invertendo

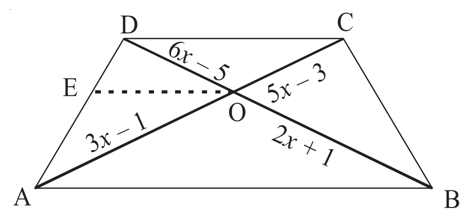
$\Rightarrow \frac{PM}{PQ} = \frac{PN}{PR}$

20. $\angle B = \angle AED$ (Given)

$\angle A = \angle A$ (Common)

$\therefore \Delta ABC \sim \Delta AED$ [AA similarity criterion]

21. Draw EO || AB, $\frac{DE}{EA} = \frac{DO}{OB}$ (In ΔADB) and $\frac{DE}{EA} = \frac{OC}{OA}$ (In ΔACD)



$\frac{3x-1}{5x-3} = \frac{2x+1}{6x-5} \quad \Rightarrow \quad x = \frac{1}{2}$ or 2

But $x = \frac{1}{2}$ is neglected because $(5x - 3)$ get negative value.

So, $x = 2$ is the required value.

22. $\frac{PX}{XQ} = \frac{PY}{YR} \quad \Rightarrow \quad \frac{1}{2} = \frac{4}{YR} \quad \Rightarrow$ YR = 8 cm

$\therefore$ PR = 8 + 4 = 12 cm

QR = $\sqrt{(12)^2 - (6)^2} = 6\sqrt{3}$ cm

23. $\Delta ABC \sim \Delta EDC$ (AA Similarity criterion)

$\frac{6}{3} = \frac{5}{CD}$

CD = 2.5 cm

24. $\Delta BOE \sim \Delta DOA$ (AA Similarity criterion)

$$\frac{BO}{DO} = \frac{BE}{DA}$$

$$\frac{1}{2} = \frac{1.5}{DA}$$

DA = 3 cm

BC = DA = 3 cm (Opposite sides of a parallelogram)

25. (*i*) 65°

(*ii*) 45°

(*iii*) 45°

(*iv*) 70°

26. In ΔCAB, $DE \parallel AB$

$$\Rightarrow \frac{DC}{AC} = \frac{CE}{BC} \quad ...(1)$$

In ΔCDB, $BD \parallel EF$

$$\frac{CF}{DC} = \frac{CE}{BC} \quad ...(2)$$

$$\Rightarrow \frac{DC}{AC} = \frac{CF}{DC}$$

$\Rightarrow DC^2 = CF \times AC$

27. In ΔCAB

$$\Rightarrow \frac{AD}{DC} = \frac{BE}{EC}$$

$\Rightarrow DE \parallel AB$ (Converse of B.P.T.)

$\Rightarrow \angle A = \angle D, \angle B = \angle E$

$\Rightarrow \angle A = \angle B$

$\Rightarrow \Delta ABC$ is isosceles.

28. In ΔPSQ

$$\frac{PB}{PQ} = \frac{PR}{PS} \quad ...(1)$$

In ΔPSC

$$\frac{PQ}{PC} = \frac{PR}{PS}$$

$$\frac{PB}{PQ} = \frac{PQ}{PC}$$

$\Rightarrow PB \times PC = (PQ)^2$

$\Rightarrow PB \times PC = 100 \text{ cm}^2$

29. $EC = BD \; (\because \Delta FEC \cong \Delta GBD)$

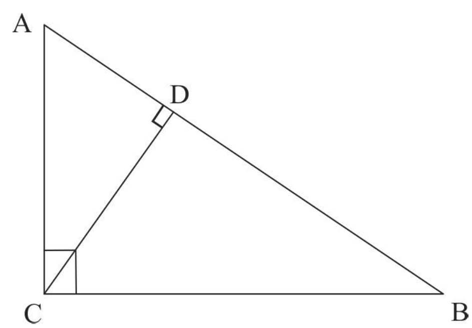
$AD = AE \; (\because \angle 1 = \angle 2)$

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$\Rightarrow DE \parallel BC$

$\Rightarrow \Delta ADE \sim \Delta ABC$

30. $\Delta ABC \sim \Delta CBD$



$\therefore BC^2 = AB.BD \quad ...(1)$

$\Delta ABC \sim \Delta ACD$

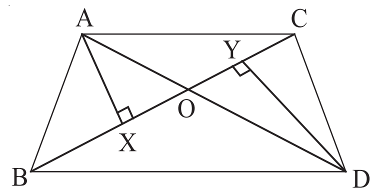
$\therefore AC^2 = AB.AD \quad ...(2)$

Divide (1) by (2), we get

$$\frac{BC^2}{AC^2} = \frac{BD}{AD}$$

31. Draw $AX \perp BC$ and $DY \perp BC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AX}{\frac{1}{2} \times BC \times DY} = \frac{AX}{DY} \quad ...(1)$$



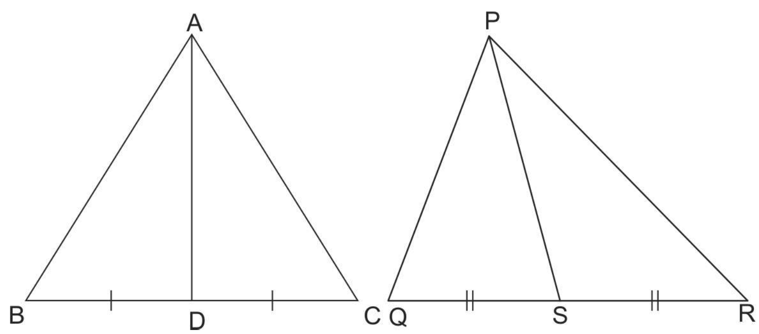
$\Delta AXO \sim \Delta DYO$ [AA similarity criterion]

$$\frac{AX}{DY} = \frac{AO}{DO} \quad ...(2) \quad \text{(C.P.S.T.)}$$

From (1) and (2), we get

$$\frac{ar\,(\Delta\,ABC)}{ar\,(\Delta\,DBC)} = \frac{AO}{DO}$$

32.



As $\Delta ABC \sim \Delta PQR$, Hence $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QS}$

In ΔABD and ΔPQS

$\frac{AB}{PQ} = \frac{BD}{QS}$ and $\angle B = \angle Q$.

$\therefore\ \Delta ABD \sim \Delta PQS$ (SAS Similarity criterion).

Hence, $\frac{AB}{PQ} = \frac{AD}{PS}$ (C.P.S.T.)

33. $\Delta BED \sim \Delta BCA$

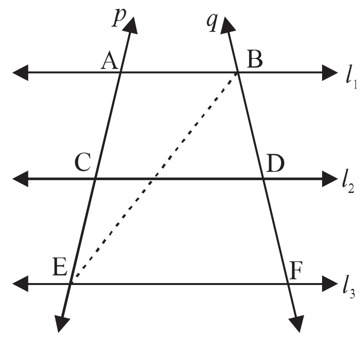
$$\frac{x}{y} = \frac{a}{a+b}$$

$$\Rightarrow x = \frac{ay}{a+b}$$

34. l1 || l2 || l3

Constr: Join BE

Proof: In ΔABE



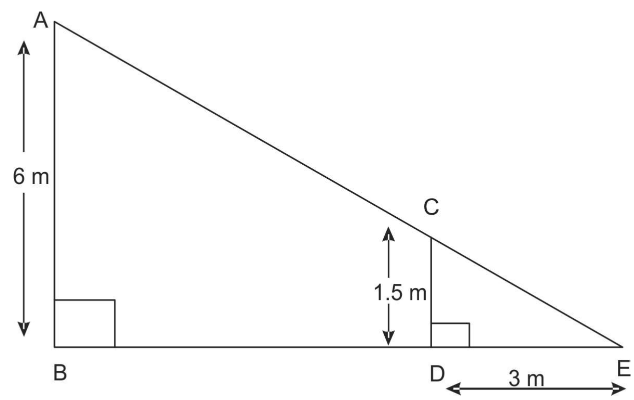
$$\frac{AC}{CE} = \frac{BX}{XE} \quad ...(1)$$

In ΔBEF

$$\frac{BX}{XE} = \frac{BD}{DF} \quad ...(2)$$

$$\Rightarrow \frac{AC}{CE} = \frac{BD}{DF}$$

35. $\Delta ABE \sim \Delta CDE$



$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\frac{6}{1.5} = \frac{3+BD}{3}$$

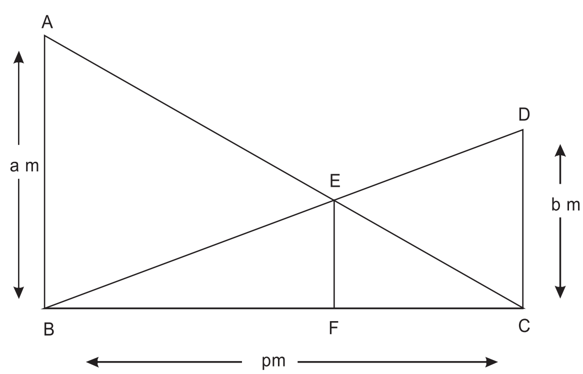
BD = 9m

36. **To prove :** EF = $\frac{ab}{a+b}$

Proof : AB || EF || DC

$\Delta EFC \sim \Delta ABC$

$$\frac{EF}{AB} = \frac{FC}{BC} \quad ...(1)$$



$\Delta BFE \sim \Delta BCD$

$$\frac{EF}{CD} = \frac{BF}{BC} \quad ...(2)$$

Adding (1) and (2), we get

$$\frac{EF}{AB} + \frac{EF}{CD} = \frac{FC+BF}{BC}$$

$$EF\left[\frac{1}{AB}+\frac{1}{CD}\right] = \frac{BC}{BC}$$

$$EF\left[\frac{1}{a}+\frac{1}{b}\right] = 1$$

$$EF = \frac{ab}{a+b}$$

37. Same as Q. 35.

38. $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, DE || BC

$\therefore \quad \angle D = \angle B$ and $\angle E = \angle C$ (Corresponding Angles)

But $\angle D = \angle E$

So, $\angle B = \angle C$

$\therefore \quad AB = AC$

So, ΔABC is an isosceles triangle.

39. In ΔOAB, $\frac{OD}{DA} = \frac{OE}{EB}$ .... (1) ($\because$ BPT)

In ΔOBC, $\frac{OE}{EB} = \frac{OF}{FC}$ .... (2) ($\because$ BPT)

From (1) and (2), we get

$$\frac{OD}{DA} = \frac{OF}{FC}$$

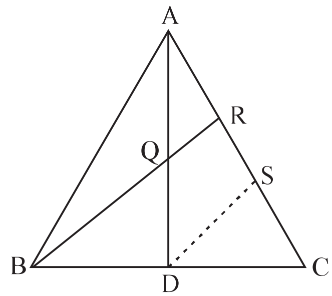
By converse of BPT, DF || AC.

40. $\Delta APB \sim \Delta DPC$ (AA Similarity criterion)

$\frac{AP}{DP} = \frac{PB}{PC}$ ($\because$ C.P.S.T.)

AP.PC = DP.PB

41. Draw PS $\parallel$ BR



In ΔCBR

PS $\parallel$ BR

$\Rightarrow$ CS = SR ....(1)

In ΔAPS

AR = RS ....(2)

From (1) and (2)

$$AR = \frac{1}{3}AC$$

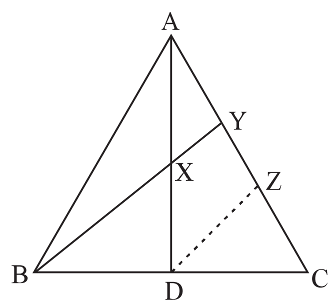
42. In ΔBCA

$$\frac{BE}{EC} = \frac{BD}{DA} \text{ (B.P.T.) and } \frac{BE}{EC} = \frac{BC}{CP} \text{ (given)}$$

$$\Rightarrow \frac{BD}{DA} = \frac{BC}{CP}$$

$\Rightarrow$ DC $\parallel$ A(Converse of B.P.T.)

43. Draw DZ $\parallel$ BY



$\Delta AXY \sim \Delta ADZ$

$$\Rightarrow \frac{AX}{AD} = \frac{XY}{DZ}$$

$\Rightarrow$ 2DZ = 5XY

Now, $\Delta CDZ \sim \Delta CBY$

$$\frac{CD}{CB} = \frac{DZ}{BY}$$

$\Rightarrow$ BX = 4XY

44. $\Delta ADE \sim \Delta ABC$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{30}{ar(\Delta ABC)} = \left(\frac{3}{9}\right)^2$$

$\therefore$ ar(ΔABC) = 270 cm^2

ar (BCFD) = ar (ΔABC) – ar (ΔADE)

= 270 – 30 = 240 cm^2

45. Given, ar ΔBXY = ar AXYC

ar (Δ ABC) = ar Δ BXY + ar AXYC

= 2 ar ΔBXY

$$\therefore \quad \frac{\text{ar}(\Delta \text{ ABC})}{\text{ar}(\Delta \text{ BXY})} = \frac{2}{1}$$

ΔABC ~ ΔXBY

$$\left(\frac{AB}{XB}\right)^2 = \frac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta BXY)}$$

$$\frac{AB}{XB} = \sqrt{2}$$

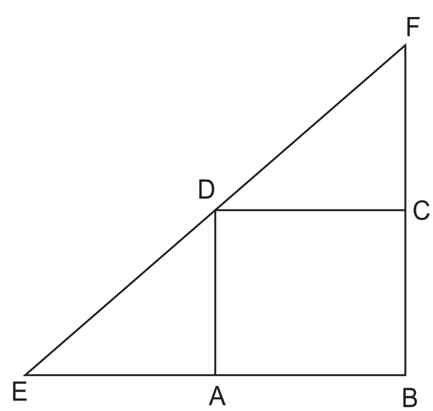
$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$1-\frac{XB}{AB} = 1-\frac{1}{\sqrt{2}}$$

$$\frac{AB-XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

46. ΔEAD ~ ΔEBF



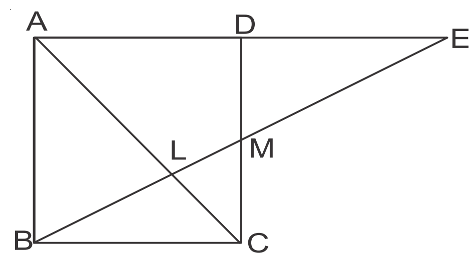
$$\frac{EA}{EB} = \frac{AD}{BF}$$

$$\Rightarrow \frac{BF}{BE} = \frac{AD}{AE} = \frac{BF-AD}{BE-AE}$$

$$= \frac{BF-BC}{BA} = \frac{CF}{DC}$$

47. Theorem 6.9 of NCERT.

48. BMC $\cong \Delta$EMD



BC = DE

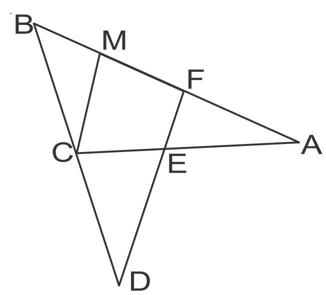
& AD = BC

$\Rightarrow$ AE = 2BC

Now, $\Delta AEL \sim \Delta CBL$

$\Rightarrow$ EL = 2BL

49. Draw CM || DF,



In ΔACM

EF || CM

$$\Rightarrow \frac{AE}{CE} = \frac{AF}{FM}$$

$\Rightarrow$ CE = MF

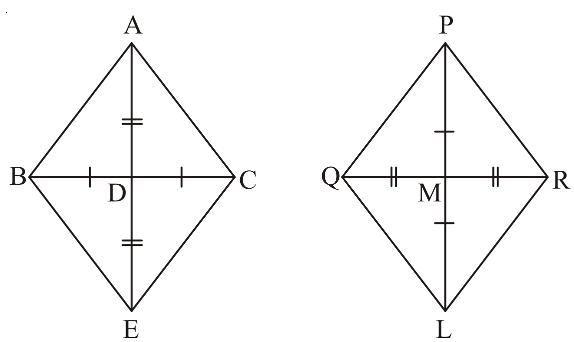
In ΔBDF

$$\frac{BD}{CD} = \frac{BF}{MF} \Rightarrow \frac{BD}{CD} = \frac{BF}{CE}$$

50. In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad ...(1)$$

Extend AD to a point E s.t. AD = DE and PM to point L s.t. PM = ML



$\therefore$ quadrilateral of ABEC and PQLR are parallelogram

($\because$ diagonals bisect each other)

$\therefore$ AC = BE, AB = EC

PR = QL, PQ = LR ....(2)

From (1) and (2)

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2\,AD}{2\,PM} = \frac{AE}{PL}$$

$\therefore$ $\Delta ABE \sim \Delta PQL$

$\therefore$ $\angle ABE = \angle PQL$ ...(3)

Similarly, $\Delta AEC \sim \Delta PLR$

$\Rightarrow$ $\angle CAE = \angle RPL$ ...(4)

$\Rightarrow$ $\angle CAB = \angle RPQ$ (from 3 and 4)

$\therefore$ In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ and } \angle CAB = \angle RPQ$$

$\therefore$ $\Delta ABC \sim \Delta PQR$

51. $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$ $(\because \Delta ABC \sim \Delta DEF)$

$$\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

Solving, we get $x = 5$

$\therefore$ AB = 9 cm BC = 12 cm AC = 15 cm

DE = 18 cm EF = 24 cm FD = 30 cm

52. $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$$

$\Rightarrow$ AB = kDE, BC = kEF, AC = kDF

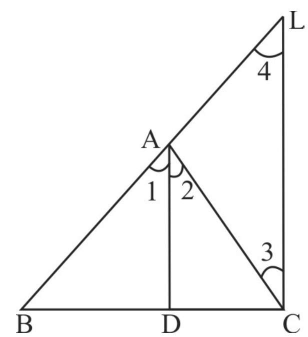
$\therefore$ AB + BC + AC = k (DE + EF + DF)

$$\therefore \quad \frac{30}{20} = \frac{9}{x} \quad \Rightarrow \quad x = 6 \text{ cm}$$

53. Constr: Produce BA upto L such that AL = AC, Join CL

Proof: In ΔACL $\angle 3 = \angle 4$

In ΔBCL



$$\frac{BD}{DC} = \frac{AB}{AL} (\because AC = AL)$$

$DA \parallel CL$

$\Rightarrow \angle 1 = \angle 4$

$\angle 2 = \angle 3$

$\Rightarrow \angle 1 = \angle 2$

Hence, AD is bisector of $\angle A$.

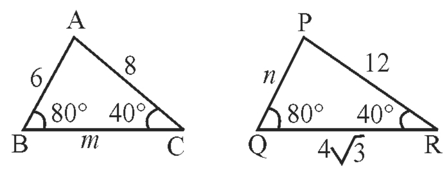
PRACTICE-TEST

Triangles

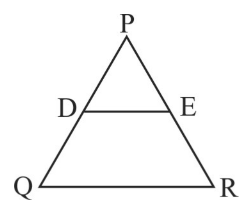
Time : 45 minutes — *M.M. : 20*

SECTION - A

1. In the given fig., $\Delta ABC \sim \Delta PQR$, then find $(m + n)$



2. In the given fig., DE || QR, PQ = 5.6 cm and PD = 1.6 cm then find PE : ER.



3. ΔABC is such that AB = 3cm, BC = 2cm and CA = 2.5 cm. In $\Delta PQR \sim \Delta ABC$ and QR = 6 cm, then perimeter of ΔDEF is _________. 1

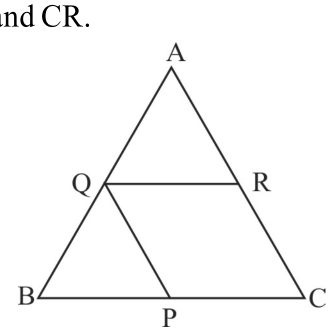
4. If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{FD}$, then

(a) $\Delta BCA \sim \Delta FDE$ (b) $\Delta FDE \sim \Delta ABC$

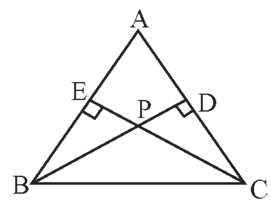
(c) $\Delta CBA \sim \Delta FDE$ (d) $\Delta FDE \sim \Delta CAB$

SECTION B

5. In the given fig., QR || BC and QP || AC. If PB = 12 cm, PC = 20 cm and AR = BQ = 15 cm, calculate AQ and CR. **2**



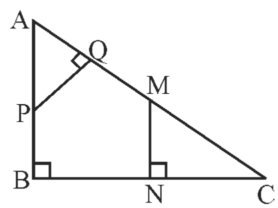
6. In the given fig., BD $\perp$ AC and CE $\perp$ AB, prove that BP × PD = EP × PC.



7. If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 3, prove that one of the parallel sideds is three times the other. **2**

SECTION C

8. In the given fig., if AB $\perp$ BC, PO $\perp$ AC andMN $\perp$ BC, prove that $\Delta APQ \sim \Delta MCN$.



9. E is a point on the side AD produced of a prallelogram ABCD and BE interects CD at F. Show that AB × BC = AE × CF. **3**

SECTION D

10. State and prove Basic Proportionality Theorem. **4**

CHAPTER

7 Co-ordinate Geometry

Key Points

1. Coordinate Geometry

Coordinate Geometry

↓

A system of geometry where the position of points on the plane is described using an ordered pair P(x, y)

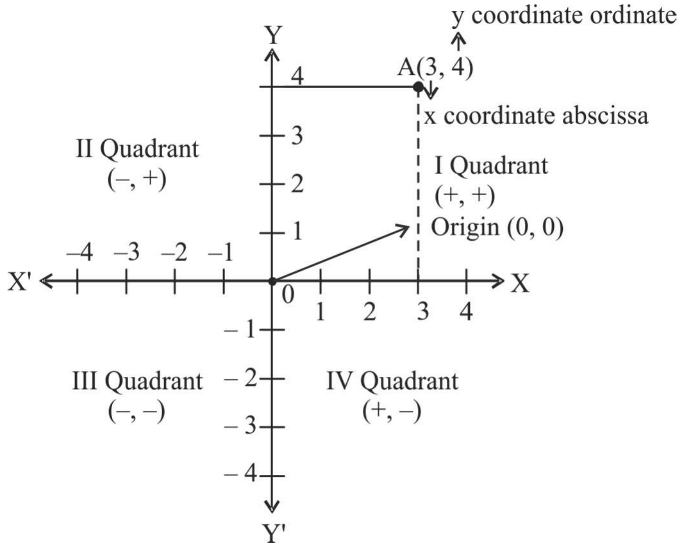
↓

The distance of a point from y-axis is called x-coordinate or abscissa	The distance of a point from x-axis is called y-coordinate or ordinate

↓

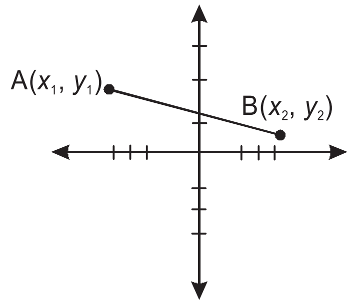
The coordinate of point on x-axis is (x, 0). The coordinate of point on y-axis is (0, y). The coordinate of point on 2-Dimensional plane i.e., cartesian plane is (x, y)

Cartesian Plane



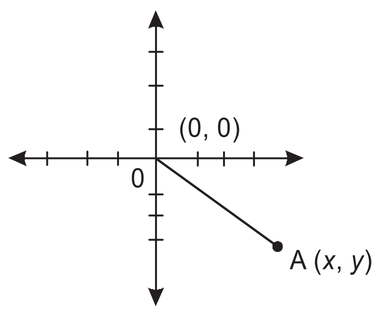
2. Distance Formula

Finding distance between two given points :



AB (Distance between A and B) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. ***Distance of a point from origin :***



Using distance formula

OA = $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

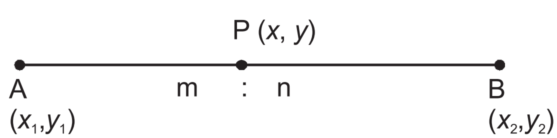
4. ***Midpoint formula :***

Coordinates of mid points of AB where $A(x_1, y_1)$ and $B(x_2, y_2)$ are : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

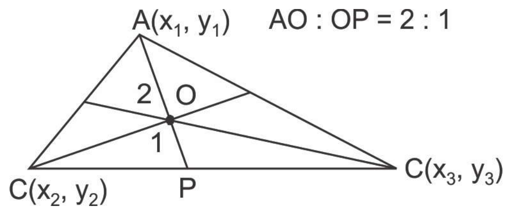
5. ***Section formula:***

The coordinates of a point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by

$$P\left(x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}\right)$$



6. ***Centroid of a triangle is given by :***



$$O\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

VERY SHORT ANSWER TYPE QUESTIONS

Fill in the blanks :

1. The distance of a point from the y-axis is called its x-coordinate or ______ .

2. The distance of a point from the x-axis is called its __________ or ordinate.

3. The point (5, 0) lies on ____ axis.

4. A point which lies on y-axis are of the form _______ .

5. A linear equation of the form $ax + by + c = 0$ when represented graphically gives a _______ .

6. The distance of a point $P(x, y)$ from the origin is ________

Multiple Choice Question :

7. P is a point on x-axis at a distance of 3 unit from y-axis to its left. The co-ordinates of P are :

(*a*) (3, 0) (*b*) (0, 3)

(*c*) (– 3, 0) (*d*) (0, – 3)

8. The distance of $P(3, -2)$ from y-axis is

(*a*) 3 units (*b*) 2 units

(*c*) – 2 units (*d*) $\sqrt{13}$ units

9. The co-ordinates of two ponts are (6, 0) and (0, – 8). The co-ordinates of the mid points are

(*a*) (3, 4) (*b*) (3, – 4)

(*c*) (0, 0) (*d*) (– 4, 3)

10. If the distance between $P(4, 0)$ and $Q(0, x)$ is 5 units, the value of x will be

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

11. The co-ordinates of the point where line $\frac{x}{a}+\frac{y}{b}=7$ intersects y-axis are

(*a*) $(a, 0)$ (*b*) $(0, b)$

(*c*) $(0, 7b)$ (*d*) $(2a, 0)$

12. The area of triangle OAB, the co-ordinates of whose vertices are A(4, 0), B(0, – 7) and O origin, is :

(*a*) 11 sq. units (*b*) 18 sq. units

(*c*) 28 sq. units (*d*) 14 sq. units

13. The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is

(*a*) 6 units (*b*) 4 units

(*c*) 3 units (*d*) 2 units

14. The co-ordinate of the point which is reflection of point (–3, 5) in x axis are

(*a*) (3, 5) (*b*) (3, –5)

(*c*) (–3, –5) (*d*) (–3, 5)

15. The co-ordinates of vertex A of ΔABC are (– 4, 2) and a point D which is mid point of BC are (2, 5). The coordinates of centroid of ΔABC are

(*a*) (0, 4) (*b*) $\left(-1, \frac{7}{2}\right)$

(*c*) $\left(-2, \frac{7}{3}\right)$ (*d*) (0, 2)

16. The distance between the line $2x + 4 = 0$ and $x - 5 = 0$ is

(*a*) 9 units (*b*) 1 unit

(*c*) 5 units (*d*) 7 units

17. The perimeter of triangle formed by the points (0, 0), (2, 0) and (0, 2) is

(*a*) 4 units (*b*) 6 units

(*c*) $6\sqrt{2}$ units (*d*) $4+2\sqrt{2}$ units

18. If the centroid of the triangle formed by (9, *a*), (*b*, – 4) and (7, 8) is (6, 8), then the value *a* and *b* are :

(*a*) $a = 4, b = 5$ (*b*) $a = 5, b = 4$

(*c*) $a = 5, b = 2$ (*d*) $a = 20, b = 2$

VERY SHORT ANSWER TYPE QUESTIONS

19. The centre of circle having end points of its diameter as (–4, 2) and (4, –3) is

(*a*) (2, –1) (*b*) (0, –1)

(*c*) $(0, -\frac{1}{2})$ (*d*) $(4, -\frac{5}{2})$ **(CBSE 2020 Basic)**

20. The distance between the points (0, 0) and $(a-b, a+b)$ is

(*a*) $2\sqrt{ab}$ (*b*) $\sqrt{2a^2+ab}$

(*c*) $2\sqrt{a^2+b^2}$ (*d*) $\sqrt{2a^2+2b^2}$ **(CBSE 2020 Standard)**

SHORT ANSWER TYPE QUESTIONS-I

21. For what value of *P*, the points (2, 1), (*p*, – 1) and (– 1, 3) are collinear.

22. Three consecutive vertices of a parallelgram are (–2, –1), (1, 0) and (4, 3). Find the co-ordinates of the fourth vertax.

23. Find the point of trisection of the line segment joining the points (1, – 2) and (– 3, 4).

24. The midpoints of the sides of a triangle are (3, 4), (4, 1) and (2, 0). Find the vertices of the triangle.

25. A circle has its centre at (4, 4). If one end of a diameter is (4, 0) then find the coordinates of the other end. **(CBSE 2020 Standard)**

26. Find the ratio in which P(4, m) divides the line segment joining the points *A*(2, 3) and *B*(6, –3). Hence find m. **(CBSE 2018)**

27. Show that the points (– 2, 3), (8, 3) and (6, 7) are the vertices of a right angle triangle.

28. Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

(CBSE 2019)

29. Find the ratio in which y-axis divides the line segment joining the points A(5, – 6) and B(– 1, – 4).

30. Find the co-ordinates of a centroid of a triangle whose vertices are (3, – 5), (– 7, 4) and (10, – 2).

31. Find the relation between x and y such that the points (x, y) is equidistant from the points (7, 1) and (3, 5).

32. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis. Also find the coordinates of the point on x-axis.

(CBSE 2019)

33. What is the value of a if the points (3, 5) and (7, 1) are equidistant from the point $(a, 0)$?

34. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3). Find the value of x.

35. A(5, 1), B(1, 5) and C(–3, –1) are the vertices of ΔABC. Find the length of median passing through A. **(CBSE 2018)**

36. Name the type of triangle formed by the points A(– 5, 6), B(– 4, – 2) and C(7, 5).

(NCERT Exempler)

37. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, – 4). How many such points are there? **(NCERT Exempler)**

38. A line intersects the y-axis and x-axis at the point P and Q. If (2, –5) is the midpoint of PQ then find the co-ordinates of P and Q. **(CBSE 2017)**

39. If A(–2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

(CBSE 2018)

40. Let P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ such that P is nearer to A. Find the co-ordinates of P and Q.

SHORT ANSWER TYPE QUESTIONS-II

41. The line segment joining the points $A(2, 1)$ and $B(5, -8)$ is trisected at the point P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k. **(CBSE 2019)**

42. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the co-ordinates of the point of intersection.

(HOTS)

43. Point A lies on the line segment XY joining X(6, – 6) and Y(– 4, – 1) in such a way that $\frac{\text{XA}}{\text{XY}} = \frac{2}{5}$. If point A also lies on the line $3x + k(y + 1) = 0$, find the value of k.

(HOTS)

44. Find the ratio in which line $x + 3y - 14 = 0$ divide in line segment joining A (–2, 4) and B(1, 7).

45. Find the centre of circle passing through (5, –8), (2, –9) and (2, 1).

46. Point P divides the line segment joining the points $A(2, 1)$ and $B(5, -8)$ such that $\frac{\text{AP}}{\text{PB}} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$. Find the value of k.

47. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal then prove that $3x = 2y$. **(CBSE 2017)**

48. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divides the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? **(CBSE 2017)**

49. If $A(-3, 2)$, $B(x, y)$ and $C(1, 4)$ are the vertices of an isosceles triangle with AB = BC. Find the value of $(2x + y)$.

50. If the point $P(3, 4)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$ then prove that $3b - 4a = 0$.

LONG ANSWER TYPE QUESTIONS-III

51. If the co-ordinates of the mid-points of the sides ofa triangle are (3, 1), (5, 6) and (–3, 2). Find the co-ordinates of its vertices and centroid. **(CBSE 2020 Standard)**

52. If $P(x, y)$ is any point on the line joining $A(a, 0)$ and $B(0, b)$ then show that $\frac{x}{a}+\frac{y}{b}$ $= 1$.

53. Find the co-ordinates of the point which divides the line segment joining the points A(2, 6) and B(10, –10) in to 4 equal points. **(CBSE-2011)**

54. Find the relation between x and y if $A(x, y)$, $B(-2, 3)$ and $C(2, 1)$ form an isosceles triangle with $AB = AC$.

55. Prove that the point $\left(x, \sqrt{1-x^2}\right)$ is at a distance of 1 unit from the origin.

56. Prove that the points (1, 2), (9, 3) and (17, 4) are collinear by section formula.

(CBSE 2017)

57. Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

58. Find the co-ordinates of the circumcenter of the triangle whose vertices are (3, 7), (0, 6) and (– 1, 5). Find the circumradius. **(HOTS)**

59. In a triangle PQR, the co-ordinates of points P, Q and R are (3, 2), (6, 4) and (9, 3) respectively. Find the co-ordinates of centroid G.

60. If co-ordinates of two adjacent vertices of a parallelogram are (3, 2) and (1, 0) and diagonats bisect each other at (–2, 5). Find the co-ordinates of the other vertices.

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. abscissa

2. y-coordinate

3. x-axis

4. $(0, y)$

5. straight line

6. $\sqrt{x^2 + y^2}$

7. (*iii*) (– 3, 0)

8. (i) 3 units

9. (ii) (3, – 4)

10. (ii) 3

11. (iii) (0, 7b)

12. (iv) 14 sq. units

13. (c) 3 units

14. (iii) (–3, –5)

15. (a) (0, 4)

16. (d) 7 units

17. (d) $\left(4+2\sqrt{2}\right)$ units

18. (d) a = 20, b = 2

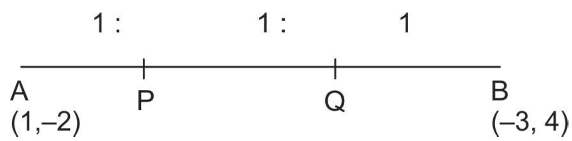
19. (c)

20. (d)

21. (1, 2)

22. 18 sq. units

23.



AP : PB = 1 : 2

AQ : QB = 2 : 1

$$P=\left(-\frac{1}{3},0\right)$$

$$Q=\left(-\frac{5}{3},2\right)$$

24. Let A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are vertices of given triangle

Let

Midpoints of AB = D(3, 4)

Midpoints of BC = E(4, 1)

Midpoints of AC = F(2, 0)

Apply Midpoint formula on AB, BC, AC

We get

$x_1 + x_2 = 6,\qquad y_1 + y_2 = 8$

$x_2 + x_3 = 8,\qquad y_2 + y_3 = 2$

$x_1 + x_3 = 4,\qquad y_1 + y_3 = 0$

Solving we get

$x_1 = 1\qquad y_1 = 3$

$x_2 = 5\qquad y_2 = 5$

$x_3 = 3\qquad y_3 = -3$

$\therefore$ A(1, 3), B(5, 5), C(3, –3)

25. (4, 8)

26. Ratio 1 : 1, m = 0

27. Show using pythagoras theorem and distance formula.

28. $(0, -2)$

29. $5 : 1$

30. $(2, -1)$

31. $x - y = 2$

32. $3 : 5$; $\left(\frac{17}{8}, 0\right)$

33. $a = 2$

34. $x = 2$

35. $\sqrt{37}$ units

36. Using distance formula, scalene triangle.

37. $x = 1, x = -15$

Two such points are there.

38. $(4, -10)$

39. $a = 1, b = 1, AB = CD = \sqrt{10}, AD = BC = \sqrt{10}$

40. $P(-1, 0), Q(-4, 2)$

41. $P(3, -2)$

Put value of $x = 3, y = -2$ is equation, then $k = -8$.

42. Let $P(x, y)$ be the point and $m : n$ is the ratio

then $x = \dfrac{6n - 2m}{m + n}, \quad y = \dfrac{3n - 5m}{m + n}$ ...(1)

From equation of line $x = 3y \Rightarrow \dfrac{x}{y} = 3$

By putting $x = 3y$ or $\dfrac{x}{y} = 3$ is (1)

$m : n = 3 : 13$

Then $P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$

43. Find $\frac{\text{XA}}{\text{AY}} = \frac{2}{3}$.

Let $A(x, y)$ is the point.

$x = 2, y = -4$

$A(2, -4)$

Put $x = 2$ and $y = -4$ in equation.

$\therefore K = 2$

44. 1 : 2

45. Centre (2, –4)

46. $K = \frac{-17}{4}$

47. $PA = PB$, Use distance formula

48. 2 : 9

49. $2x + y = 1$

50. $3b - 4a = 0$ proved by using distance formula.

51. A(–1, 7), B(–5, –3), C(11, 5), co-ordinate of centroid $\left(\frac{5}{3}, 3\right)$

52. Prove by section formula.

53. (4, 2), (6, –2) and (8, –6)

54. Prove by distance formula.

55. Prove by distance formula.

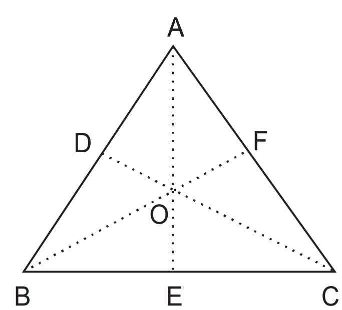
56. $k = \frac{1}{2}$, $k = -2$

58. Find co-ordinates of mid points of AB, BC, CA

then DO = OE = OF

then (circumcentre) $O(x, y) = \left(1, \frac{13}{2}\right)$

circumradius $AO = \frac{\sqrt{17}}{2}$.



59. $G(x, y) = (6, 3)$

ar Δ PQG = $\frac{3}{2}$ sq. units

ar Δ PRG = $\frac{3}{2}$ sq. units

60. Other vertices $(-5, 10)$ and $(-7, 8)$

PRACTICE-TEST

Coordinate Geometry

Time : 45 Minutes **M.M. : 20**

SECTION - A

1. x axis divides the line segment joining A(2, –3) and B(5, 6) in the ratio

(i) 2: 3 (ii) 3:5

(iii) 1 : 3 (iv) 2 : 1 **1**

2. What is the distance between the points A(c, 0) and B(0, $-c$) **1**

3. The distance of point P(– 6, 8) from the origin is _______ . **1**

4. Find the value of 'a' so that the point (3, a) lies on the line segment $2x - 3y = 5$. **1**

SECTION B

5. Find the point on y-axis which is equidistant from (–5, –2) and (3, 2) **2**

6. If the points A(8, 6) and B(x, 10) lie on the circle whose centre is (4, 6) then find the value of x. **2**

7. Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0). **2**

SECTION C

8. Show that the points A(–3, 2), B(– 5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus. **3**

9. Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y. **3**

SECTION D

10. If the point P divides the line segment joining the points A(–2, –2) and B(2, – 4) such that $\frac{AP}{AB} = \frac{3}{7}$, then find the coordinate of P. 4

CHAPTER 8

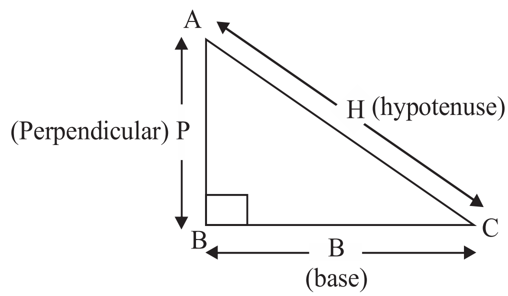
Introduction to Trigonometry

KEY POINTS

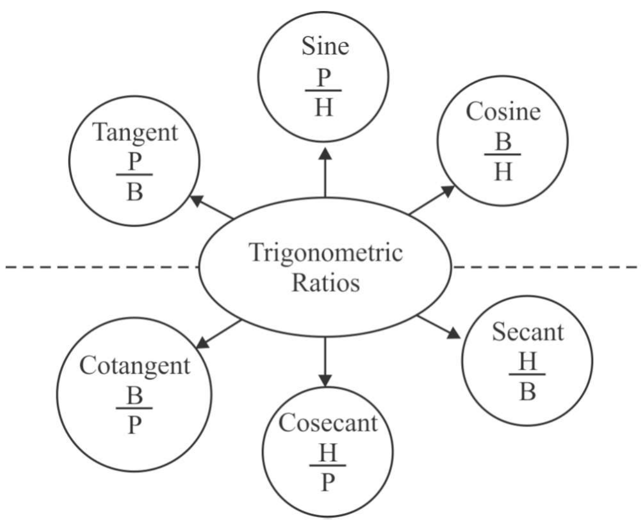
- A branch of mathematics which deals with the problems related to right angled triangles. It is the study of relationship between the sides and angles of a right angled triangle.

  Note : For $\angle A$, Perpendicular is BC and base is AB.

  For $\angle C$, Perpendicualr is AB and Base is BC.



Trigonometric Ratios of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.



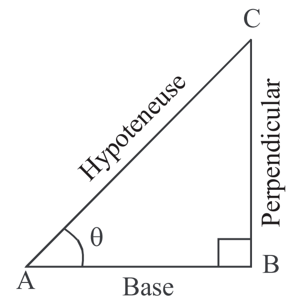
where
P $\rightarrow$ perpendicular
B $\rightarrow$ Base
H $\rightarrow$ Hypotenuse

Mind Trick: To learn the relationship of sine, cosine and tangent follow this sentence.

Some **P**eople **H**ave **C**urly **B**rown **H**air **T**hrough **P**roper **B**rushing

$\sin A = \frac{P}{H}$ $\quad$ $\cos A = \frac{B}{H}$ $\quad$ $\tan A = \frac{P}{B}$

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Reciprocal ratios:

$$\sin\theta = \frac{1}{\text{cosec }\theta}, \quad \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

3. $$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

4. **Identities**

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. **Trigonometric ratios of some specific angles**

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\sin\theta = \cos\theta$, find the value of θ

2. Find the value of $\tan^4\theta + \cot^4\theta$, if $\sin\theta - \cos\theta = 0$

3. Find the value of $\tan\theta + \cot\theta$, if $\tan^2\theta - 3\tan\theta + 1 = 0$

4. If $\tan\theta = \frac{4}{3}$ then find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

5. If $3x = \operatorname{cosec}\theta$ and $\frac{3}{x} = \cot\theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$

6. If $x = a\sin\theta$ and $y = a\cos\theta$ then find the value of $x^2 + y^2$

7. If $\cos A = \frac{3}{5}$, find the value of $4 + 4\tan^2 A$

8. Find the value of $9\sec^2 A - 9\tan^2 A$

9. Express $\sec\theta$ in terms of $\cot\theta$

10. If $x = a\sec\theta$, $y = b\tan\theta$, then find the value of $b^2x^2 - a^2y^2$.

11. Find the value of $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$, if $\tan\theta = \dfrac{4}{3}$.

12. Find the value of $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$

13. Given $\tan\theta = \dfrac{1}{\sqrt{3}}$, find the value of $\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$. (CBSE, 2010)

14. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\tan^2\theta + \cot^2\theta$.

15. If $5\tan\theta - 4 = 0$, then value of $\dfrac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is

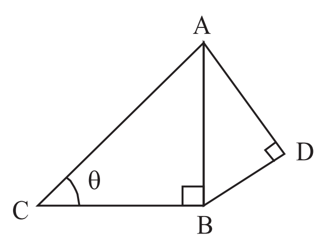
(a) $\dfrac{5}{3}$ (b) $\dfrac{5}{6}$ (c) 0 (d) $\dfrac{1}{6}$

16. $3\tan^2\theta - 3\sec^2\theta + 4$ is equal to

(a) 3 (b) 2

(c) 1 (d) 0

17. In Fig. if $AD = 4$ cm, BD = 3 cm and CB = 12 cm. then $\cot\theta =$



(a) $\dfrac{12}{5}$ (b) $\dfrac{5}{12}$

(c) $\dfrac{13}{12}$ (d) $\dfrac{12}{13}$

18. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then $x^2 + y^2$ is

(a) 25 (b) 45 (c) 7 (d) 49

19. If $\sin\theta = \frac{a}{b}$, then the value of $\sec\theta + \tan\theta$ is

(a) $\sqrt{\frac{a+b}{a-b}}$ (b) $\frac{a+b}{a-b}$ (c) $\sqrt{\frac{b+a}{b-a}}$ (d) $\frac{b+a}{b-a}$

SHORT ANSWER TYPE QUESTIONS (1)

Prove that :

20. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

21. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \text{Sec}\,\theta$

22. If $x = p\sec\theta + q\tan\theta$ & $y = p\tan\theta + q\sec\theta$ then prove that $x^2 - y^2 = p^2 - q^2$

23. If $7\sin^2\theta + 3\cos^2\theta = 4$ then show that $\tan\theta = \frac{1}{\sqrt{3}}$

24. If $\sin(A-B) = \frac{1}{2}$, $\cos(A+B) = \frac{1}{2}$ then find the value of A and B.

25. Find the value of $\cos\theta$, if $\sec\theta + \tan\theta = 5$

26. If $3\cot A = 4$, find the value of $\frac{\text{cosec}^2 A + 1}{\text{cosec}^2 A - 1}$.

27. Find the value of $\tan^3\theta + \cot^3\theta$, if $\tan\theta + \cot\theta = 2$.

28. Find the value of $\tan\theta$, if $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$.

(CBSE 2011)

29. In ΔABC, right angled at B, AB = 5 cm and $\angle ACB = 30°$. Find BC and AC.

30. Show that : $\frac{1-\sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

31. Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, $\theta \le 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS

Prove that :

32. $\dfrac{\tan A + \text{Sec}\,A - 1}{\tan A - \text{Sec}\,A + 1} = \dfrac{1 + \text{Sin}\,A}{\text{Cos}\,A}$

33. $\dfrac{1}{\sec x - \tan x} - \dfrac{1}{\cos x} = \dfrac{1}{\cos x} - \dfrac{1}{\sec x + \tan x}$

34. $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta = \sec\theta\,\text{cosec}\,\theta + 1$ **(CBSE 2019)**

35. $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

36. $\sec A\,(1 - \sin A)\,(\sec A + \tan A) = 1$

37. If $\sec\theta = x + \dfrac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$

38. If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$

39. Prove that $\cos\theta = \dfrac{p^2 - 1}{p^2 + 1}$, if $p = \text{cosec}\,\theta + \cot\theta$.

40. Show that: $x^2 + y^2 + z^2 = r^2$

if $x = r\cos\alpha\sin\beta$, $y = r\cos\alpha\cos\beta$ and $z = r\sin\alpha$ prove that

41. Find the value of $\sin^{10}\theta + \text{cosec}^{19}\theta$, if $\sin\theta + \text{cosec}\,\theta = 2$.

42. Prove that: $2\sec^2 x - \sec^4 x - 2\text{cosec}^2 x + \text{cosec}^4 x = \cot^4 x - \tan^4 x$

43. Find the value of cosec θ, *if* $\text{cosec}\theta - \cot\theta = \dfrac{1}{3}$

44. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

45. Evaulate : $\dfrac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\text{cosec}\,30° + \sec 60° - \cot^2 30°}$

46. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$ (CBSE, 2001 C)
Prove that : $a^2 + b^2 = m^2 + n^2$

LONG ANSWER TYPE QUESTIONS

Prove That:

47. $\left(1+\dfrac{1}{\tan^2\theta}\right)\left(1+\dfrac{1}{\cot^2\theta}\right) = \dfrac{1}{\sin^2\theta - \sin^4\theta}$

48. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

49. $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A \cos A$

50. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$ then show that $n(m^2 - 1) = 2m$

51. Prove that: $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta$

52. Prove that :

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

53. If $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2 + n^2)\operatorname{Cos}^2\beta = n^2$

54. **Prove that :**

$$\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} = 1$$

55. Prove that : $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

56. Prove that: $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{\sin\theta}{1-\cos\theta}$

57. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$ **(CBSE 2020)**

58. Prove $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \sec^2 A + \tan^2 A - 2\sec A\tan A$ **(CBSE 2020 Basic)**

59. Prove $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ **(CBSE 2020 Basic)**

60. If $\cos(A+B)=\sin(A-B)=\frac{1}{2}$, $0<A+B<90°$ and $A>B$ then find the value of **A** and B. **(CBSE 2020 Basic)**

61. If $\tan\theta+\sin\theta=m$, $\tan\theta-\sin\theta=n$, then prove that $m^2-n^2=4\sqrt{mn}$.

(CBSE 2020 Standard)

62. Prove that : $l^2m^2(l^2+m^2+3)=1$

If $l=\operatorname{cosec}x-\sin x, m=\sec x-\cos x$ **(CBSE 2020 Standard)**

63. Prove $\dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\dfrac{1-\sin\theta}{\cos\theta}$ **(CBSE 2020 Standard)**

64. Prove that $\dfrac{(1+\sin x-\cos x)^2}{(1+\sin x+\cos x)^2}=\dfrac{1-\cos x}{1+\cos x}$ **(CBSE 2019)**

65. Prove that $\dfrac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}=2+\dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$ **(CBSE 2019)**

66. If $4\tan\theta=3$ then find the value of $\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}$ **(CBSE 2018)**

67. Prove that $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\sec\theta+\tan\theta$ **(CBSE 2018)**

68. Prove that $\dfrac{1}{1+\sin^2\theta}+\dfrac{1}{1+\cos^2\theta}+\dfrac{1}{1+\sec^2\theta}+\dfrac{1}{1+\operatorname{cosec}^2\theta}=2$

69. Prove that $\dfrac{\tan^3\theta}{1+\tan^2\theta}+\dfrac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$

70. If cosec $=4x+\dfrac{1}{16x}$, prove that $\operatorname{cosec}\theta+\cot\theta=8x$ or $\dfrac{1}{8x}$

ANSWERS AND HINTS

1. $45°$
2. 2
3. 3
4. 7
5. $\frac{1}{3}$
6. a^2
7. $\frac{100}{9}$
8. a^2b^2
9. $\sqrt{\frac{1+\cot^2\theta}{\cot\theta}}$
10. $0°$
11. $\frac{16}{9}$
12. $\tan^2\theta$
13. $\frac{1}{2}$
14. 10/3
15. (c)
16. (iii) -1
17. (a)
18. (a)
19. (iii) $\sqrt{\frac{b+a}{b-a}}$
20. LHS = $\sec^2\theta\,(\sec^2\theta - 1)$

    RHS = $\tan^2\theta\,(\tan^2\theta + 1)$

    Use $1 + \tan^2\theta = \sec^2\theta$
21. Relationalise and proceed in LHS
22. Squaring both sides of x and y and subtracting.
23. Divide both sides by $\cos^2\theta$
24. A = 45°, B = 15°
25. $\cos\theta = 5/13$
26. $\frac{17}{8}$
27. 2
28. $\sqrt{2}-1$

29. AC = 10, BC = $5\sqrt{3}$, use Pythagoras theorem

30. Substitute values of sin 60° and cos 60° and solve

31. 60°

Note : 32 to 38 use trigonometric identities and prove (based on Ex. 8.4 of NCERT)

39. $\sqrt{3}$

41. – 1

42. 2

43. $\operatorname{cosec} \theta = \frac{5}{3}$

44. $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$

Square both sides and get $1 + 2 \cos \theta \sin \theta = 2 \cos^2 \theta$

$\Rightarrow 2 \cos \theta \sin \theta = 2 \cos^2 \theta - 1$ ...(1)

Now square $(\cos \theta - \sin \theta)^2$ and get

$(\cos \theta - \sin \theta)^2 = 1 - 2 \cos \theta \sin \theta$ ...(2)

Substitute (1) in (2)

45. 9.

46. Find m^2 and n^2 and add

Note : Q47 to Q50 Use identities to prove

51. 0

52. Rationalise $\frac{1}{\operatorname{cosec} \theta + \cot \theta}$ in LHS and proceed, use $\frac{1}{\sin \theta} = \operatorname{cosec} \theta$.

Rationalise $\frac{1}{\operatorname{cosec} \theta - \cot \theta}$ on RHS and proceed, use $\frac{1}{\sin \theta} = \operatorname{cosec} \theta$.

53. Find m^2 and n^2 and substitute in LHS.

54. Take common $\sin^2 \theta$ in Numerator and $\cos^2 \theta$ in Denominator of 2nd term on LHS and replace 1 by $\sin^2 \theta + \cos^2 \theta$.

55. 0

56. $\frac{2}{3}$

57. $(\sin\theta + \cos\theta) = \sqrt{3}$

square both sides and get value of $\dfrac{1}{\sin\theta \times \cos\theta}$

Change $\tan\theta + \cot\theta$ into $\sin\theta$ and $\cos\theta$ proceed.

58. Change $\cot A = \dfrac{\cos A}{\sin A}$, take $\cos A$ common from Numerator and Denominator,

Rationalise remaining term and change into sec A and tan A.

59. LHS = $\dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$, write $1 = \sin^2\theta + \cos^2\theta$ and proceed.

60. $\cos(A + B) = \dfrac{1}{2} = \cos 60°$

$\sin(A - B) = \dfrac{1}{2} = \sin 30°$

$\Rightarrow \left.\begin{array}{l} A+B=60° \\ A-B=30° \end{array}\right]$ Solve these equations

A = 45°, B = 15°

61. Find m^2 and n^2 substitute in $m^2 - n^2$ and substitute m and n in $4\sqrt{mn}$

62. $\dfrac{2}{3}$

65. Convert $\cot\theta$ and $\text{cosec}\,\theta$ into $\sin\theta$ and $\cos\theta$
and use $\sin^2\theta = 1 - \cos^2\theta$

66. Divide Numerator and Denominator by $\cos\theta$, and use $\sec\theta = \sqrt{1+\tan^2\theta}$
or use pythagoras theorem and trigonometeric ratios,

Ans. $\dfrac{13}{11}$

67. Same as Q 59.

PRACTICE-TEST

Introduction to Trigonometry

Time : 45 Minutes **M.M.: 20**

SECTION-A

1. If $\sin\theta = \frac{4}{5}$ what is the value of $\cos\theta$. **1**

2. Find the value of $\tan^4\theta + \cot^4\theta$, if $\tan\theta + \cot\theta = 2$ **1**

3. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$ then find the value of $5\left(x^2 - \frac{1}{x^2}\right)$

4. If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is **1**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

SECTION-B

5. If $5\tan\theta = 4$ then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ **2**

6. Find the value of $5\sin\theta - 3\cos\theta$ if $3\sin\theta + 5\cos\theta = 5$ **2**

7. Prove that $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \operatorname{cosec}\alpha$ **2**

SECTION-C

8. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$ **3**

9. Prove that $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ **3**

SECTION-D

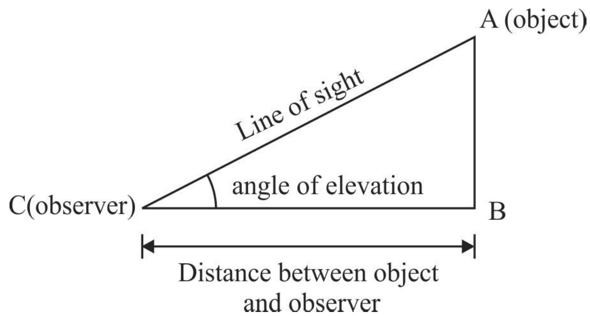
10. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1-\sin\theta}$. **4**

CHAPTER

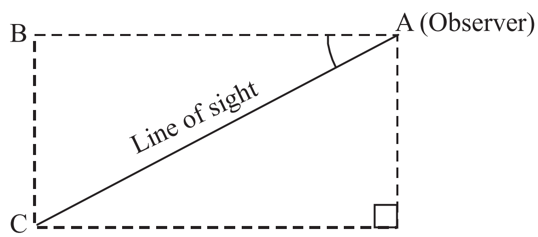
9 Some Applications of Trigonometry

KEY POINTS

- **Angle of Elevation:** Let AB be height of object. C is the observer looking upto to A (the top of AB). AC is called the line of sight and $\angle ACB$ is angle of elevation.



- **Angle of Depression :** Let A is the observer looking at C (the object) from a height BC. AC is line of sight and $\angle BAC$ is angle of depression.



- If the observer moves towards the object the angle of elevation increases and if the observer moves away from the object, the angle of elevation decreases.
- Numerically, angle of elevation is equal to angle of depression (both are measured with the same horizontal parallel planes).

VERY SHORT ANSWER TYPE QUESTIONS

1. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is :

(a) 45° (b) 30° (c) 60° (d) 90°

2. The tops of the poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, then l =

(*a*) 26 m (b) 16 m (c) 12 m (d) 10 m

3. A pole of height 6 m casts a shadow $2\sqrt{3}$ m long on the ground. the angle of elevation of the sun is (CBSE 2017)

(a) 30° (b) 60° (c) 45° (d) 90°

4. A ladder leaning aginast a wall makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, then the length of the ladder is —

(CBSE 2016)

(a) 3 m (b) 4 m (c) 5 m (d) 6 m

5. If a tower is 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then the angle of elevation of the sun is: (CBSE, 2017)

(a) 30° (b) 45° (c) 60° (d) 90°

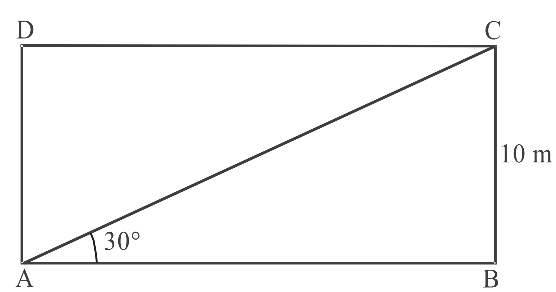
6. A tower is 50 m high. When the sun's altitude is 45° then what will be the length of its shadow?

7. The length of shadow of a pole 50 m high is $\frac{50}{\sqrt{3}}$ m. find the sun's altitude.

8. Find the angle of elevation of a point which is at a distance of 30 m from the base of a tower $10\sqrt{3}$ m high.

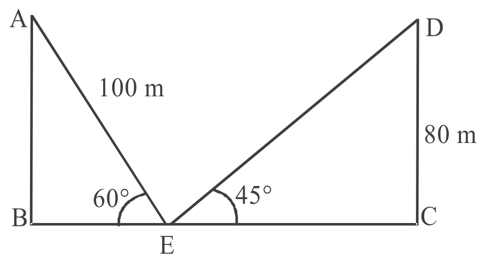
9. A kite is flying at a height of $50\sqrt{3}$ m from the horizontal. It is attached with a string and makes an angle 60° with the horizontal. Find the length of the string.

10. In the given figure find the perimeter of rectangle ABCD.

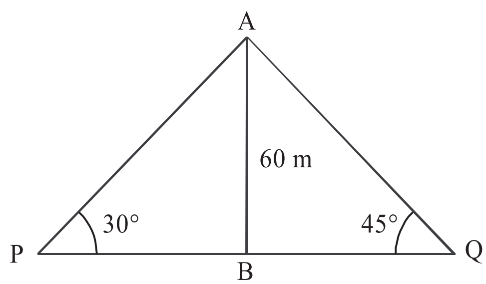


SHORT ANSWER TYPE QUESTIONS

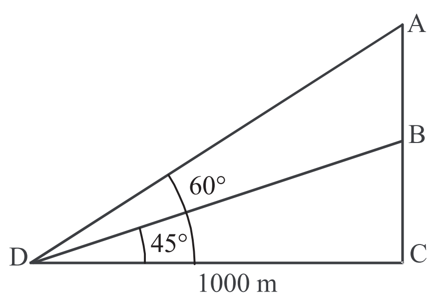
11. In the figure, find the value of BC.



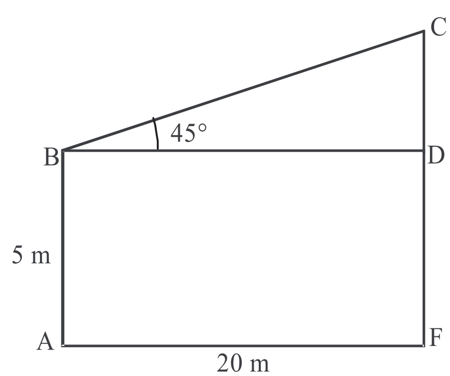
12. In the figure, two persons are standing at the opposite direction P & Q of the tower. If the height of the tower is 60 m then find the distance between the two persons.



13. In the figure, find the value of AB.



14. In the figure, find the value of CF.



15. If the horizontal distance of the boat from the bridge is 25 m and the height of the bridge is 25 m, then find the angle of depression of the boat from the bridge.

16. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing. (True / False)

17. If a man standing on the deck of a ship 3 m above the surface of sea observes a cloud and its reflection in the sea, then the angle of elevation of the cloud is equal to the angle of depression of its reflection. (True / False)

18. The string of a kite is 150 m long and it makes an angle 60° with the horizontal. Find the height of the kite above the ground. (Assume string to be tight)

19. The shadow of a vertical tower on level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower.

(Use $\sqrt{3}$ = 1.73)

20. An aeroplane at an altitude of 200 m observes angles of depression of opposite points on the two banks of the river to be 45° and 60°, find the width of the river. (Use $\sqrt{3}$ = 1.732)

21. The angle of elevation of a tower at a point is 45°. After going 40 m towards the foot of the tower, the angle of elevation of the tower becomes 60°. Find the height of the tower. (Use $\sqrt{3}$ = 1.732)

22. The upper part of a tree broken over by the wind makes an angle of 30° with the ground and the distance of the foot of the tree from the point where the top touches the ground is 25 m. What was the total height of the tree?

23. A vertical flagstaff stands on a horizontal plane. From a point 100 m from its foot, the angle of elevation of its top is found to be 45°. Find the height of the flagstaff.

24. The length of a string between kite and a point on the ground is 90 m. If the string makes an angle α with the level ground and $\sin\alpha = \frac{3}{5}$. . Find the height of the kite. There is no slack in the string.

25. An aeroplane, when 3000 m high, passes vertically above another plane at an instant when the angle of elevation of two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes. (Use $\sqrt{3}$ = 1.732)

26. A 7 m long flagstaff is fixed on the top of a tower on the horizontal plane. From a point on the ground, the angle of elevation of the top and the bottom of the flagstaff are 45° and 30° respectively. Find the height of the tower.

(Use $\sqrt{3}$ = 1.732)

27. From the top of a 7 m high building, the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 45°. Find the height of the tower. **(CBSE 2020)**

28. Anand is watching a circus artist climbing a 20m long rope which is tightly stretched and tied from the top of vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.

LONG ANSWER TYPE QUESTIONS

29. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

(Use $\sqrt{3}$ = 1.73) (CBSE 2020)

30. A man standing on the deck of a ship, 10 m above the water level observes the angle of elevation of the top of a hill as 60° and angle of depression of the bottom of the hill as 30°. Find the distance of the hill from the ship and height of the hill.

31. From a window 60 m high above the ground of a house in a street, the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are 60° and 45° respectively. Show that the height of opposite house is $60(1+\sqrt{3})$ metres.

32. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 3600 $\sqrt{3}$ m, find the speed in km/hour of the plane.

33. A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird. (Use $\sqrt{3}$ = 1.732)

34. The shadow of a tower standing on a level ground is found to be 30 m longer when the sun altitude is 30° longer when the sun altitude is 30° than when it is 60°. Find the height of the tower.

35. The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building. **(CBSE 2020)**

36. An observer from the top of a light house, 100 m high above sea level, observes the angle of depression of a ship, sailing directly towards him, changes from 30° to 60°. Determine the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ = 1.732)

37. The angles of elevation and depression of the top and bottom of a light house from the top of a building 60 m high are 30° and 60° respectively. Find

(i) The difference between the height of the light house and the building.

(ii) distance between the light house and the building.

38. A fire in a building 'B' is reported on telephone in two fire stations P an Q, 20 km apart from each other on a straight road. P observes that the fire is at an angle of 60° to the road, and Q observes, that it is at an angle of 45° to the road. Which station should send its team to start the work at the earliest and how much distance will this team has to travel?

39. A 1.2m tall girl spots a balloon on the eve of Independence Day, moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the girl at an instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon.

40. The angle of elevation of the cloud from a point 10 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of lake. **(CBSE 2020)**

41. Two pillars of equal heights stand on either side of a roadway 150 m wide. From a point on the roadway between the pillars, the angles of elevation of the top of the pillars are 60° and 30°. Find the height of pillars and the position of the point. **(CBSE, 2011)**

42. The angle of elevation of the top of tower from certain point is 30°. If the observer moves 20 m towards the tower the angle of elevation of the top increases by 15°. Find the height of the tower.

43. A moving boat is observed from the top of a 150 m high cliff moving away form the cliff. The angle of depression of the boat changes form 60° to 45° in 2 minutes. Find the speed of the boat in m/h.(Take $\sqrt{3}$ = 1.732)

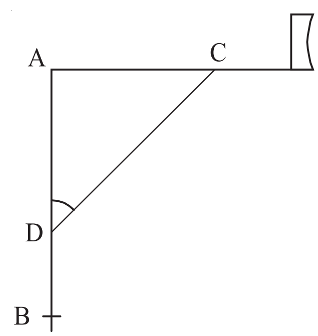
44. From the top of a 120 m high tower a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as 60° and 45°. Find the distance between the cars. (Use $\sqrt{3}$ = 1.732)

45. A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

(CBSE 2020)

46. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in the figure. If AC = 1.5 m long and CD = 3 m, find (*i*) tan θ (*ii*) sec θ + cosec θ. **(CBSE 2020)**

47. At a point on level ground, the angle of elevation of a vertical tower is found to be α such that $\tan\alpha = \frac{1}{3}$. After walking 200 m towards the tower, then angle of elevation β becomes such that $\tan\beta = \frac{3}{4}$, find the height of the tower.



48. A vertically straight tree, 20m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height of the ground did the tree break? θ

49. If the angle of elevations of a cloud from a point *h* meters above a lake be 30° and the angle of depression of its reflection in the lake be 60°. Prove that the height of cloud is 2h, also find the distance of observer from cloud.

50. The angles of elevation of the top of a tower of height h meter from two points P and Q at a distance of xm and y m from the base of the tower respectively and in the same straight line with it, are 60° and 30°, respectively prove that height of tower be $\sqrt{xy}\ m$.

51. Two poles of heights 18 m and 30 m stand vertically on the ground. The tops of two poles are connected by a wire, which is inclined to the horizontal at an angle of 60°. Find the length of wire and the distance between the poles.

52. The angles of depression of the top and bottom of a 10 m tall pole from the top of a transimission tower are 45° and 60° respectively. Find the height of the transmission tower and the distance between the pole and tower.

53. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making and angle of 30° with it. The height of the breaking point from the ground is 10 m. Find the total height of the tree.

ANSWERS AND HINTS

1. (b)	**2.** (c)
3. (b)	**4.** (c)
5. (c)	**6.** 50 m
7. 60°	**8.** 30°
9. 100 m	**10.** $20\left(\sqrt{3}+1\right)$ m
11. 130 m	**12.** $60\left(\sqrt{3}+1\right)$ m
13. $1000\left(\sqrt{3}-1\right)$m	**14.** 25 m
15. 45°	**16.** False
17. False	**18.** $75\sqrt{3}$ m
19. 13.65 m	**20.** 315.46 m

21. 94.64 m

22. $25\sqrt{3}$ m

23. 100 m

24. 54 m

25. 1268 m

26. 9.562 m

27. $7(\sqrt{3}+1)$m

28. 10 m

29. 2.184 m

30. $10\sqrt{3}$ m, 40 m

32. 864 km/hr

33. 29.28 m

34. $15\sqrt{3}$ m

35. 20 m

36. 115.46 m

37. 20 m, $20\sqrt{3}$ m

38. Station P, 7.4 km (approx)

39. $58\sqrt{3}m$

40. 20 m

41. height = 64.95 m, distance (Position) = 112.5 m from the pillar having angle of elevation 60°

42. $10(\sqrt{3}+1)\ m$

43. 1902 m/h (approx.)

44. 189.28 m

45. h = $20(\sqrt{3}-1)$ m

46. (*i*) $\tan\theta = \frac{1}{\sqrt{3}}$

(*ii*) $\sec\theta + \text{cosec}\,\theta = \frac{2}{\sqrt{3}}+2$

47. h = 120 m

48. $40(2-\sqrt{3})$m

49. 2 h

51. Length of wire = $8\sqrt{3}$ m distance = $4\sqrt{3}$ m

52. $h = 5\sqrt{3}(\sqrt{3}+1)$m

53. Height of tree = 30m

PRACTICE-TEST

Some Applications of Trigonometry

Time : 45 Minutes ***M.M.: 20***

SECTION-A

1. A pole which is 6 m high cast a shadow $2\sqrt{3}$ on the ground. What is the sun's angle of elevation. **1**
2. The height of a tower is 100 m. When the angle of elevation of sun is 30°, then what is the shadow of the tower? **1**
3. The angle of elevation of the sun, when the shadow of a pole h meters high is $\sqrt{3}\ h$ is.

   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90° **1**
4. An observer 1.5 metre tall is 20.5 metre away from a tower 22 metres high. The angle of elevation of the top of the tower from the eye of the observer is,

   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 0° **1**

SECTION-B

5. From a point on the ground 20 m away from the foot of a tower the angle of elevation is 60°. What is the height of tower? **2**
6. The ratio of height and shadow of a tower is $1:\frac{1}{\sqrt{3}}$ .What is the angle of elevation of the sun? **2**
7. The angle of elevation of the top of a tower is 30°. If the height of the tower is tripled, then prove that the angle of elevation would be doubled. **2**

SECTION-C

8. The tops of the two towers of height x and y standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

3

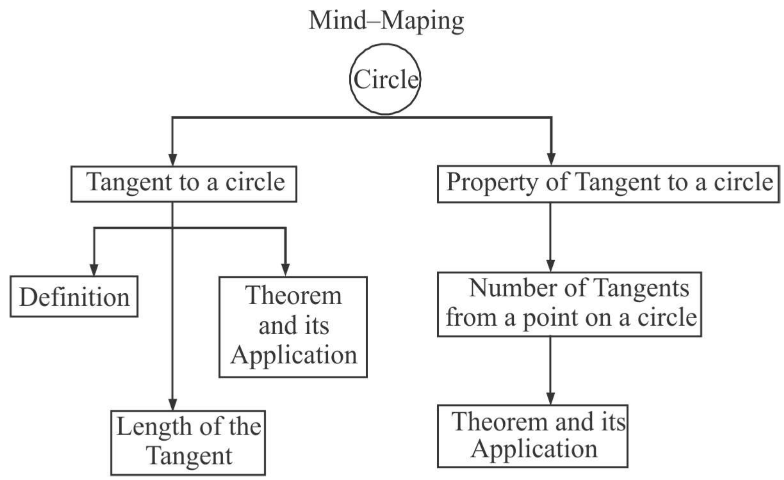
9. The angle of elevation of the top of a rock from the top and foot of a 100 m high tower are 30° and 45° respectively. Find the height of the rock. **3**

SECTION-D

10 A man standing on the deck of a ship, 10 m above the water level observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and height of the hill. **4**

CHAPTER

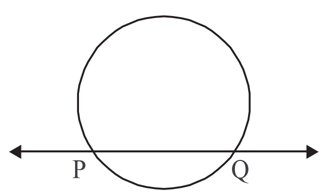
10 Circles



KEY POINTS

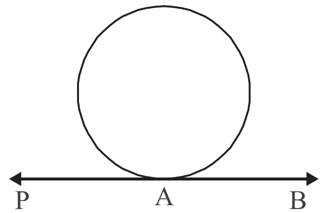
1. A **circle** is a collection of all those points in a plane which are at a constant distance from a fixed point. The fixed point is called the **centre** and fixed distance is called the **radius**.

2. **Secant:** A line which intesects a circle in two distinct points is called a secant of the circle.



3. **Tangent**: It is a line that intersects the circle at only one point. The point where tangent touches the circle is called the point of contact.

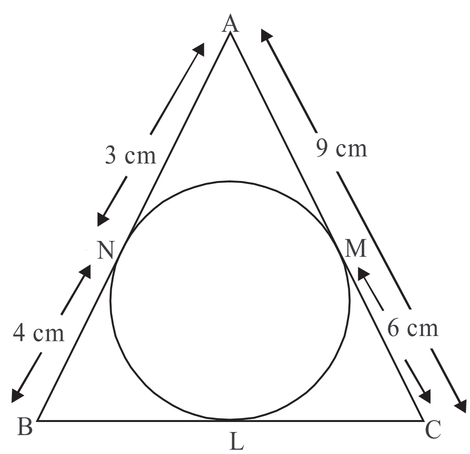
   Here A is the point of contact.



4. **Number of Tangent**: Infinitely many tangents can be drawn on a circle.
5. **Number of Secant**: There are infinitely many secants which can be drawn to a circle.
6. The proofs of the following theorems can be asked in the examination:–
   (i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
   (ii) The lengths of tangents drawn from an external point to a circle are equal.
7. The tangent to a circle is a special case of the secant.
8. There is no tangent to a circle passing through a point lying inside the circle.
9. There is one and only one tangent to a circle passing through a point lying on the circle.
10. There are exactly two tangents to a circle through a point lying outside the circles.

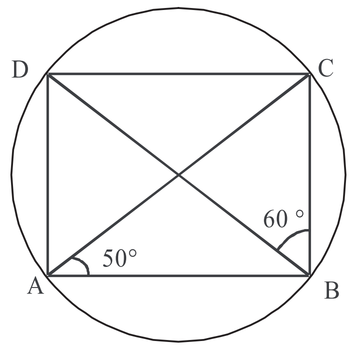
VERY SHORT ANSWER TYPE QUESTIONS

1. In fig., ΔABC is circumscribing a circle. Find the length of BC.

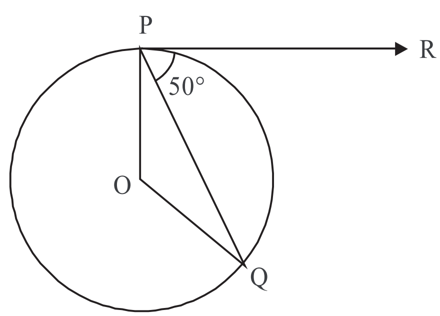


2. The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle.

3. In fig., ABCD is a cyclic quadrilatreral. If $\angle BAC = 50°$ and $\angle DBC = 60°$, then find $\angle BCD$.



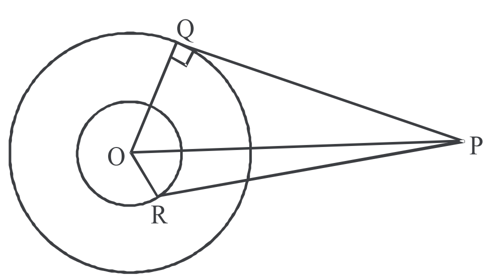
4. In figure, O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angles of 50° with PQ. Find $\angle POQ$.



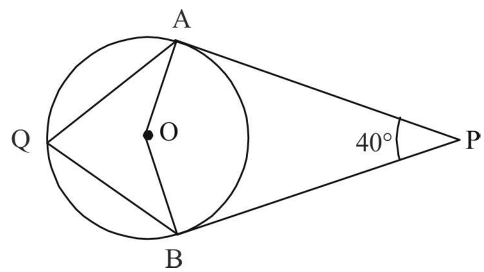
5. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

   If radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of that circle which is tangent to the other circle.

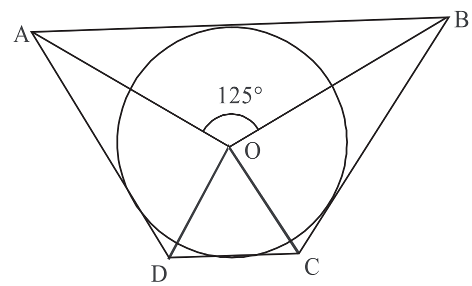
7. In the given figure, PQ is tangent to outer circle and PR is tangent to inner circle. If PQ = 4cm, OQ = 3 cm and OR = 2 cm then find the length of PR.



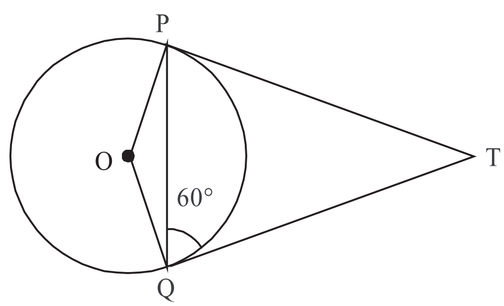
8. In the given figure, O is the centre of the circle, PA and PB are tangents to the circle then find $\angle AQB$. **(CBSE 2016)**



9. In the given figure, If $\angle AOB = 125°$ then find $\angle COD$.



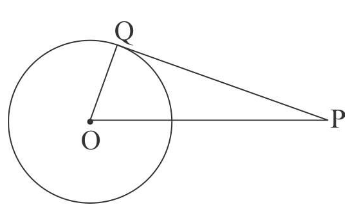
10. If two tangent TP and TQ are drawn from an external point T such that $\angle TQP = 60°$ then find $\angle OPQ$.



11. How many tangents can a circle have?

12. A tangent to a circle intersects it in _________ point.

13.



If PQ is a tangent then find the value of $\angle POQ + \angle QPO$.

14. Choose the correct Answer.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$ cm

15. A circle can have ________ parallel tangents at the most.

16. The common point of a tangent to a circle and radius of the circle is called ________.

17. Find the distance between two points of contact of two parallel tangents to a given circle of radius 9 cm.

18. Find the radius of a circle, if distance between two parallel tangents be 10 cm.

19. How many common tangents can be drawn to two circles touching internally.

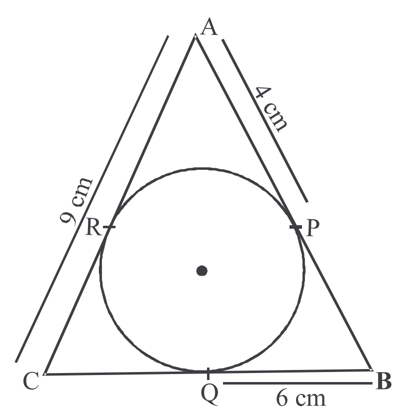
SHORT ANSWER TYPE QUESTIONS

20. If diameters of two concentric circles are d_1 and d_2 ($d_2 > d_1$) and c is the length of chord of bigger circle which is tangent to the smaller circle. Show that $d_2^2 = c^2 + d_1^2$.

21. The length of tangent to a circle of radius 2.5 cm from an external point P is 6 cm. Find the distance of P from the nearest point of the circle.

22. TP and TQ are the tangents from the external point T of a circle with centre O. If $\angle OPQ = 30°$ then find the measure of $\angle TQP$.

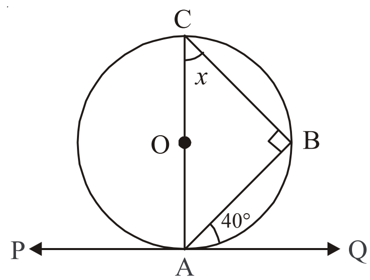
23. In the given fig. AP = 4 cm, BQ = 6 cm and AC = 9 cm. Find the semi perimeter of ΔABC.



24. A circle is drawn inside a right angled triangle whose sides are a, b, c where c is the hypotenuse, which touches all the sides of the triangle. Prove $r = \frac{a+b-c}{2}$ where r is the radius of the circle.

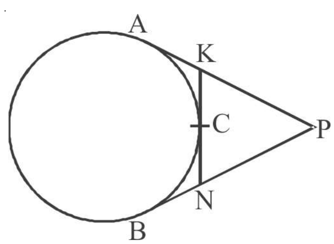
25. Prove that in two concentric circles the chord of the larger circle which is tangent to the smaller circle is bisected at the point of contact.

26. In the given Fig., AC is diameter of the circle with centre O and A is the point of contact, then find x.

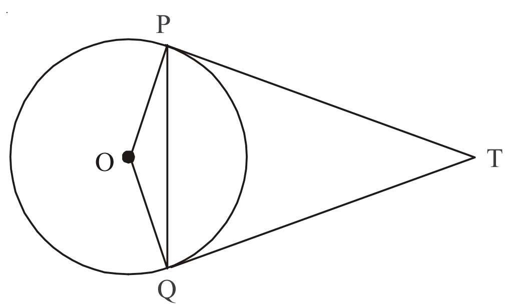


27. In the given fig. KN, PA and PB are tangents to the circle. Prove that:

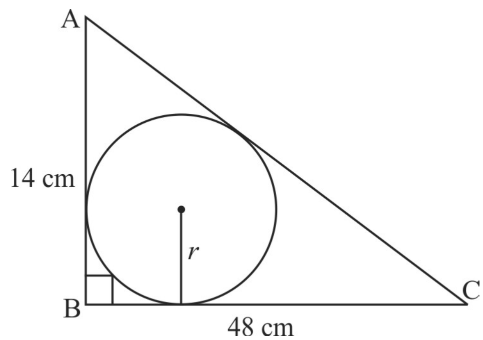
KN = AK + BN.



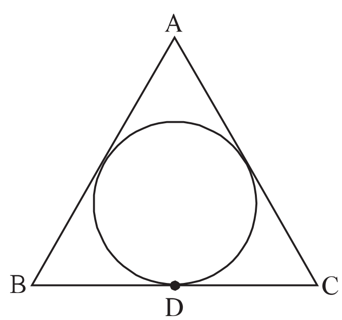
28. In the given fig. PQ is a chord of length 6 cm and the radius of the circle is 6 cm. TP and TQ are two tangents drawn from an external point T. Find $\angle$PTQ.



29. In the given figure, ABC is a triangle in which $\angle B = 90°$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find the radius (r) of in the circle.

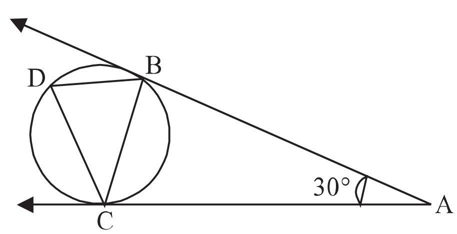


30. If the inscribed circle of the ΔABC touches BC at D. Prove that AB – BD = AC – CD.

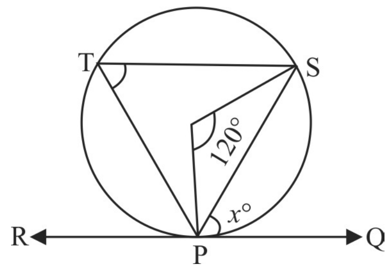


31. From a point P which is at distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn, then find the area of the quadrilateral PQ OR.

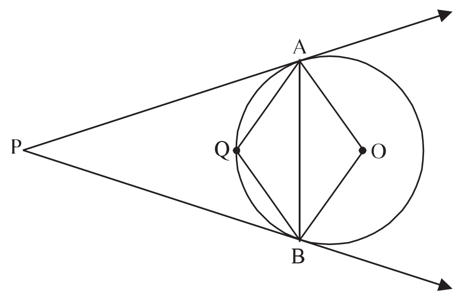
32. In the given figure, tangents AC and AB are drawn to a circle from a point A such. That $\angle BAC = 30°$, a chord BD is drawn parallel to the tangent AC, find $\angle DBC$.



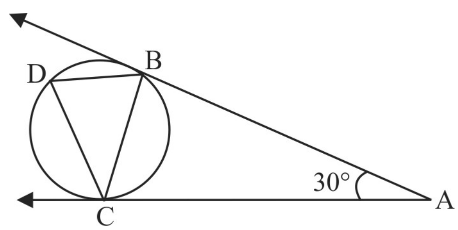
33. Find the value of $x°$.



34. PA and PB are tangents to the circle with centre at O. If $\angle APB = 70°$, then find $\angle AQB$.

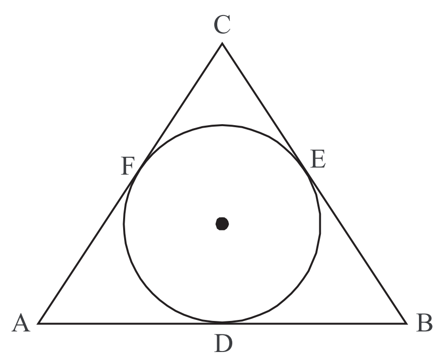


35. In figure, CD is a tangent and AB is a drameter of the circle centrel at O. If $\angle DCB = 30°$, then find $\angle ADC$.

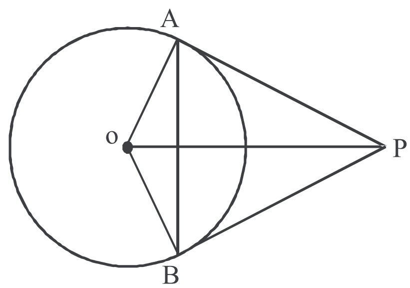


LONG ANSWER TYPE QUESTIONS

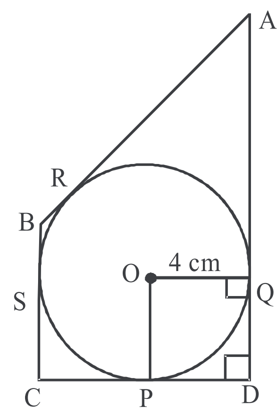
36. In the given figure find AD, BE, CF where AB = 12 cm, BC = 8 cm and AC = 10 cm.



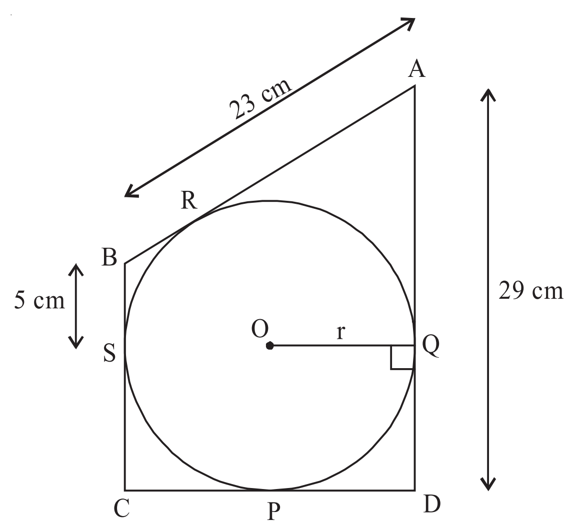
37. In the given fig. OP is equal to the diameter of the circle with centre O. Prove that ΔABP is an equilateral triangle.



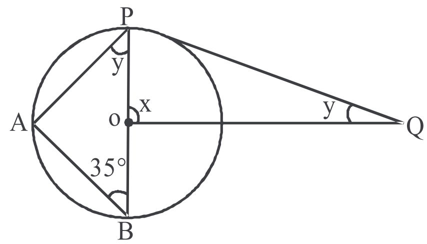
38. In the given fig., find PC. If AB = 13 cm, BC = 7 cm and AD = 15 cm.



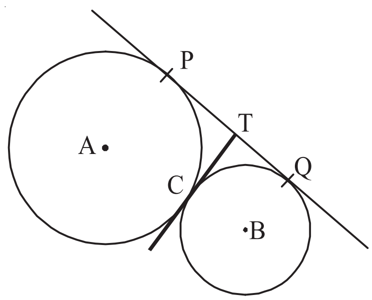
39. In the given figure, find the radius of the circle.



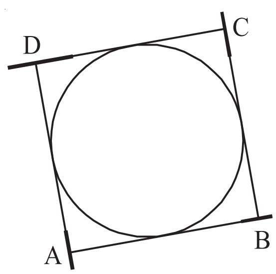
40. In the given fig. PQ is tangent and PB is diameter. Find the values of angles x and y.



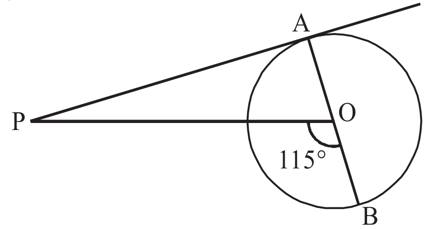
41. In given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



42. In the given figure, a circle touches all the four sides of a quadrilateral ABCD. If AB = 6 cm, BC = 9 cm and CD = 8 cm, then find the length of AD.

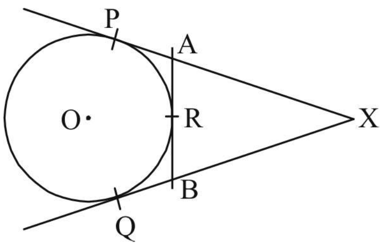


43. In figure, PA is a tangent from an external point P to a circle with centre O, If $\angle POB = 115°$. Find $\angle APO$.

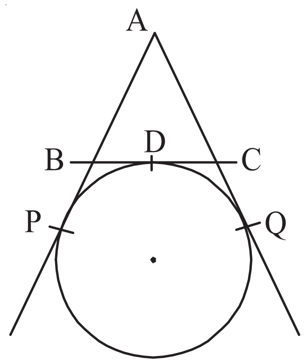


44. In figure, XP and XQ are tangents from X to the circle with centre O, R is a point on the circle and AB is tangent at R. Prove that :

XA + AR = XB + BR



45. In the given figure, find the perimeter of ΔABC, if AP = 12 cm.



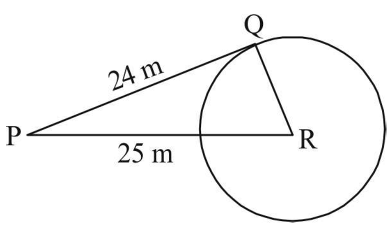
ANSWERS AND HINTS

1. Since length of both the tangents from a point outside the circle is equal, So

$$BN = BL, \quad CM = CL$$

$$BL + CL = BC = 10 \text{ cm}$$

2.



By Pythagoras Theorem, QR = 7 cm.

3. Angle in the same segment are euqal.

- DC is the chord so $\angle DAC = \angle DBC = 60°$.
- The sum of the opposite angles of a cyclic quadrilateral is 180°.

So $\angle BCD = 70°$

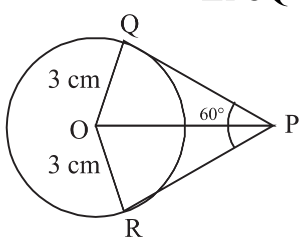
4. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

So, $\angle RPO = 90°$

$\angle OPQ = \angle OQP = 40°$

$\angle POQ = 100°$

5.



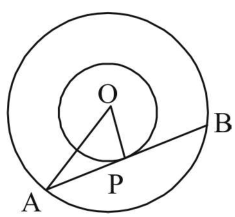
$\Delta QPO \cong \Delta RPO$

$\Rightarrow \quad \angle QPO = \angle RPO = \dfrac{60°}{2} = 30°$

In ΔQPO, $\angle OQP = 90°$(Tangent is perpendicular at the point of contact).

$$\tan 30° = \frac{OQ}{QP} \Rightarrow QP = 3\sqrt{3} \text{ cm}$$

6.



In ΔAOP, right angled at P.

$$OA^2 = AP^2 + OP^2 \Rightarrow (5)^2 = AP^2 + 4^2 \Rightarrow AP^2 = 9$$

$\Rightarrow \quad AP = 3$

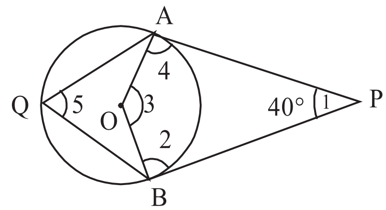
$\therefore \quad AB = 6$ cm $\quad (\because OP \perp AB$ so OP bisects AB)

7. In $\Delta PQO \quad (4)^2 + (3)^2 = (OP)^2$

$5 = OP$

In ΔPRO, $(5)^2 = (2)^2 + (PR)^2$

$PR = \sqrt{21}$ cm

8. 

In Quadrilateral OAPB

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

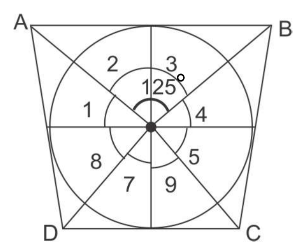
$\angle 1 + \angle 3 = 180°$

$\angle 3 = 140°$

Now, $\angle 3 = 2 \angle 5$

$\angle 5 = 70°$ or $\angle AQB = 70°$

9. $\left.\begin{array}{l} \angle 1 = \angle 2 \\ \angle 3 = \angle 4 \\ \angle 5 = \angle 6 \\ \angle 7 = \angle 8 \end{array}\right]$ (CPCT) of their corresponding triangles.



$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

or $\angle AOB + \angle COD = 180°$

or $\angle COD = 55°$

10. $\angle OQT = 90°$ (Angle between tangent & radius)

$\angle PQO = 30°$

$\angle PQO = \angle OPQ = 30°$

11. Infinitely many

12. One

13. 90°

14. d($\sqrt{119}$ cm)

15. Two

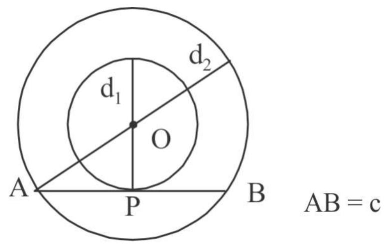
16. Point of Contact

17. 18 cm

18. 5cm

19. 1

20.



$$AB = c$$

$$AO^2 = OP^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2 = AP^2$$

$$\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AP$$

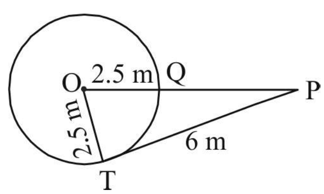
$$2\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AB$$

$$\sqrt{(d_2)^2 - (d_1)^2} = c$$

$$(d_2)^2 - (d_1)^2 = c^2$$

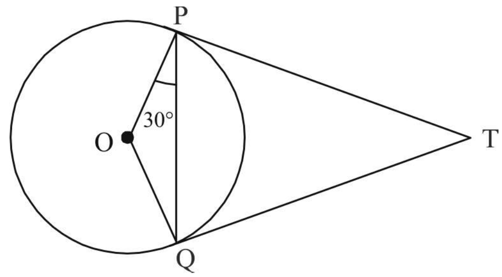
$$d_2{}^2 = c^2 + d_1{}^2$$

21.



$$(OP)^2 = (OT)^2 + (PT)^2$$
$$(OP)^2 = (2.5)^2 + (6)^2$$
$$= 42.25$$
$$(OP)^2 = (6.5)^2 \quad \Rightarrow \quad OP = 6.5 \text{ cm}$$
$$QP = 4 \text{ cm}$$

22.

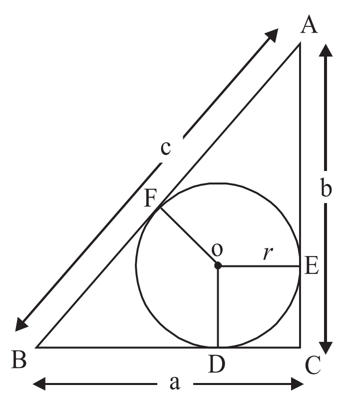


$$\angle OQP = \angle OPQ = 30°$$
$$\angle OQT = 90° \text{ (Angle between radius and tangent)}$$
$$\angle TQP = \angle OQT - \angle OQP$$
$$= 90° - 30° = 60°$$

23.

$$AP = AR = 4 \text{ cm}$$
$$CR = CQ = (9 - 4) \text{ cm} = 5 \text{ cm}$$
$$\text{Semi perimeter} = \frac{1}{2}[AC + AB + BC]$$
$$= \frac{1}{2}[9 + 10 + 11] = 15 \text{ cm}$$

24.



$$b - r = AF, \quad a - r = BF$$

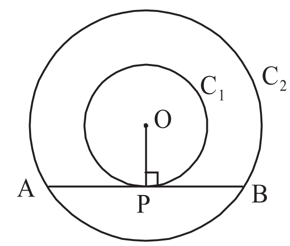
or, $AB = c = AF + BF = b - r + a - r$

This gives, $r = \dfrac{a + b - c}{2}$

25. Join OP

AB is tangent to circle C_1 at P and OP is radius

$$OP \perp AB$$



AB is chord of circle C_2 and $OP \perp AB$.

Therefore OP is the bisector of the chord AB as the perpendicular from the centre bisects the chord i.e,

$$AP = BP$$

26.

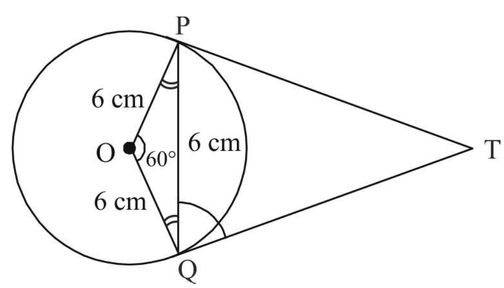
$$\angle OAB = 50°$$
$$x + \angle B + \angle OAB = 180°$$
$$x + 90° + 50° = 180°$$
$$x = 40°$$

27.

$$AK = KC$$
$$BN = NC$$
$$\therefore \quad KN = KC + NC = AK + BN$$

28.

$$\angle POQ + \angle PTQ = 180°$$
$$60° + \angle PTQ = 180°$$
$$\angle PTQ = 120°$$



29. $r = 6$ cm

31. 60 cm^2

32. $\angle DBC = 75°$

33. $x = 60°$

34. $\angle AQB = 125°$

35. $\angle ADC = 120°$

36.

$$AC = AF + FC = 10 \text{ cm} \quad \ldots(1)$$
$$AB = AD + DB = 12 \text{ cm} \quad \ldots(2)$$

$$BC = BE + CE = 8 \text{ cm} \quad \ldots(3)$$

$$\begin{bmatrix} BD & = & BE \\ AD & = & AF \\ CF & = & CE \end{bmatrix} \quad \ldots(4)$$

$$AC = AD + FC = 10 \text{ cm} \quad \ldots(5)$$

$$AB = AD + DB = 12 \text{ cm} \quad \ldots(6)$$

$$BC = BD + CF = 8 \text{ cm} \quad \ldots(7)$$

Add (5, 6, 7)

$$2(AD + FC + DB) = 30$$

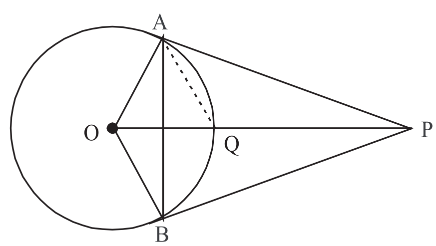
$$AD + FC + DB = 15$$

Substitute values from (1), (2) & (3)

and find. AD = 7 cm, BE = 5 cm, CF = 3 cm.

37. $OP = 2r$

$\Rightarrow \quad OQ = QP = r$



Consider ΔAOP in which $OA \perp AP$ and OP is the hypotenuse.

$$OQ = AQ = OA$$

(Mid point of hypotenuse is equidistance from the vertices).

$\Rightarrow$ OAQ is an equilitateral triangle.

$\Rightarrow \quad \angle AOQ = 60°$

Consider right angled triangle OAP

$$\angle AOQ = 60°$$

$$\angle OAP = 90° \quad \Rightarrow \quad \angle APO = 30°$$

$$\angle APB = 2\angle APO = 2 \times 30° = 60°$$

$$PA = PB \text{ (tangents)}$$

$\Rightarrow \quad \angle PAB = \angle PBA$

$$\angle APB = 60°$$

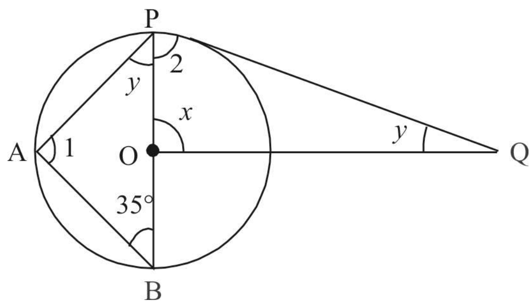
$$\angle PAB = \angle PBA = \frac{180° - 60°}{2} = 60°$$

$\therefore$ Δ ABP is an equilateral triangle.

38. PC = 5 cm

39. 11 cm

40.



In ΔABP, $\angle 1 = 90°$ (Angle in semi-circle)

$$\angle 1 + 35° + \angle y = 180°$$
$$90° + 35° + \angle y = 180°$$
$$\angle y = 55°$$

ΔOPQ, $\angle 2 = 90°$ (Angle between tangent and radius)

$$\angle 2 + \angle x + \angle y = 180°$$
$$90° + \angle x + 55° = 180°$$
$$\angle x = 35°$$

42. AD = 5 cm

43. 25°

45. 24 cm

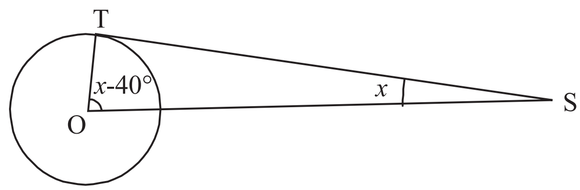
PRACTICE-TEST

CIRCLES

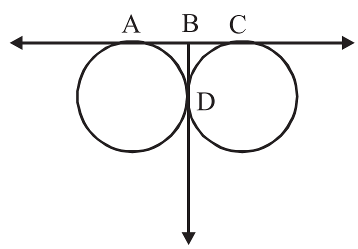
Time : 45 Minuts ***M.M.: 20***

SECTION-A

1. In the given figure find x, where ST is the tangent. **1**

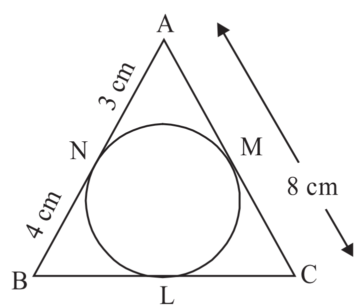


2. In the given figure if AC = 9 cm, find BD.**1**



3. In the given figure, ΔABC is circumscribing a circle, then find the length of BC.

1

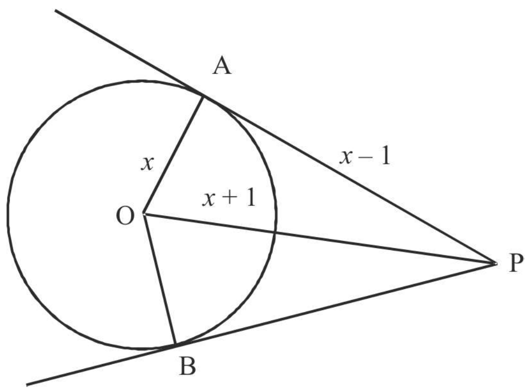


4. From the external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$. **1**

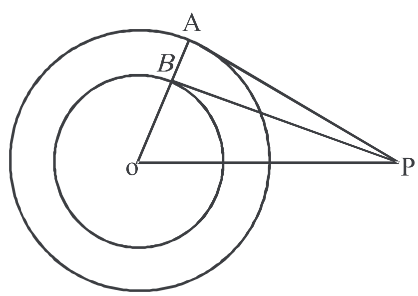
SECTION-B

5. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O is 60° then find the length of OP.

6. In the following figure find x. **2**

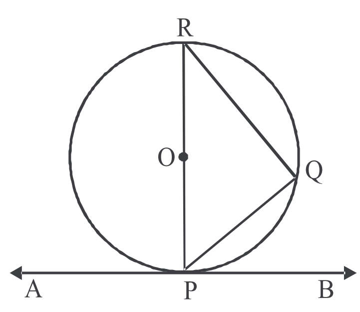


7. Two concentric circle with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle as shown in the figure. If AP = 10 cm. Find BP **2**

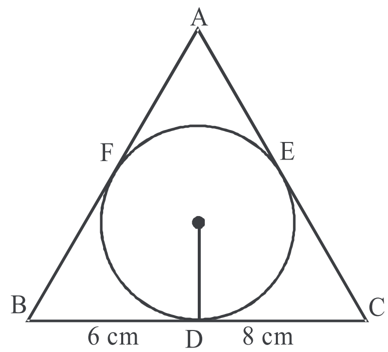


SECTION-C

8. In the given figure, AB is a tangent to a circle with centre O. Prove $\angle BPQ = \angle PRQ$. **3**

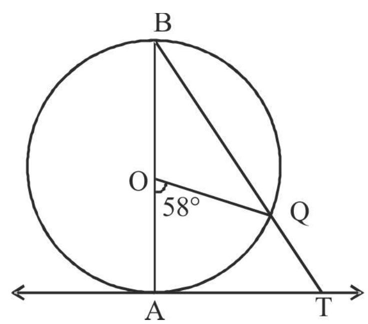


9. In the given figure ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segment BD and DC into which BC is divided by the point of contact D are of length 6 cm and 8 cm respectively, find side AB if the *ar*(ΔABC) = 63 cm² **3**



SECTION-D

10. AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$ find $\angle ATQ$. **4**



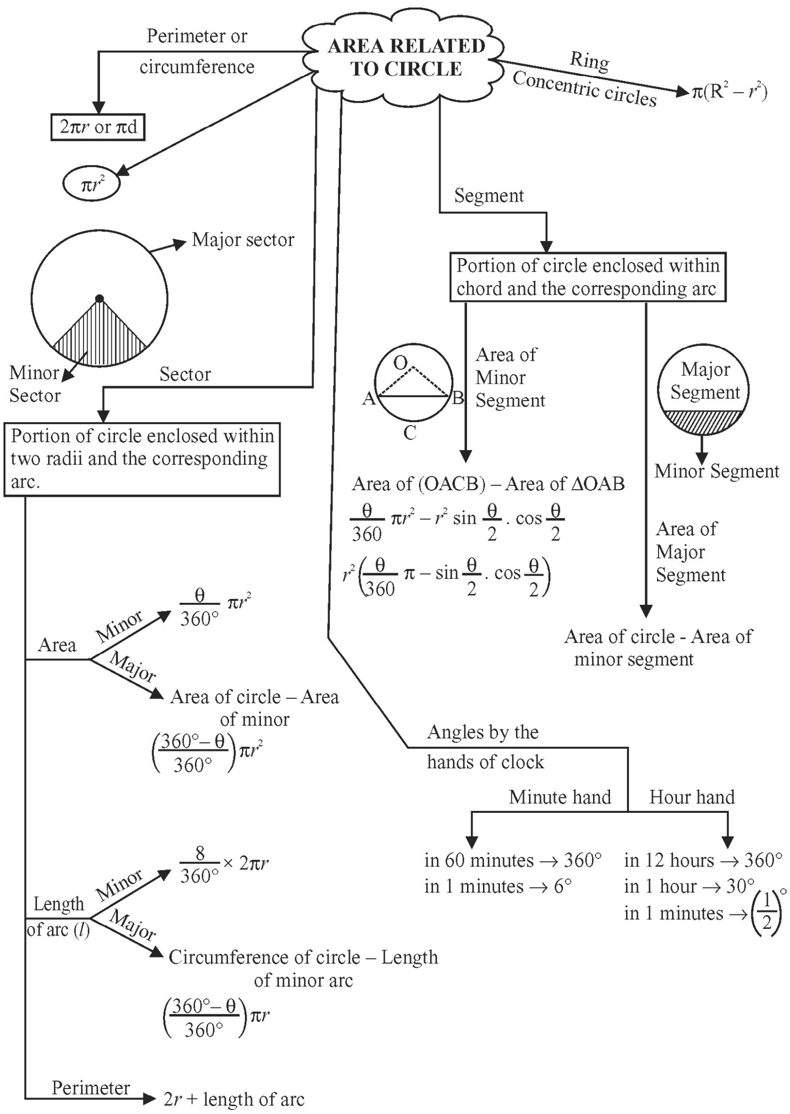
CHAPTER

11 Areas Related to Circles

TOPICS

Perimeter and Area of a circle.

Area of sector and segment of a circle.



KEY POINTS

Circle: A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains the same. The fixed point is called the centre and the constant distance is known as the radius of the circle.

If r is radius of a circle, then

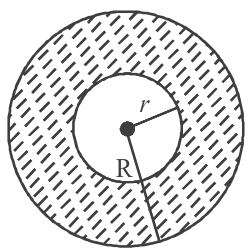
(i) Circumference = $2\pi r$ or πd where d = 2r is the diameter of the circle

(ii) Area = $\boldsymbol{\pi r^2}$ or $\dfrac{\pi d^2}{4}$

(iii) Area of semi circle = $\dfrac{\pi r^2}{2}$

(iv) Area of quadrant of a circle = $\dfrac{\pi r^2}{4}$

Area enclosed by two concentric circles: If R and r are radii of two concentric circles, then area enclosed by the two circles = $\pi R^2 - \pi r^2$



$= \pi (R^2 - r^2)$

$= \pi (R + r) (R - r)$

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

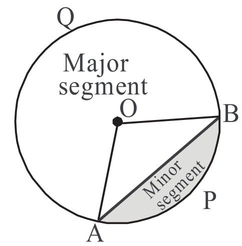
(ii) If two circles touch externally, then distance between their centres is equal to the sum of their radii.

(iii) Distance covered by rotating wheel in one revolution is equal to the circumference of the wheel.

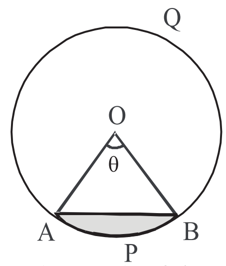
(iv) The number of revolutions completed by a rotating wheel in

$$\text{one minute} = \frac{\text{Distance moved in one minute}}{\text{Circumference of the wheel}}$$

Segment of a Circle: The portion (or part) of a circular region enclosed between a chord and the corresponding arc is called a segment of the circle. In adjacent fig. APB is minor segment and AQB is major segment.

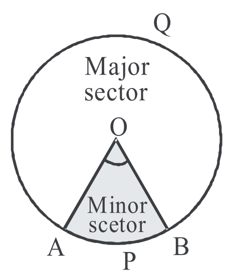


Area of segment APB = Area of the sector OAPB – Area of ΔOAB



Sector of a circle: The portion (or part) of the circular region enclosed by the two radii and the corresponding arc is called a sector of the circle.

In adjacent figure OAPB is minor sector and OAQB is the major sector.



$$\text{Area of the sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{1}{2} \times \text{length of arc} \times \text{radius} = \frac{1}{2} lr$$

$$\text{Length of an arc of a sector of angle } \theta = \frac{\theta}{360^\circ} \times 2\pi r$$

(i) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(ii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(iii) Angle described by minute hand in 60 minutes = 360°

Angle described by minute hand in one minute $= \frac{360°}{60} = 6°$

Thus minute hand rotates through an angle of 6° in one minute

(iv) Angle described by hour hand in 12 hours = 360°

Angle described by hour hand in one hour $= \frac{360°}{12} = 30°$

Angle described by hour hand in one minute $= \frac{30°}{60} = \left(\frac{1}{2}\right)^{\circ}$

Thus, hour hand rotates through an angle of $\left(\frac{1}{2}\right)^{\circ}$ in one minute.

VERY SHORT ANSWER QUESTIONS

1. If the diameter of a semi circular protactor is 14 cm, then find its perimeter.

2. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

3. Find the area of the circle 'inscribed' in a square of side *a* cm.

4. Find the area of a sector of a circle whose radius is *r* and length of the arc is *l*.

5. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 kms.

6. If the area of a circle is 616 cm², then what is its circumference?

7. What is the area of the circle that can be inscribe in a square of side 6 cm?

8. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 24 cm and 7 cm?

9. A wire can be bent in the form of a circle of radius 35 cm. If it is bent in the form of a square, then what will be its area?

10. What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 3π cm?

11. Write the formula for the area of a sector of angle θ (in degrees) of a circle of radius *r*.

12. If the circumference of two circles are in the ratio 2:3, what is the ratio of their areas?

13. If the difference between the circumference and radius of a circle is 37 cm, then find the circumference of the circle. (Use $\pi = \frac{22}{7}$)

14. If diameter of a circle is increased by 40%, find by how much percentage its area increases?

15. The minute hand of a clock is 6 cm long. Find the area swept by it between 11:20 am and 11:55 am.

16. The perimeter of a sector of a circle of radius 14 cm is 68 cm. Find the area of the sector. (CBSE 2020)

17. The circumference of a circle is 39.6 cm. Find its area.

(Use $\pi = \frac{22}{7}$) (CBSE 2020)

18. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use $\pi = \frac{22}{7}$)

19. Area of a sector having length of corresponding arc 'l' and radius 'r' is ______.

20. Circumference of a circle of radius s is ___________ .

21. Area of a circle of radius is _______ ,

22. Length of an arc of a sector of a circle with radius r and angle θ is _______ .

23. Area of a sector with radius r and angle with degrees measure θ is _______ .

24. Area of segment of a circle = Area of the corresponding sector ________ .

SHORT ANSWER TYPE QUESTIONS (1)

25. Find the area of a quadrant of a circle whose circumference is 22 cm.

(Use $\pi = \frac{22}{7}$)

26. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?

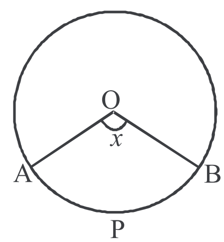
27. If a square is inscribed in a circle, what is the ratio of the area of the circle and the square?

28. Find the area of a circle whose circumference is 44 cm. **(CBSE 2020)**

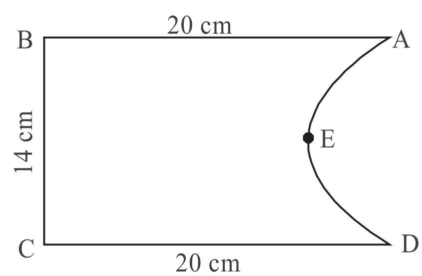
29. If the perimeter of a circle is equal to that of square, then find the ratio of their areas.

30. What is the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal?

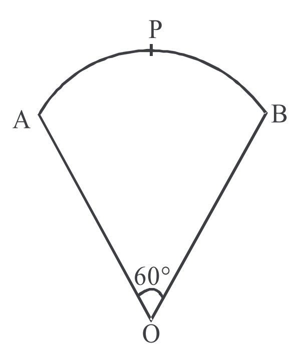
31. In fig., O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.



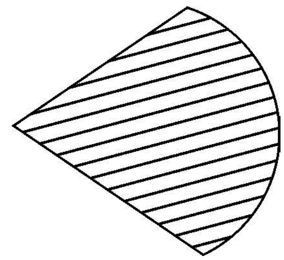
32. Find the perimeter of the given fig, where AED is a semicircle and ABCD is a rectangle. **(CBSE 2015)**



33. In fig. OAPBO is a sector of a circle of radius 10.5 cm. Find the perimeter of the sector.



34. A Japenese fan can be made by sliding open its 7 small sections, each of which is in the form of sector of a circle having central angle of 15°. If the radius of this fan is 24 cm, find the length of the lace that is required to cover its entire boundary. (Use $\pi = 22/7$)



(CBSE 2014)

35. The perimeter of a sector of circle of radius 6.3 cm is 25.8 cm. Find the area of the sector.

36. Find the area of a circle in which a square of area 64 cm^2 is inscribed.

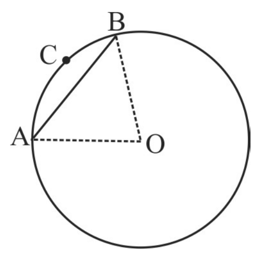
37. Find the area of a circle which is inscribed in a square of area 64 cm^2.

SHORT ANSWER TYPE II QUESTIONS

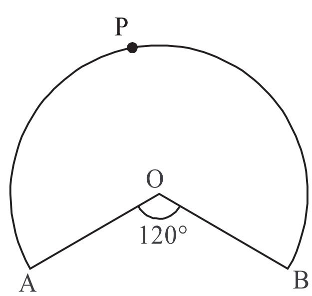
38. Area of a sector of a circle of radius 36 cm is 54π cm^2 . Find the length of the corresponding arc of the sector.

39. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am to 6:40 am.

40. Find the area of the segment bounded by a chord AB and the arc ACB of the circle with centre O having radius 7 cm and sector angle equal to 90°, as shown in the fig.



41. In fig, OAPB is a sector of a circle of radius 3.5 cm with the centre at O and $\angle AOB = 120°$. Find the length of OAPBO.



42. Circular footpath of width 2 m is constructed at the rate of ₹ 20 per square meter, around a circular park of radius 1500 m. Find the total cost of construction of the foot path. (Take $\pi = 3.14$)

43. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm. Calculate the speed of cycle.

44. In a circle with centre O and radius 4 cm, and of angle 30°. Find the area of minor sector and major sector AOB. (Use $\pi = 3.14$)

45. Find the area of the largest triangle that can be inscribed in a semi circle of radius r unit. (NCERT Exemplar)

46. In a square park of side 8 m two goats are tied at opposite vertices with a rope of length 1.4 m and a cow is tied in the centre with a rope of length 2.1m. Calculate the area of park which cannot be grazed by them.

47. A sector of 100° cut off from a circle contains area 70.65 cm². Find the radius of the circle. (Use $\pi = 3.14$)

48. The hour and minute hand of a 12 hour clock are 3.5 cm and 7 cm long respectively. Find the sum of distance travelled by their tips in a day. $\left(\text{use } \pi = \frac{22}{7}\right)$

49. A square water tank has its each side equal to 40 m. There are four semi circular grassy plots all around it. Find the cost of turfing the plot at Rs 1.25 per sq. m. (Use $\pi = 3.14$)

50. Length of a chord of a circle of a radius of 4 cm is 4 cm. Find the area of the sector and segment formed by the chord.

51. Find the area of the minor segment of a circle of radius 21 cm, when the angle of the corresponding sector is 120°.

52. A piece of wire 11 cm long is bent into the form of an arc of a circle subtending an angle of 45° at its centre. Find the radius of the circle.

53. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

54. A pendulum swings through an angle of 45° and describes an arc of 22 cm in length. Find the length of the pendulum. $\left(\text{use } \pi = \frac{22}{7}\right)$

54. Two circles touch externally. The sum of their area is 130π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

LONG ANSWER TYPE QUESTIONS

55. Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

56. Find the number of revolutions made by a circular wheel of area 6.16 m^2 in rolling a distance of 572 m.

57. Three horses are tied at the vertices of a triangular park of sides 35 m, 84 m and 91 m with the help of a rope of length 14 m each. Calculate the ratio of the area which can be grazed to the area which can't be grazed.

58. Two circle touch each other internally. The sum of their area is 116π cm^2 and distance between their centres is 6 cm. Find the radii of the circles. **(CBSE = 2017)**

59. You are required to create a model of a circular wall clock and paste the numbers from 1 to 12 on its dial. What is the angle made at the centre between 3 and 7? Find the area of this region, if the length of the minute hand is 21 cm. **(CBSE)**

ANSWERS AND HINTS

1. $\pi r + d = \frac{22}{7} \times 7 + 14 = 36$ cm

2. $2\pi r = \pi r^2 \Rightarrow$ diameter = 4 units

3. Side of the square is equal to diameter of the circle,

$$\pi r^2 = \pi \times \frac{a^2}{4} \text{ (side} = a\text{, radius} = \frac{a}{2}\text{)}$$

4. $l = \frac{\theta}{360°} \times 2\pi r$, Area $= \frac{\theta}{360°} \times \pi r^2 = \frac{l \times \pi r^2}{2\pi r} = \frac{lr}{2}$ sq. units

5. $\frac{\text{distance}}{\text{circumference}} = \frac{11 \times 1000 \times 7 \times 100}{2 \times 22 \times 25} = 7000$

6. $\pi r^2 = 616 \Rightarrow r = 14$ cm or $2\pi r = 88$ cm

7. Side of the square is equal to the diameter of the circle

$\Rightarrow$ r = 3 cm or $\pi r^2 = \pi(3)^2 = 9\pi$ cm^2.

8. $\pi R^2 = \pi r_1^2 + \pi r_2^2 \Rightarrow R = 25$ and diameter = 50 cm.

9. $2\pi r = 2 \times \frac{22}{7} \times 35 = 220$ cm , Side of square $\frac{220}{4} = 55$ cm

Area of square = 55 × 55 = 3025 cm^2

10. $l = \frac{\theta}{360°} \times 2\pi r \quad \Rightarrow \quad 3\pi = \frac{\theta}{360°} \times 2\pi \times 6 \quad \Rightarrow \quad \theta = 90°$

11. $\frac{\theta}{360°} \times \pi r^2$

12. $\frac{2\pi r_1}{2\pi r_2} = \frac{2}{3} \Rightarrow r_1 = \frac{2}{3} r_2$ or $\frac{\pi r_1^2}{\pi r_2^2} = \frac{\left(\frac{2}{3} r_2\right)^2}{r_2^2} = 4:9$

13. $(2\pi r - r) = 37$ or $r = 7$, $2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm

14. 96%

15. $\frac{210° \times 22 \times 6 \times 6}{360° \times 7} = 66$ cm^2 ($\theta = 210°$) (11: 20 to 11: 55 = 35 minutes)

16. 280 cm^2

17. 124.74 cm^2

18. 10.27 cm^2

19. $A = \frac{1}{2} lr$

20. $2\pi r$

21. πs^2

22. $\frac{\theta}{360°} \times 2\pi r$

23. $\frac{\theta}{360°} \times \pi r^2$

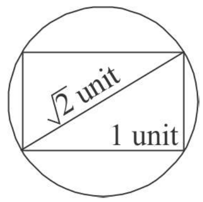
24. Area of the corresponding triangle

25. $2\pi r = 22,\ r = \dfrac{7}{2}$

Area of quadrant $= \dfrac{\pi r^2}{4} = \dfrac{22 \times 7 \times 7}{7 \times 4 \times 2 \times 2} = 9.625 \text{ cm}^2$

26. $l = \dfrac{\theta}{360°} \times 2\pi r \Rightarrow 5\pi = \dfrac{\theta}{360°} \times 2\pi \times 10 \Rightarrow \theta = 90°$

27.



If side of square is 1 unit, by Pythagoras Theorem

Diameter $\sqrt{2}$ unit.

Area of square = 1 × 1 = 1 sq units.

Area of Circle $= \pi r^2 = \pi \times \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{2}}{2} = \dfrac{\pi}{2} = \dfrac{11}{7}$

Required ratio = 11 : 7

28. 154 cm^2

29. $2\pi r = 4$ unit or $\dfrac{2\pi r}{4 \text{ unit}} = \dfrac{\text{Perimeter of circle}}{\text{Perimeter of square}}$ (Let side of square = 1 unit)

$$r = \frac{7}{11} \text{ unit}$$

$$\frac{\pi r^2}{1} = \frac{22}{7} \times \frac{7}{11} \times \frac{7}{11} = \frac{14}{11} \quad \text{or} \quad 14 : 11$$

30. Area of equilateral triangle $= \dfrac{\sqrt{3}}{4} a^2$

Area of circle $= \pi \left(\dfrac{a}{2}\right)^2$

Required ratio $= \sqrt{3} : \pi$

31. $\dfrac{\theta}{360°} \pi r^2 = \pi r^2 \times \dfrac{5}{18}$

$\theta = 100°$

32. 20 cm + 14 cm + 20 cm + πr

20 cm + 14 cm + 20 cm + $\frac{22}{7} \times 7 = 76$ cm

33. $\frac{\theta}{360°} \times 2\pi r = \frac{60 \times 2 \times 22 \times 105}{360° \times 7 \times 10} = 11$ cm

Perimeter = 10.5 + 10.5 + 11 cm = 32 cm

34. $\theta = 7 \times 15° = 105°$

$l = \frac{\theta}{360°} 2\pi r = 44$ cm

Length of lace = $l + 2r$

= 44 + 48 = 92 cm

35. Perimeter of sector = $l + 2r$

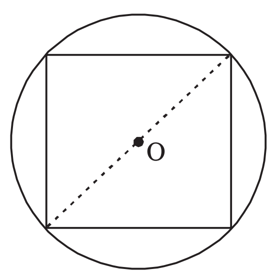
l = 25.8 – 12.6 = 13.2 cm

$\frac{\theta}{360°} \times 2\pi r = l$

Area of sector = $\frac{\theta}{360°} \pi r^2$

Area of sector = 41.58 cm^2

36. d = Diagonal of square

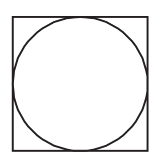


d = side $\sqrt{2} = 8\sqrt{2}$ cm

$r = 4\sqrt{2}$ cm

Area = $\pi R^2 = 32\pi$ cm^2

37. Diameter of circle = Side of square



$\therefore r = 4$ cm

Area = 16π cm^2

38. $54\,\pi = \frac{\theta \times \pi \times 36 \times 36}{360°}$

$\theta = 15°$

$l = \frac{\theta}{360°} \times 2\pi r = \frac{15° \times 2 \times \pi \times 36}{360°} = 3\,\pi$ cm

39. Area $= \frac{\theta}{360°} \times \pi r^2 = \frac{210° \times 22 \times 5 \times 5}{360° \times 7} = \frac{1650}{36} = 45 \cdot \frac{5}{6} \text{cm}^2$

($\theta = 210°$ in 35 minutes)

40. Area of sector = area of sector – area of ΔAOB

$= \frac{77}{2} - \frac{49}{2}$

$= 14 \text{ cm}^2$

41.

$$l = \frac{240° \times 2 \times 22 \times 35}{360° \times 7 \times 10}$$

$= 14.67$

Length of OAPBO = 14.6 + 3.5 + 3.5

= 21.67 cm

42.

$$\pi (r_2^2 - r_1^2) = \pi[(1502)^2 - (1500)^2] \times 20$$

$= 3.14 [(1502)^2 – (1500)^2] \times 20$

= ₹ 377051.2

43. Circumference of cycle = 2πr

$= 2 \times \frac{22}{7} \times 30$ cm

= 188.57 cm

Speed of cycle $= \frac{18857 \times 140 \times 60}{100 \times 1000}$

= 15.84 km/h

44. Area of Minor sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30°}{360°} \times 3.14 \times 4 \times 4 \text{ cm}^2$

$= 4.19 \text{ cm}^2$ (approx.)

Area of major sector $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{330°}{360°} \times 3.14 \times 4 \times 4$$

$$= 46.1 \text{ cm}^2 \quad \text{(approx)}$$

45. $\text{Area of } \Delta = \frac{1}{2} \text{base} \times \text{height}$

$$= \frac{1}{2} AB \times OC$$

$$= \frac{1}{2} 2r \times r = r^2 \text{ square unit}$$

46. Grazing area of Goats = 2 × area of quadrants

$$= 2 \times \frac{22}{7} \times 1.4 \times 1.4 \times \frac{1}{4} = 3.08\text{m}^2$$

Grazing area of cow = Ar. of circle

$$= \frac{22}{7} \times 2.1 \times 2.1 = 13.86\text{m}^2$$

Area which can't be grazed = Area of square – total grazing area

$= 64 – 16.94 = 43.06 \text{ m}^2$

47. $$\frac{7065}{100} = \frac{100° \times 314 \times r^2}{360° \times 100}$$

$$\frac{7065 \times 360}{100 \times 314} = r^2$$

$$9 = r$$

$$r = 9 \text{ cm.}$$

48. Distance by minute hand in 1 day = $24 \times 2\pi R$
Distance by hour hand in 1 day = $2 \times 2\pi r$
Total distance travelled by tips of both hands = $24 \times 2\pi R + 2 \times 2\pi R$
= 1056 + 44
= 1100 cm

49. Four semicircluar means 2 circles ,

Area of 2 circles $= 2\pi r^2$

$$= 2 \times 3.14 \times 20 \times 20$$

$$= 2512 \text{ sq.m}$$

$$\text{Total cost} = 2512 \times 1.25$$
$$= ₹\ 3140$$

50. Length of chord = radius

$\therefore$ Angle of sector = 60°

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{8\pi}{3}\text{cm}^2$$

Area of segment = Area of sector - Area of triangle

$$= \frac{8\pi}{3} - \frac{\sqrt{3}}{4}r^2$$

$$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)\text{cm}^2$$

51. Area of the segment = Area of sector – Area of Δ

$$\text{Area of sector} = \frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21 = 462\ \text{cm}^2$$

$$\text{Area of }\Delta = \frac{441}{4}\sqrt{3}\ \text{cm}^2$$

$$\text{Area of segment} = \left(462 - \frac{441}{4}\sqrt{3}\right)\ \text{cm}^2$$

$$= \frac{21}{4}\left(88 - 21\sqrt{3}\right)\text{cm}^2$$

52.
$$l = \frac{\theta}{360°} \times 2\pi r$$

$$11 = \frac{45°}{360°} \times \frac{2 \times 22 \times r}{7}$$

$$14 = r$$

$$r = 14\ \text{cm}$$

53. $2\pi r = 2r + 16.8$

$$2 \times \frac{22}{7}r - 2r = \frac{168}{10} \quad \text{or} \quad 2r\left(\frac{22}{7} - 1\right) = \frac{168}{10}$$

or, $$2r\left(\frac{15}{7}\right) = \frac{168}{10} \quad \text{or} \quad r = \frac{168 \times 7}{10 \times 2 \times 15} = \frac{1176}{300} = 3.92 \text{ cm}$$

54. $$l = \frac{\theta}{360^\circ} \times (2\pi r)$$

$$22 = \frac{45}{360^\circ} \times 2 \times \frac{22}{7} \times r$$

$$r = 28$$

$\Rightarrow$ Length of pendulum = 28 cm

55. $$\pi r_1^2 + \pi r_2^2 = 130\,\pi \Rightarrow r_1^2 + r_2^2 = 130 \qquad \ldots(1)$$

$\Rightarrow$ $$r_1 + r_2 = 14 \qquad \ldots(2)$$

Substitute the value of r_1 from (2) in (1) and solve.

$$2r_2{}^2 - 28\,r_2 + 66 = 0$$

$$r_2^2 - 14r_2 + 33 = 0 \qquad \text{(Neglecting – ve)}$$

$$r_2 = 11 \text{ cm and } r_1 = 3 \text{ cm}$$

56. $$\pi r^2 = \frac{616}{100} \quad \text{or} \quad r^2 = 1.96 \quad \text{or} \quad r = 1.4 \text{ m}$$

$$2\pi r = 2 \times \frac{22}{7} \times \frac{14}{10} = \frac{616}{100} = 8.8 \text{ m}$$

Number of revolutions $= \dfrac{572}{8.8} = 65$

57. Grazing area of Horses $= \dfrac{180^\circ}{360^\circ} \times \dfrac{22}{7} \times (14)^2 = 308\text{m}^2$

Area of triangular park $= \dfrac{1}{2} \times 35 \times 84 = 1470\text{m}^2$

Area which can't be grazed $= 1162\text{m}^2$

Grazing Area : Area can't be grazed = 308 : 1162

= 22 : 83

58. $R^2 + r^2 = 116$ ...(1)

$R - r = 6$ ...(2)

Sequence both sides and solving, we get

$2Rr = 80$ ...(3)

Addign and solving (1) and (3)

$R + r = 14$ ...(4)

Solving (2) and (4)

$R = 10$ cm, $r = 4$ cm

59. Angle made minute hand in 1 minute = 6°

$\therefore$ Angle made between 3 to 7 = 6° × 20 = 120°

Area = $\frac{\theta}{360°}\pi R^2 = 462\text{cm}^2$

PRACTICE-TEST

AREAS RELATED TO CIRCLES

Time : 45 Minutes **M.M.: 20**

SECTION-A

1. If the area of sector is $\frac{7}{18}$ of the area of the circle. Find the measure of central angle of the sector. **1**

2. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is: **1**

   (a) 48 cm (b) 31 cm (c) 25 cm (d) 17 cm

3. The area of sector whose perimeter is four times its radius of measure r units is_________. **1**

4. If the area of a sector of a circle bounded by an arc of length 5 π cm is equal to 20 π cm^2, then find the radius of the circle.

SECTION-B

5. The perimeter of a sector of circle of radius 5.7 cm is 27.2 cm. Find the area of the sector. **2**

6. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand between 6:10 pm and 6:45 pm. **2**

7. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular cardboard of dimensions 16 cm × 8 cm. Find the area of the remaining cardboard. **2**

SECTION-C

8. The length of a rope by which a cow is tied is increasd from 12m to 19m. How much more area can the cow graze now? (Use $\pi = 22/7$) **3**

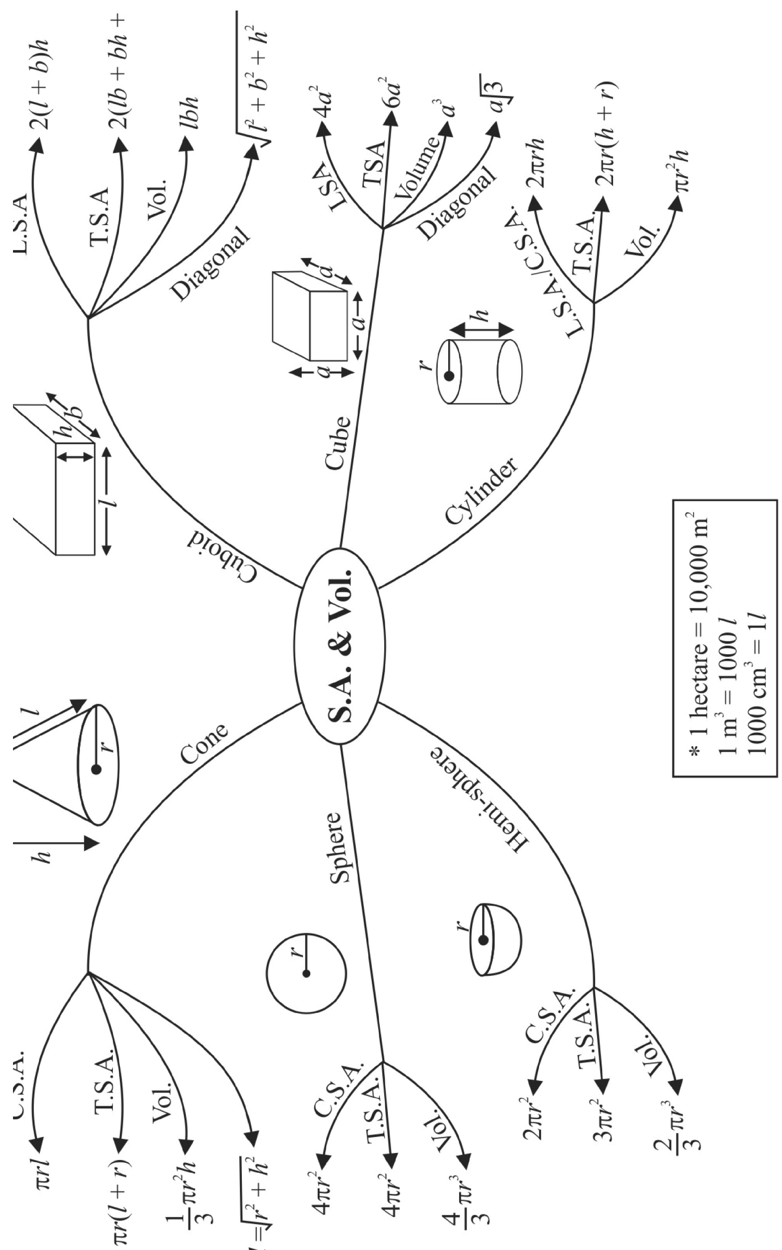
9. A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment. (Use $\pi = 22/7$) **3**

SECTION-D

10. Find the area of minor and major segments of a circle of radius 42 cm, if the length of the arc is 88 cm. **4**

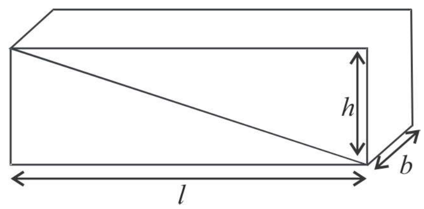
CHAPTER

12 Surface Areas and Volumes



KEY POINTS

1. **Cuboid:** 3-D shapes like a book, a match box, an almirah, a room etc. are called Cuboid.



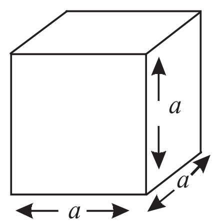
For cuboid length = l, breadth = b, height = h

Volume $= \boldsymbol{l \times b \times h}$

Lateral surface area of solid cuboid $= 2\boldsymbol{h}(\boldsymbol{l} + \boldsymbol{b})$

Total surface area of solid cuboid $= 2(\boldsymbol{lb} + \boldsymbol{bh} + \boldsymbol{hl})$

2. **Cube:** 3-D shapes like ice-cubes, dice etc. are called cube.



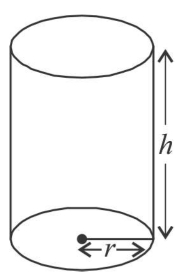
In cube, length = breadth = height = a

Volume $= \boldsymbol{a^3}$

Lateral surface area of solid cube $= 4\boldsymbol{a^2}$

Total surface area of solid cube $= 6\boldsymbol{a^2}$

3. **Cylinder**: 3-D shapes like jars, circular pillars, circular pipes, rood rollers etc. are called cylinder.



(a) For right circular cylinder solid, base radius = $\boldsymbol{r}$, height = $\boldsymbol{h}$

Volume = $\boldsymbol{\pi r^2 h}$

Curved surface area of solid cylinder = $\boldsymbol{2\pi rh}$

Total surface area of solid cylinder = $\boldsymbol{2\pi r (r + h)}$

(b) For right circular cylinder (Hollow)

external radius = R

internal radius = r

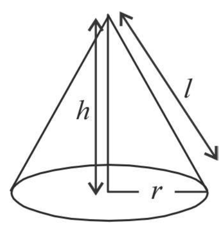
height = h

Volume = $\boldsymbol{\pi(R^2 - r^2)h}$

Curved surface area = $2\pi(\boldsymbol{R} + \boldsymbol{r})\boldsymbol{h}$

Total surface area = $\boldsymbol{2\pi(R + r)\, h} + 2\pi(\boldsymbol{R}^2 - \boldsymbol{r}^2)$

4. Cone: 3-D shapes like conical tents, ice-cream cone etc. are called Cone.



For right circular cone,

base radius = $\boldsymbol{r}$

height = $\boldsymbol{h}$

slant height = $\boldsymbol{l}$

$\boldsymbol{l} = \sqrt{h^2 + r^2}$

Volume = $\frac{1}{3}$ $\boldsymbol{\pi r^2 h}$

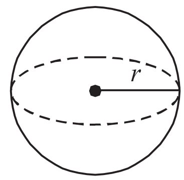
Curved surface area of solid cone = $\boldsymbol{\pi rl}$

Total surface area of solid cone = $\boldsymbol{\pi r (r + l)}$

It may be noted that if radius and height of a cone and cylinder are same then

3 × volume of a cone = volume of right circular cylinder

5. **Sphere**: 3-D shapes like cricket balls, footballs etc. are called sphere.

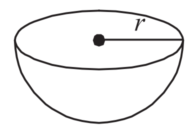


(a) For sphere : Radius = $\boldsymbol{r}$

Volume = $\frac{4}{3}\boldsymbol{\pi r^3}$

surface area = $\boldsymbol{4\pi r^2}$

(b) For Hemisphere (solid): Radius = $\boldsymbol{r}$

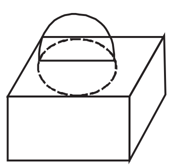
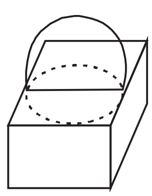


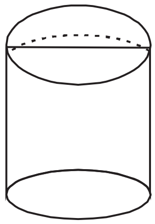
Volume = $\frac{2}{3}\boldsymbol{\pi r^3}$

Curved surface area = $\boldsymbol{2\pi r^2}$

Total surface area = $\boldsymbol{3\pi r^2}$

COMBINATION OF SOLIDS-I

Figure	Surface Area of Resultant Fig.	Volume of Resultant Fig.
 Cuboid & H.sph.	T.S.A$_{\text{cuboid}}$ + C.S.A$_{\text{h.sp.}}$ - Area of circle	Vol.$_{\text{cube}}$ + Vol.$_{\text{h.sphere}}$
 Cube & H.Sph.	T.S.A$_{\text{cube}}$ + C.S.A$_{\text{h.sph.}}$ - Area of circle	Vol.$_{\text{cube}}$ + Vol.$_{\text{h.sphere}}$



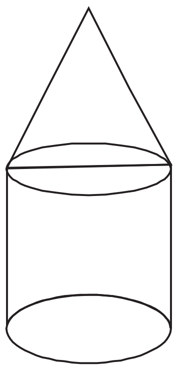
Case I → when cylinder is hollow

$C.S.A_{cyl.} + C.S.A_{h.sph.}$ — $Vol._{cyl.} + Vol._{h.sphere}$

Case II → when cylinder is solid

$C.S.A_{cyl.} + C.S.A_{h.sph.} +_{h.sph..} + Ar.\ of\ base$

Cyl. & H.Sph.



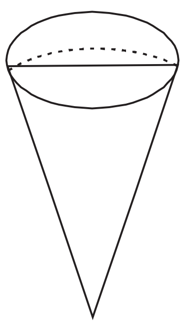
Case I → when cylinder is hollow

$C.S.A_{cyl.} + C.S.A._{cone}$ — $Vol._{cyl.} + Vol._{cone}$

Case II → when cylinder is solid

$C.S.A_{cyl.} + C.S.A_{cone} + Ar.\ of\ base$

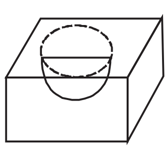
Cyl. & Cone

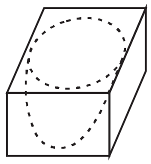


$C.S.A_{cone} + C.S.A_{h.sp.}$ — $Vol._{cone} + Vol._{h.sphere}$

Cone & H.Sph.

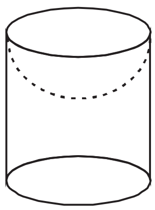
SURFACE AREA OF RESULTANT FIGURE

Figure	Surface Area of Resultant Fig.	Volume of Resultant Fig.
 H.sph. curved out of cuboid	$T.S.A_{cuboid} + C.S.A_{h.sp.}$ -Area of circle	$Vol._{cuboid} - Vol._{h.sphere}$



T.S.A_{cube} + C.S.$A_{h.sph.}$ - Area of circle Vol.$_{cube}$ – Vol.$_{h.sphere}$

H.Sph. curved out of cube



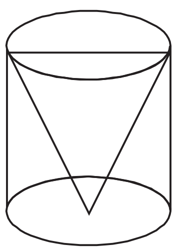
Case I → hollow cylinder

C.S.$A_{cyl.}$+C.S.$A_{h.sph.}$ Vol.$_{cyl.}$ – Vol.$_{h.sphere}$

Case II → Solid cylinder

C.S.$A_{cyl.}$+ C.S.$A_{h.sph.}$+ Ar. of circle

H.Sph. depression in cylinder



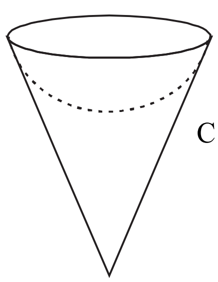
Case I → when cylinder is hollow

C.S.$A_{cyl.}$+C.S.A.$_{cone}$ Vol.$_{cyl.}$ – Vol.$_{cone}$

Case II → when cylinder is solid

C.S.$A_{cyl.}$+C.S.A_{cone}+Ar. of base

Conical depression in cylinder



C.S.A_{cone} + C.S.$A_{h.sp.}$ Vol.$_{cone}$ – Vol.$_{h.sphere}$

H.Sph. depression in cone

VERY SHORT ANSWER TYPE QUESTIONS

1. Match the following:

Column I	**Column II**
(a) Surface area of a sphere	(i) $2\pi rh$
(b) Total surface area of a cone	(ii) $\frac{1}{3}\pi r^2 h$
(c) Volume of a cuboid	(iii) $2\pi r(r + h)$
(d) Volume of hemisphere	(iv) $\frac{1}{3}\pi h(r^2 + R^2 + rR)$
(e) Curved surface area of a cone	(v) $\pi r\,(r + l)$
(f) Total surface area of hemisphere	(vi) $l \times b \times h$
(g) Curved surface area of a cylinder	(vii) $\frac{2}{3}\pi r^3$
(h) Volume of a cone	(viii) πrl
(i) Total surface area of a cylinder	(ix) $3\pi r^2$
(j) Volume of a frustum of a cone	(x) $4\pi r^2$

2. Fill in the Blanks:

(i) The total surface area of cuboid of dimension $a \times a \times$ b is __________.

(ii) The volume of right circular cylinder of base radius r and height 2r is ______________.

(iii) The total surface area of a cylinder of base radius r and height h is_____________.

(iv) The curved surface area of a cone of base radius r and height h is_______________.

(v) If the height of a cone is equal to diameter of its base, the volume of cone is_________________ .

(vi) The total surface area of a solid hemisphere of radius r is____________.

(vii) The curved surface area of a hollow cylinder of outer radius R, inner radius r and height h is __________ .

(viii) If the radius of a sphere is doubled, its volume becomes__________times the volume of original sphere.

(ix) If the radius of a sphere is halved, its volume becomes__________times the volume of original sphere. **(NCERT Exemplar)**

3. Write 'True' or 'False' in the following:

(*i*) Two identical solid hemispheres of equal base radius r are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

(*ii*) A solid cylinder of radius *r* and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $(4\pi rh + 4\pi r^2)$.

(iii) A solid cone of radius *r* and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r(\sqrt{r^2 + h^2} + 3r + 2h)$.

(iv) A solid ball is exactly fitted inside the cubical box of side 'a'. The volume of the ball is $\frac{4}{3}\pi a^2$.

4. The total surface area of a solid hemisphere of radius *r* is

(a) πr^2 (b) $2\pi r^2$ (c) $3\pi r^2$ (d) $4\pi r^2$

5. The volume and the surface area of a sphere are numerically equal, then the radius of sphere is

(a) 0 units (b) 1 unit (c) 2 units (d) 3 units

6. A cylinder, a cone and a hemisphere are of the same base and of the same height. The ratio of their volumes is

(a) 1:2:3 (b) 2:1:3 (c) 3:1:2 (d) 3:2:1

7. A solid sphere of radius '*r*' is melted and recast into the shape of a solid cone of height '*r*'. Then the radius of the base of cone is

(a) 2r (b) r (c) 4r (d) 3r

8. Three solid spheres of diameters 6 cm, 8 cm and 10 cm are melted to form a single solid sphere. The diameter of the new sphere is

(a) 6 cm (b) 4.5 cm (c) 3 cm (d) 12 cm

9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is:

(a) 12 cm (b) 14 cm

(c) 15 cm (d) 18 cm

10. Find total surface area of a solid hemi-sphere of radius 7cm.

11. Volume of two spheres is in the ratio 64 : 125. Find the ratio of their surface areas.

12. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volumes of cylinder to that of the cone.

13. If the volume of a cube is 1331 cm^3, then find the length of its edge.

14. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes? (CBSE 2020)

SHORT ANSWER TYPE QUESTION (TYPE-I)

15. How many cubes of side 2 cm can be cut from a cuboid measuring (16cm × 12cm × 10cm)?

16. Find the height of largest right circular cone that can be cut out of a cube whose volume is 729 cm^3.

17. Two identical cubes each of volume 216 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?

18. Two cones with same base radius 8 cm and height 15 cm are joined together along with their bases. Find the surface area of the shape so formed. **(NCERT exampler)**

19. The total surface area of a right circular cone is 90π cm^2. If the radius of the base of the cone is 5 cm, find the height of the cone. **(CBSE - 2011)**

20. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE 2020)**

21. Find the volume of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

SHORT ANSWER TYPE QUESTION (TYPE-II)

22. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere. **(CBSE-2012)**

23. Find the depth of a cylindrical tank of radius 10.5 cm, if its capacity is equal to that of a rectangular tank of size 15 cm × 11 cm × 10.5 cm.

24. Volume of two spheres are in the ratio 64:27, find the ratio of their surface areas. **(CBSE-2012)**

25. A petrol tank is a cylinder of base diameter 28 cm and length 24 cm filted with conical ends each of axis length 9 cm. Determine the capacity of the tank.

26. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

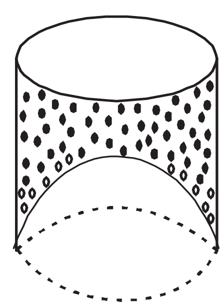
27. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE 2019)**

28. The diameter of a roller 120 cm long is 64 cm. If it takes 500 complete revolutions to level a playground, determine the cost of levelling it at the rate of 30 paise per square meter. **(CBSE 2013)**

29. The sum of the radius of base and height of a solid right circuler cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 square cm., find the volume of the cylinder. (Use $\pi = 22/7$) **(CBSE-2016)**

30. A juice seller was serving his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. [Use $\pi = 3.14$]

(NCERT, CBSE 2019, 2009)



31. The internal and external diameters of a hollow hemispherical vessel are 12 cm and 16 cm respectively. If the cost of painting 1 cm^2 of the surface area is ₹ 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

(CBSE 2019)

32. Suresh decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is ₹ 40 per metre, find the amount by which Suresh helped the centre. **(CBSE 2017)**

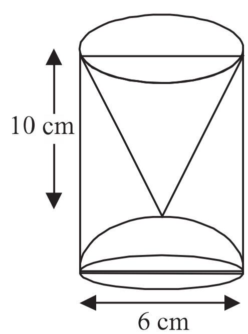
33. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

LONG ANSWER TYPE QUESTIONS

34. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 gm mass. (Use $\pi = 3.14$) **(NCERT, CBSE 2019)**

35. A right cylindrical container of radius 6 cm and height 15 cm is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone. **(CBSE 2019)**

36. A wooden article as shown in the fig. was made from a cylinder by scooping out a hemisphere from one end and a cone from the other end. Find the total surface area of the remaining article. **(NCERT, CBSE 2019)**



37. The height of a solid cylinder is 15 cm and its diameter is 7 cm. Two equal conical holes of radius 3 cm and height 4 cm are cut off. Find the volume and surface area of the solid.

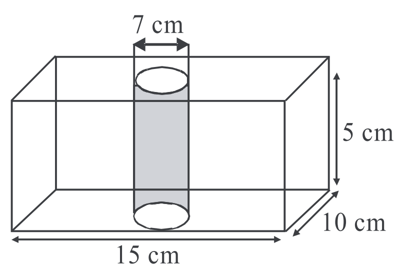
38. If h, c and V respectively represent the height, curved surface area and volume of a cone, prove that **(CBSE 2015)**

$$c^2 = \frac{3\pi V h^3 + 9V^2}{h^2}$$

39. A solid wooden toy is in the form of a hemi-sphere surmounted by a cone of same radius. The radius of hemi-sphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemi-spherical part of the toy at the rate of ₹ 10 per cm^2.

$\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE, 2015)**

40. In the given figure, from a cuboidal solid metalic block of dimensions 15 cm × 10 cm × 5 cm a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE – 2015)**



41. A solid toy is the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the toy.

42. A tent is in the shape of a right circular cylinder upto a height of 3 m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of ₹ 80 per m^2.

43. The difference between outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long is 88 cm^2. If the volume of the metal used in making the cylinder is 176 cm^3. Find the outer and inner diameters of the cylinder.

(HOTS)

44. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid. **(CBSE 2020)**

45. A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to edge of the cube. Determine the volume of the remaining block. **(CBSE 2020)**

ANSWERS AND HINTS

1. (a) (x) $4\pi r^2$ (b) (v) $\pi r\,(r + l)$

(c) (vi) $l \times b \times h$ (d) (vii) $\frac{2}{3}\pi r^3$

(e) (viii) πrl (f) (ix) $3\pi r^2$

(g) (i) $2\pi rh$ (h) (ii) $\frac{1}{3}\pi r^2 h$

(i) (iii) $2\pi r(r + h)$ (j) (iv) $\frac{1}{3}\pi h(r^2 + R^2 + rR)$

2. (i) $2a^2 + 4ab$ (ii) $2\pi r^3$

(iii) $2\pi r(r + h)$ (iv) $\pi r\sqrt{r^2 + h^2}$

(v) $\frac{2}{3}\pi r^3$ (vi) $3\pi r^2$

(vii) $2\pi h(R + r)$ (viii) 8

(ix) $\frac{1}{8}$

3. (i) False (ii) False

(iii) False (iv) False

4. (c) $3\pi r^2$ **5.** (d) 3 units

6. (c) 3 : 1 : 2 **7.** (a) 2r

8. (d) 12 cm

9. (b) 14 cm

10. 462 cm^2 **11.** 16 : 25

12. 3 : 1 **13.** 11 cm

14. 3 : 1

15. No. of cubes = $\frac{16 \times 12 \times 10}{2 \times 2 \times 2} = 240$

16. Side of cube = $\sqrt[3]{729} = 9\text{cm}$

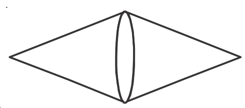
Height of largest cone = Side of cube = 9 cm

17. Side of cube = $\sqrt[3]{216} = 6$ cm

Length, breadth and height of new cuboid is 12 cm, 6 cm and 6 cm respectively.

Surface area of cuboid = 2[12 × 6 + 6 × 6 + 6 × 12] = 360 cm^2

18. $l = \sqrt{r^2 + h^2}$



$l = 17$

Area = $2\pi rl$ = 854.85 cm^2

19. $\pi r\,(l + r) = 90\pi$

$l = 13$ cm

$h = \sqrt{l^2 - r^2}$

$h = 12$ cm

20. Let the height and radius of cylinder be x cm and x cm respectively.

Volume of cylinder = $\frac{176}{7}$ cm^3

$\frac{22}{7} \times (x)^2 \times x = \frac{176}{7}$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$ cm

21. d = 4.2 cm; r = 2.1 cm

h = 4.2 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of cone = 19.4 cm^3 (approx)

22. Radius of sphere = 3 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

= 113.14 cm^3

23. Capacity of cylindrical tank = Capacity of rectangular tank

$$\frac{22}{7} \times (10.5)^2 \times h = 15 \times 11 \times 10.5$$

h = 5 cm

24. $$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27}$$

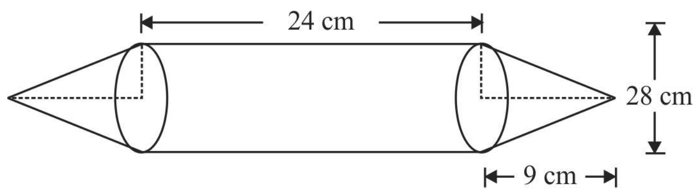
$\Rightarrow R^3 : r^3 = 64 : 27$

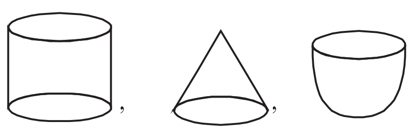
$\Rightarrow R : r = 4 : 3$

$4\pi R^2 : 4\pi r^2 = R^2 : r^2 \Rightarrow 4^2 : 3^2 = 16 : 9$

25. Capacity of tank = Volume of cylindrical part + 2 × Volume of conical part

= 18480 cm^2



26. 

Radius = r, height = r

$\text{Volume}_{\text{cylinder}} : \text{Volume}_{\text{cone}} : \text{volume}_{\text{hemisphere}}$

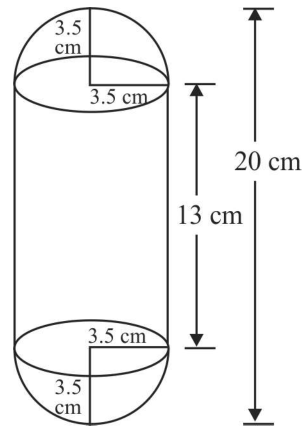
Req. Ratio = $\pi r^3 : \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3$

$= 1 : \frac{1}{3} : \frac{2}{3}$

$= 3 : 1 : 2$

27. Height of cylinder = 20 – 3.5 – 3.5 = 13 cm

Volume of solid = Volume of cylindrical part + 2 × Volume of hemispherical part



$$= \frac{22}{7} \times (3.5)^2 \times 13 + 2 \times \frac{2}{3} \times \frac{22}{7}(3.5)^3$$

$$= 680\frac{1}{6} \text{ cm}^3$$

28. r = 32 cm; h = 120 cm

Area covered in 1 revolution

= C.S.A. of roller

= 2 πrh

= 24137.14 cm^2

Area covered in 500 rev.

= 1206.86 m^2

Cost of levelling = Area × Rate

= ₹ 1206.86 × 0.3

= ₹ 362.06

29. $r + h = 37$

$2\pi r(r + h) = 1628$

r = 7 cm

h = 30 cm

Volume = $\pi r^2 h$

Volume = 4620 cm^3

30. Apparent capacity = $3.14 \times \left(\frac{5}{2}\right)^2 \times 10 = 196.25$ cm^3.

Actual capacity = Volume of cylindrical part – Volume of hemispherical part

$$= 196.25 - \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3$$

$$= 163.54 \text{ cm}^3 \text{ approx}$$

31. $r = 6$cm; R = 8 cm

S.A. of vessel = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi \times 228 = 715.92$ cm^2

Total cost = S.A. × Rate

= ₹ 3579.60/-

32. $r = 7$cm; $h = 24$m

$l = 25$ m

S.A. of tent = πrl

= 550m^2

Area of 10 tents = 5500 m^2

Total cost = Area × Rate

= Area × ₹$\frac{40}{2}$

= ₹ 1,10,000

33. $r = 7$cm; $h = 14$ cm

$l = \sqrt{245} = 15.65\text{cm}$

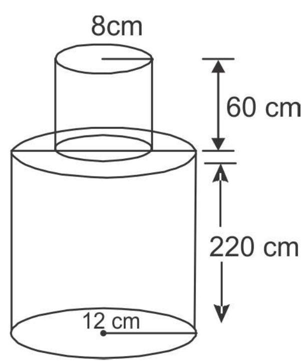
S.A. of remainging solid

= T.S.A. of cube + C.S.A. of cone – Area of circle

$= 6a^2 + \pi rl - \pi r^2$

= 1366.3 cm^2

34.



Volume of solid = $3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$

$= 111532.8$ cm^3

Mass of the pole = $111532.8 \times \frac{8}{1000}$ kg

= 892.2624 kg

35. Let radius of conical section be r cm.

$\therefore$ Height of conical section be 4r cm.

According to the question

10 × Volume of ice-cream in 1 cone = Volume of cylindrical container

$$10 \times \left[\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right] = \pi(6)^2 \times 15$$

$$r = 3 \text{ cm}$$

36. $r = 3$ cm

S.A. of article = C.S.A.$_{\text{cylinder}}$ + C.S.A.$_{\text{sphere}}$ + C.S.A.$_{\text{cone}}$

S.A. = $2\pi r\text{H} + 2\pi r^2 + \pi rl$

$= \pi r(2\text{H} + 2r + l)$

$= 3\pi(20 + 6 + \sqrt{58})$

$= \pi(78 + 3\sqrt{58})\text{cm}^2$

37. Surface area of solid = C.S.A.$_{\text{cyl.}}$ + 2 Area of Ring + 2C.S.A.$_{\text{cone}}$

$$= 2\pi\left[\frac{7}{2} \times 15 + 6.5 \times 0.5 + 15\right]$$

$$= 2 \times \frac{22}{7} \times 70.75 = \frac{3113}{7}$$

$= 444.7\text{cm}^2$ (approx.)

38. $\text{V} = \frac{1}{3}\pi R^2 h$

$$\Rightarrow R^2 = \frac{3\text{V}}{\pi h} \qquad \text{...(1)}$$

Now, $c = \pi \text{R}l$

$c^2 = \pi^2 R^2 l^2$

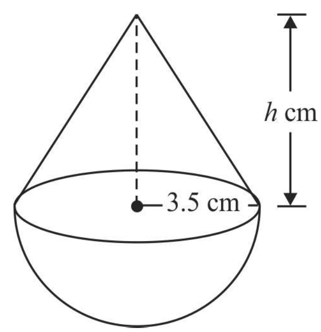
$c^2 = \pi^2 R^2 (h^2 + R^2)$

$$c^2 = \pi^2 \frac{3V}{\pi h}\left(h^2 + \frac{3V}{\pi h}\right)$$

$$c^2 = \frac{3\pi^2 V(\pi h^3 + 3V)}{\pi^2 h^2}$$

$$c^2 = \frac{3\pi V h^3 + 9V^2}{h^2}$$

39. Volume of toy = $\frac{1001}{6}$ cm^3



$$\frac{2}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3 + \frac{1}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 \times h = \frac{1001}{6}$$

$$h = 6 \text{ cm}$$

Area of hemispherical part of toy

$$= 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 = 77 \text{ cm}^2$$

Cost of painting = 77 × 10 = ₹ 770

40. Surface of the remaining block = TSA of cuboidal block + CSA of cylinder – Area of two circular bases

$$= 2(15 \times 10 + 10 \times 5 + 15 \times 5) + 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$= 583$ cm^2

41. Volume of toy = Volume of cylindrical part + Volume of hemispherical part + Volume of conical part

$$= \frac{22}{7}\times(2.1)^2\times 12 + \frac{1}{3}\times\frac{22}{7}\times(2.1)^2\times 7 + \frac{2}{3}\times\frac{22}{7}\times(2.1)^3$$

$= 218.064$ cm^3

42. Slant height = $\sqrt{(14)^2 + (10.5)^2}$ = 17.5 m

Surface area of tent = $2\times\frac{22}{7}\times 3\times 14 + \frac{22}{7}\times 14\times 17.5$

$= 1034$ m^2

Cost of cloth = 1034 × 80 = ₹ 82720

43. Let inner and outer radius of hallow cylinder be r cm and R cm respectively.

Difference between Outer and Inner CSA = 88 cm^2

$$2 \times \frac{22}{7} \times 14 \times [R - r] = 88$$

$$R - r = 1 \qquad ...(1)$$

Volume of hollow cylinder = 176 cm^3

$$\frac{22}{7} \times 14 \times [R^2 - r^2] = 176$$

$$R^2 - r^2 = 4$$

$$(R - r)(R + r) = 4$$

$$R + r = 4 \qquad ...(2) \quad [\because \text{ from (1)}]$$

From (1) and (2), we get

$$R = 2.5 \text{ cm and } r = 1.5 \text{ cm}$$

$\therefore$ Outer and inner diameter are 5 cm and 3 cm respectively.

44. Height of cone = 9.5 – 3.5 = 6 cm

$$\text{Volume of solid} = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 + \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6$$

$$= 166.83 \text{ cm}^3 \text{ approx}$$

45. Radius of hemisphere = $\frac{21}{2}$ = 10.5 cm

$$\text{Volume of remaining block} = (21)^3 - \frac{2}{3} \times \frac{22}{7} \times (10.5)^3$$

$$= 6835.5 \text{ cm}^3$$

PRACTICE-TEST

SURFACE AREAS AND VOLUMES

Time : 45 Minutes | ***M.M.: 20***

SECTION-A

1. The total surface area ofa hemisphere of radius $2r$ is ________ **1**

2. The radius of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is **1**

(a) 4.2 cm (b) 8.4 cm

(c) 2.1 cm (d) 1.05 cm

3. The volume of a cube is $1l$. Find the length of the side of the cube. **1**

4. Volume of two cubes are in the ratio 27 : 125. The ratio of their surface areas is_______. **1**

SECTION-B

5. A cube and a sphere have equal total surface area. Find the ratio of the volume of sphere and cube. **2**

6. Two cubes, each of side 8cm are joined end to end. Find the surface area of the resulting figure. **2**

7. The volume of a hemi-sphere is 2156 cm^3. Find its curved surface area. **2**

SECTION-C

8. A circus tent is in the shape of a cylinder surmounted by a conical roof. If the common diameter is 56 m, the height of the cylindrical portion is 6 m and the height of the roof from the ground is 30 m, find the area of the canvas used for the tent. **3**

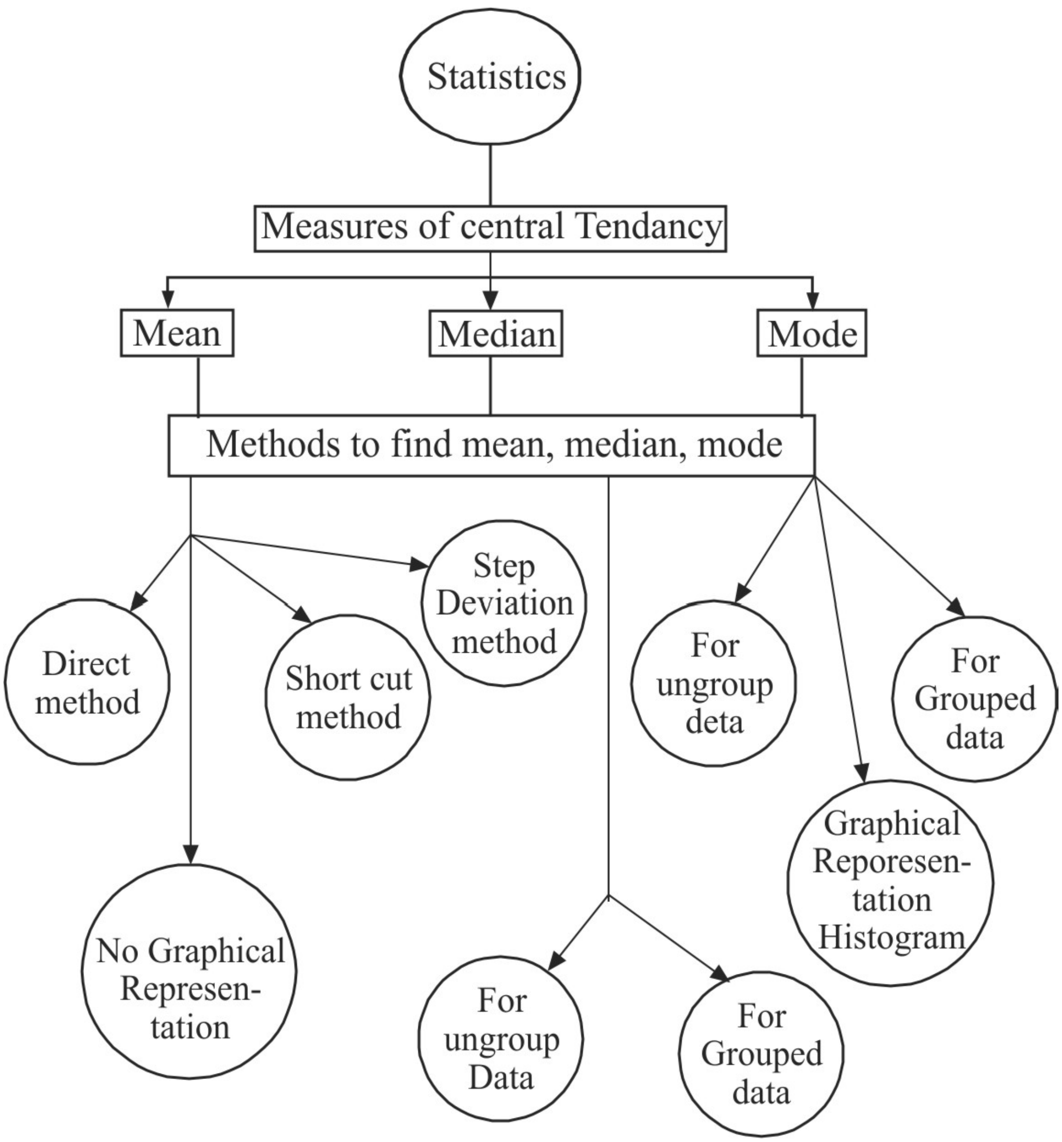
9. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole of radius $\frac{3}{2}$ cm and depth $\frac{8}{9}$ cm is drilled in the cylinder. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape. **3**

SECTION-D

10. A decorative block is made up by joining a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 4 cm. Find the cost of painting it at a price of ₹ 2.5 per cm^2. **4**

CHAPTER

13 Statistics



KEY POINTS:

1. **Mean $(\bar{x})$**

(a) For raw data, $\bar{x} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + ... + x_n}{n}$

i.e. $\bar{x} = \frac{\text{sum of observations}}{\text{no of observations}}$

(b) For Grouped data

(i) For small calculation, we apply Direct method

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

(ii) If calculations are tedius or observations are large, then we apply short cut/ Assumed Mean method or step Deviation method

Short cut/Assumed Mean Method

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}, \; a \rightarrow \text{assumed mean}$$

$$d_i = x_i - a$$

Step Deviation Method

$$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h, \; u_i = \frac{d_i}{h}, \; h \rightarrow \text{class size}$$

2. **Median**

(a) For ungrouped data, we first arrange data in ascending or descending order.

Count number of times say 'n'. If n is odd, then Median = $\left(\frac{n+1}{2}\right)^{th}$ observation

If n is even, then Median = $\frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$ obsevation

(b) For grouped data

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

(3) Mode $= l + \dfrac{(f_1 - f_o)}{(2f_1 - f_o - f_2)} \times h$ (For grouped data)

For ungrouped data mode is the most frequent observation.

NOTES:

1. Empirical relationship between three measures of central tendency:

mode = 3 median – 2 mean.

2. If class interval is discontinuous, then make it continuous by subtracting 0.5 from Lower Limit and adding 0.5 to upper limit.

3. x_i = class mark = $\dfrac{\textit{Upper Limit} + \textit{Lower Limit}}{2}$

4. h = class size = Upper Limit – Lower limit

5. Modal class → A class interval having maximum frequency.

6. Median class → A class interval is which cumulative frequency is greater than and nearest to $\dfrac{n}{2}$ $(n = \Sigma f_i)$

8. If mean of $x_1, x_2, ..., x_n$ is $\overline{x}$ then

(a) Mean of $kx_1, kx_2, ..., kx_n$ is $k\overline{x}$

(b) Mean of $\dfrac{x_1}{k}, \dfrac{x_2}{k}, ..., \dfrac{x_n}{k}$ is $\dfrac{\overline{x}}{k}$

(c) Mean of $x_1 + k, x_2 + k, ..., x_n + k$ is $\overline{x} + k$

(d) Mean of $x_1 - k, x_2 - k, ..., x_n - k$ is $\overline{x} - k$

9. If mean of n_1 observation is $\overline{x}_1$ and mean of n_2 observation is $\overline{x}_2$ then their combined

$$\text{Mean} = \frac{n_1\,\overline{x}_1 + n_2\,\overline{x}_2}{n_1 + n_2}$$

10. $\Sigma x_i = n\,\overline{x}$

11. Range = Highest observation – Lowest observation

12. Graphical Representation of Mode is a Histogram.

VERY SHORT ANSWER TYPE(I) QUESTIONS

1. What is the mean of first 12 prime numbers?

2. The mean of 20 numbers is 18. If 2 is added to each number, what is the new mean?

3. The mean of 5 observations 3, 5, 7, x and 11 is 7, find the value of x.

4. What is the median of first 5 natural numbers?

5. What is the value of x, if the median of the following data is 27.5?
24, 25, 26, $x + 2$, $x + 3$, 30, 33, 37

6. What is the mode of the observations 5, 7, 8, 5, 7, 6, 9, 5, 10, 6?

7. The mean and mode of a data are 24 and 12 respectively. Find the median.

8. Write the class mark of the class 19.5 – 29.5.

9. Multiple Choice Question

(i) If the class intervals of a frequency distribution are 1 – 10, 11 – 20, 21 – 30, ..., 51 – 60, then the size of each class is:

(a) 9 (b) 10 (c) 11 (d) 5.5

(ii) If the class intervals of a frequency distribution are 1 – 10, 11 – 20, 21 – 30 ...., 61 – 70, Then the upper limit of 21 – 30 is:

(a) 21 (b) 30

(c) 30.5 (d) 20.5

(iii) Consider the frequency distribution.

Class	0 – 5	6 – 11	12 – 17	18 – 23	24 – 29
Frequency	13	10	15	8	11

The upper limit of median class is :

(a) 17 (b) 17.5 (c) 18 (d) 18.5

(iv) Daily wages of a factory workers are recorded as:

Daily wages (in ₹)	121 – 126	127 – 132	133– 138	139 – 144	145 – 150
No. of workers	5	27	20	18	12

The lower limit of Modal class is:

(a) ₹ 127 (b) ₹ 126 (c) ₹ 126.50 (d) ₹ 133

(v) For the following distribution

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

The sum of Lower limits of the median class and modal class is (CBSE 2020)

(a) 15 (b) 25 (c) 30 (d) 35

(vi) The median and mode respectively of a frequency distribution are 26 and 29. Then, its mean is (CBSE 2020)

(a) 27.5 (b) 24.5 (c) 28.4 (d) 25.8

10. Find the class-marks of the classes 10-25 and 35-55. (CBSE 2020)

11. Fill in the blank

(a) Mode = 3_________ – 2 _________

(b) Arithmetic mean of all factors of 20 is _________ .

(c) The cumulative frequency of a given class is obtained by adding the frequencies of all the classes ___________.

(d) The mode of a frequency distribution is determined graphically by _________.

(e) If the mode is 8 and mean is also 8, then median will be _________ .

(f) The measure of central tendency which cannot be determined graphically is _________ .

(g) If the class marks of a continuous frequency distribution are 22, 30, 38, 46, 54, 62 then the class corresponding to class mark 46 is _________ .

(h) Construction of cumulative frequency distribution table is useful in determining _________ .

(i) The step deviation formula for finding mean is _________ .

(j) The formula to find median of grouped data is _________ .

(k) The formula to find mode of grouped data is _________ .

(l) The Range of the observations 255, 125, 130, 160, 185, 170, 103 is _______ .

(m) Class mark = $\frac{1}{2}$ (________ + ________) .

(n) The median of Ist ten prime numbers is ________ .

(o) The assumed mean method to find mean is ________ .

SHORT ANSWER TYPE QUESTIONS (I)

12. The mean of 11 observation is 50. If the mean of first Six observations is 49 and that of last six observation is 52, then find sixth observation.

13. Find the mean of following distribution:

x	12	16	20	24	28	32
f	5	7	8	5	3	2

14. Find the median of the following distribution:

x	10	12	14	16	18	20
f	3	5	6	4	4	3

15. Find the mode of the following frequency distribution:

Class	0–5	5–10	10 –15	15–20	20–25	25–30
Frequency	2	7	18	10	8	5

16. Convert the following deistribution in frequency distribution:

Marks	**No. of students**
Less than 20	0
Less than 30	4
Less than 40	16
Less than 50	30
Less than 60	46
Less than 70	66
Less than 80	82
Less than 90	92
Less than 100	100

17. Write the following data into less than cummulative frequency distribution table :

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	7	9	6	8	10

18. Find mode of the following frequency distribution :

Class Interval	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
Frequency	25	34	50	42	38	14

(CBSE 2018 - 19)

19. What is the median of the following data? **(CBSE 2011)**

x	10	20	30	40	50
f	2	3	2	3	1

20. Mean of a frequency distribution ($\bar{x}$) is 45. If $\Sigma f_i = 20$ find $\Sigma f_i x_i$

(CBSE 2011)

21. Find the mean of the following distribution : **(CBSE 2020)**

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

22. Find the mode of the following data : (CBSE 2020)

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120-140
Frequency	6	8	10	12	6	5	3

23. Compute the mode for the following frequency distribution: (CBSE 2020)

Size of items (in cm)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28
Frequency	5	7	9	17	12	10	6

SHORT ANSWER TYPE QUESTIONS (II)

24. If the mean of the following distribution is 54, find the value of P.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	7	P	10	9	13

25. Find the median of the following frequency distribution :

C.I.	0–10	10–20	20–30	30–40	40–50	50–60
f	5	3	10	6	4	2

26. The median of following frequency distribution is 24 years. Find the missing frequency x.

Age (In years)	0–10	10–20	20–30	30–40	40–50
No. of persons	5	25	x	18	7

27. Find the median of the following data:

Marks	Below 10	Below 20	Below 30	Below 40	below 50	Below 60
No. of student	0	12	20	28	33	40

28. Find the mean weight of the following data:

Weight (In kg.)	30–35	35–40	40–45	45–50	50–55	55–60
No. of Students	2	4	10	15	6	3

29. Find the mode of the following data:

Height (In cm)	Above 30	Above 40	Above 50	Above 60	Above 70	Above 80
No. of plants	34	30	27	19	8	2

30. The following table represent marks obtained by 100 students in a test:

Marks obtained	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
No. of students	14	16	28	23	18	8	3

Find mean marks of the students. **(CBSE 2018 -19)**

31. The following table represent pocket allowance of children of a colony. The mean pocket allowance is ₹ 18. Find the missing frequency.

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
No. of children	3	6	9	13	k	5	4

(CBSE – 2018)

32. Find mode of the following frequency distribution:

Class Interval	0–20	20–40	40–60	60–80	80–100
No. of Students	15	18	21	29	17

The mean of above distribution is 53. Use Empirical formula to find approximate value of median.

LONG ANSWER TYPE QUESTIONS

33. The mean of the following data is 53, Find the values of f_1 and f_2.

C.I	0–20	20–40	40–60	60–80	80–100	Total
f	15	f_1	21	f_2	17	100

34. If the median of the distribution given below is 28.5, find the values of x and y.

C.I	0–10	10–20	20–30	30–40	40–50	50–60	Total
f	5	8	x	15	y	5	60

35. The median of the following distribution is 35, find the values of a and b.

C.I	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Total
f	10	20	a	40	b	25	15	170

36. Find the mean, median and mode of the following data:

C.I	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	2	3	6	7	14	12	4	2

37. The rainfall recorded in a city for 60 days is given in the following table:

Raifall (in cm)	0–10	10–20	20–30	30–40	40–50	50–60
No. of Days	16	10	8	15	5	6

Calulate the median rainfall.

38. Find the mean of the following distribution by step- deviation method:

Daily Expenditure (in ₹)	100–150	150–200	200–250	250–300	300–350
No. of Households	4	5	12	2	2

39. The distribution given below show the marks of 100 students of a class:

Marks	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
No. of Students	4	6	10	10	25	22	18	5

Find the median marks of the above distribution.

40. The annual profit earned by 30 factories in an industrial area is given below:

Profit (₹ in lakh)	No. of Factories
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3
More than or equal to 40	0

Find the median of the above data.

41. Find the mean and median of the following distribution:

(CBSE 2018 -19)

Class Interval	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	7	5	8	10	6	6	8

42. If mean of the given distribution is 65.6 find the missing frequency.

(CBSE 2017)

Class Interval	10 – 30	30 – 50	50 – 70	70 – 90	90 – 110	110 – 130	Total
Frequency	5	8	f_1	20	f_2	2	50

43. The mode of the frequency distribution is 36. Find the missing frequency (f).

(CBSE 2020)

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	8	10	f	16	12	6	7

44. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f. **(CBSE 2020)**

Class Interval	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	3	6	9	13	f	5	4

45. The following table gives production yield per hectare of wheat of 100 farms of a village : **(CBSE 2020)**

Production Yield	40 – 45	45 – 50	50 – 55	55 – 60	60 – 55	65 – 70
Frequency	4	6	16	20	30	24

Find the mode of the above data.

46. Find the unknown entries a, b, c, d, e, f in the following distribution of heights of students in a class: **(CBSE 2020)**

Height (in cm	150-155	155-160	160-165	165-170	170-175	175-180
Frequency	12	b	10	d	e	2
Cummulative Frequency	a	25	c	43	48	f

Find the mode of the above data.

ANSWERS AND HINTS

1. 16.4 approx.

2. 20

3. 9

4. 3

5. $x = 25$

6. 5

7. Median = 20

8. 24.5

9. (i) B (First make intervals continuous, Then find class size)

(ii) C

(iii) B

(iv) C

(v) B $\begin{bmatrix} \text{Modal class } 15-20 \\ \text{Median class } 10-15 \end{bmatrix}$

(vi) B

10. 17.5 and 45

11. (a) 3 Median – 2 mean

(b) 7

(c) Preceding the given classes

(d) Histogram

(e) 8

(f) Mean

(g) 42 – 50 (as difference b/w 2 consecutive observation is 8

$\therefore$ Subtract $\frac{8}{2}$ form 46 for Lower Limit and Add $\frac{8}{2}$ to 46 for upper Limit)

(h) Median

(i) $\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

(j) Median $= l + \left(\dfrac{\dfrac{n}{2} - cf}{f}\right) \times h$

(k) Mode $= l + \dfrac{(f_1 - f_o)}{(2f_1 - f_o - f_2)} \times h$

(l) Range = 255 – 103 = 152

(m) $\dfrac{1}{2}$ (upper limit + Lower limit)

(*n*) 12.9

(o) $\overline{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

12. 56

13. 20

14. 14

15. 12.89 approx.

16.

Marks	**No. of students**
10-20	0
20-30	4
30-40	12
40-50	14
50-60	16
60-70	20
70-80	16
80-90	10
90-100	8

17.

Marks	**No. of students**
less than 10	7
less than 20	16
less than 30	22
less than 40	30
less than 50	40

18.

Class Interval	**Frequency**
25 – 30	25
30 – 35	$34 = f_0$
$\boxed{35 – 40}$	$50 = f_1$
40 – 45	$42 = f_2$
45 – 50	38
50 – 55	14

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h = 35 + \frac{(50 - 34)}{(100 - 34 - 42)} \times 5 = 35 + \frac{16 \times 5}{24}$$

$= 35 + 3.33 = 38.33$ approx.

19.

x_i	f_i	cf
10	2	2
20	3	5
30	2	7
40	3	10
50	1	11
Total	11	

N = 11 (odd)

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{ observation} = 6\text{th observation} = 30$$

20. $\bar{x} = \dfrac{\Sigma f_i\, x_i}{\Sigma f_i} \Rightarrow 45 = \dfrac{\Sigma f_i\, x_i}{20} \Rightarrow \Sigma f_i x_i = 900$

21. 8.15

22. 62.5

23. 14.46 cm

24. 11

25. 27

26. 25

27. 30

28. 46

29. 63.75 cm

30.

Mark	x_i	d_i	u_i	f_i	$f_i u_i$
30 – 35	32.5	– 15	– 3	14	– 42
35 – 40	37.5	– 10	– 2	16	– 32
40 – 45	42.5	– 5	– 1	28	– 28
45 – 50	47.5 = a	0	0	23	0
50 – 55	52.5	5	1	18	18
55 – 60	57.5	10	2	8	16
60 – 65	62.5	15	3	3	9
				110	–59

$$\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 47.5 - \frac{59}{110} \times 5 = 47.5 - 2.68 = 44.82$$

31. (Make Table just like Q. 30)

$$\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$18 = 18 + \frac{(k-8)}{40+k} \times 2$$

$$2k - 16 = 0$$

$$k = 8$$

32. $$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

$$= 60 + \frac{(29 - 21)}{(2 \times 29 - 21 - 17)} \times 20 = 68$$

Mode = 3 Median – 2 mean

68 = 3 Median – 2 × 53

$$\frac{68 + 106}{3} = \text{Median}$$

Median = 58

33. $f_1 = 18, f_2 = 29$

34. $x = 20, y = 7$

35. $a = 35$, $b = 25$

36. Mean = 32, median = 33, mode = 34.39 approx.

37. Median = 25 cm

38. Mean = ₹ 211

39. Median = 24

40. Median = ₹ 17.5 lakhs.

41. Mean = 51.92, Median = 65

42.

C.I	f_i	x_i	$f_i x_i$
10 – 30	5	20	100
30 – 50	8	40	320
50 – 70	f_1	60	$60f_1$
70 – 90	20	80	1600
90 – 110	f_2	100	$100f_2$
110 – 130	2	120	240
	$\mathbf{35 + f_1 + f_2}$		$\mathbf{2260 + 60 f_1 + 100 f_2}$

$35 + f_1 + f_2 = 50 \quad \Rightarrow \quad f_1 + f_2 = 15$ ....(1)

$$\bar{x} = \frac{\Sigma fixi}{\Sigma fi}$$

$$65.6 = \frac{2260 + 60\, f_1 + 100\, f_2}{50}$$

$\Rightarrow \; 3 f_1 + 5 f_2 = 51$ ...(2)

Solve (1) & (2) $\quad f_1 = 12, f_2 = 3$

43. $f = 10$

44. $f = 8$

45. Mode = 63.125

46. $a = 12$, $b = 13$, $c = 35$, $d = 8$, $e = 5$, $f = 50$

PRACTICE-TEST

Statistics

Time : 45 Minutes **M.M. : 20**

SECTION-A

1. Find the mean of first 10 natural numbers. **1**

2. The range of the data 14, 27, 29, 61,45,15, 9,18 is **1**

(a) 61 (b) 52

(c) 47 (d) 53

3. In a continuous frequency distribution, the median of the data is 24. If each item is increased by 2, then find the new median. **1**

4. For a frequency distribution, mean, median and mode are connected by the relation. **1**

(a) mode = 3 mean – 2median (b) mode = 2 median – 3mean

(c) mode = 3 median – 2mean (d) mode = 3 median + 2 mean

SECTION-B

5. The mean of 10 observations is 42. If each observation in the data is decreased by 12, then find the new mean of the data. **2**

6. The mean of 10 numbers is 15 and that of another 20 number is 24 then find the mean of all 30 observations. 2

7. The mileage (km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below: **2**

Mileage (in km/l)	10 – 12	12 – 14	14 – 16	16 – 18
No. of cars	7	12	18	13

Find the mean mileage.

SECTION-C

8. Life time of 400 fans are given in the following frequency distribution table:

Life time	2000-2400	2400-2800	2800-3200	3200-3600	3600-4000
No. of fans	5	15	20	23	17

Find the median number of fans. **3**

9. The mode of the following data is 36. Find the value of x. **3**

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	x	16	12	6	7

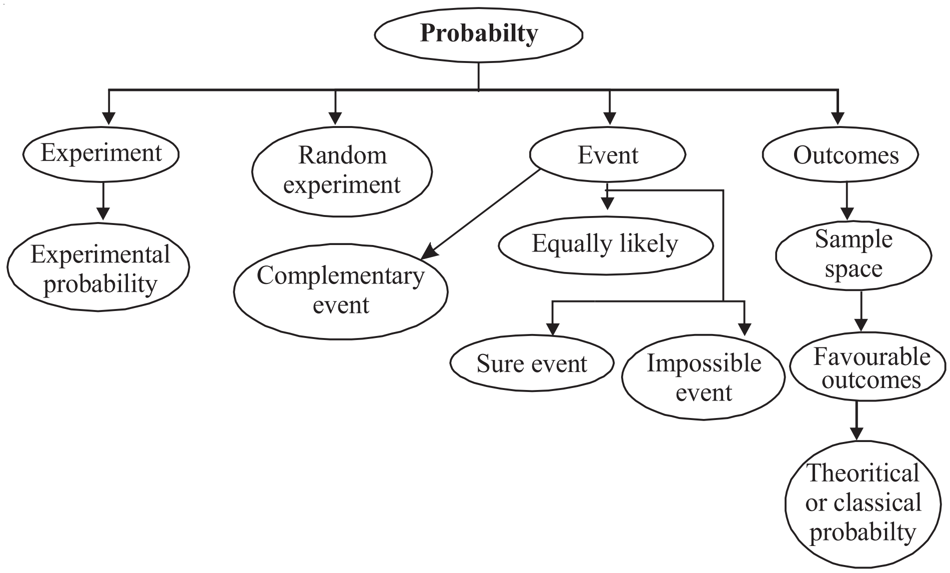
SECTION-D

10. The median of the following data is 28. Find the values of x and y, if the total frequency is 50. **4**

Marks	0-7	7-14	14-21	21-28	28-35	35-42	42-49
No. of	3	x	7	11	y	16	9

CHAPTER

14 Probability



KEY POINTS:

1. Probability is a quantitative measure of likelihood of occurrence of an event.
2. Probability of an event $E = \dfrac{\text{Number of outcomes favourable to E}}{\text{Total number of outcomes}}$
3. $0 \le P(E) \le 1$
4. If $P(E) = 0$, then it is an impossible event.
5. If $P(E) = 1$, then it is sure event.
6. If E is an event, then not E($\overline{E}$) is called complementary event.
7. $P(\overline{E}) = 1 - P(E) \Rightarrow P(E) + P(\overline{E}) = 1$
8. Probability of an event is never negative.
9. Sample space (S) : The collection of all possible outcomes of an event.

Examples of Sample space

1. When one coin is tossed, then S = H, T
2. When two coins are tossed, then S = HH, TT, HT, TH
3. When three coins are tossed, then S = HHH, TTT, HTT, THT, TTH, THH, HTH, HHT
4. When four coins are tossed, then S = HHHH, TTTT, HTTT, THTT, TTHT, TTTH, HHHT, HHTH, HTHH, THHH, HTHT, THTH, TTHH, HHTT, THHT, HTTH.

1 coin	2coins	3 coins	4 coins
2 outcomes	2×2 outcomes	$2 \times 2 \times 2 = 8$ outcomes	$2 \times 2 \times 2 \times 2 = 16$ outcomes

1. When a die is thrown once, then S = 1, 2, 3, 4, 5, 6, n(S) = 6
2. When two dice are thrown together or A die is thrown twice, then

$$
\begin{aligned}
S = \ & (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\
& (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\
& (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\
& (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\
& (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\
& (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \\
n(S) = \ & 6 \times 6 = 36
\end{aligned}
$$

3. When 3 dice are thrown or a die is thrown thrice then

$$
\begin{aligned}
n(S) = \ & 6 \times 6 \times 6 = 216, \\
n(S) \rightarrow \ & \text{no. of outcomes in sample space}
\end{aligned}
$$

Playing cards n(s) = 52

Red Cards (26) — Heart (13), Diamond (13)

Black Cards (26) — Spade (13), Class (13)

Each suit contains 1 ace, 1 king, 1 Queen, 1 jack and nine number cards 2, 3, 4, 5, 6, 7, 8, 9, 10

Face card 12
4 king, 4 Queen & 4 Jack

Non face card 40
36 number cards + 4 aces

VERY SHORT ANSWER TYPE QUESTIONS

1. Fill in the Blanks

(a) The probability of an event is greater than or equal to .... and is less than or equal to .... **[NCERT]**

(b) The probability of an impossible event is ....

(c) The probability of an event that is certain to happen is .... and such an event is called .... **[NCERT]**

(d) The sum of probabilities of all the elementary events of an experiment is .... **[NCERT]**

(e) Probability of an event E + probability of the event not E is equal to .... **[NCERT]**

(f) If probability of winning a game is $\frac{4}{9}$, then the probability of its losing is ....

(g) If coin is tossed twice, then the number of possible outcomes is ....

(h) If a die is thrown twice, then the number of possible outcomes is ....

2. State True/False

(a) The probability of an event can be negative.

(b) The probability of an event is greater than 1.

3. Multiple Choice Questions

(i) Which of the following cannot be the probability of an event? **[NCERT]**

(A) 0.7 (B) $\frac{2}{3}$ (C) -1.5 (D) 15%

(ii) Which of the following can be the probability of an event?

[NCERT Exemplar]

(A) -0.04 (B) 1.004 (C) $\frac{18}{23}$ (D) $\frac{8}{7}$

(iii) An event is very unlikely to happen, its probability is closest to

[NCERT Exemplar]

(A) 0.0001 (B) 0.001 (C) 0.01 (D) 0.1

(iv) Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is:

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{4}{9}$ (D) $\frac{2}{5}$

(v) When a die is thrown, the probability of getting an odd number less than 3 is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 0

(vi) Rashmi has a die whose six faces show the letters as given below:

A	B	C	D	A	C

If she throws the die once, then the probability of getting C is:

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$

(vii) A card is drawn from a well shuffled pack of 52 playing cards. The event E is that the card drawn is not a face card. The number of outcomes favourable to the event E is:

(A) 51 (B) 40 (C) 36 (D) 12

4. Choose the correct answer from the given four options

(i) If the probability of an even is 'p' then probability of its complementary event will be:

(A) p – 1 (B) p (C) 1 – p (D) $1-\frac{1}{p}$

(ii) P(Winning) = $x/12$, P(Losing) = 1/3. Find x **[CBSE 2014]**

(A) 6 (B) 8 (C) 7 (D) 9

(iii) The probability of a number selected at random from the numbers 1, 2, 3, ... 15 is a multiple of 4 is: **(CBSE 2020)**

(A) $\frac{4}{15}$ (B) $\frac{2}{15}$ (C) $\frac{1}{15}$ (D) $\frac{1}{5}$

(iv) The probability that a non-leap year selected at random will contains 53 Mondays is:

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

(v) A bag contains 6 red and 5 blue balls. One ball is drawn at random. The probability that the ball is blue is:

(A) $\frac{2}{11}$ (B) $\frac{5}{6}$ (C) $\frac{5}{11}$ (D) $\frac{6}{11}$

(vi) One alphabet is chosen from the word MATHEMATICS. The probability of getting a vowel is:

(A) $\frac{6}{11}$ (B) $\frac{5}{11}$ (C) $\frac{3}{11}$ (D) $\frac{4}{11}$

(vii) Two coins are tossed simultaneously. The probability of getting at most one head is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

5. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king.

6. Out of 250 bulbs in a box, 35 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.

7. Non Occurance of any event is 3:4. What is the probability of Occurance of this event?

8. If 29 is removed from (1, 4, 9, 16, 25, 29), then find the probability of getting a prime number.

9. A card is drawn at random from a deck of playing cards. Find the probability of getting a face card.

10. In 1000 lottery tickets, there are 5 prize winning tickets. Find the probability of winning a prize if a person buys one ticket.

11. One card is drawn at random from a pack of cards. Find the probability that it is a black king. **(CBSE 2020)**

12. A die is thrown once. Find the probability of getting a perfect square.

13. Two dice are rolled simultaneously. Find the probability that the sum of the two numbers appearing on the top is more than and equal to 10.

14. Find the probability of multiples of 7 in 1, 2, 3, ...,33, 34, 35.

15. If a pair of dice is thrown once, then what is the probability of getting a sum of 8? **(CBSE 2020)**

16. A letter of English alphabet is chosen at random. Determine the probability that chosen letter is a consonant. **(CBSE 2020)**

17. If the probability of winning a game is 0.07, what is the probability of losing it? **(CBSE 2020)**

SHORT ANSWER TYPE QUESTIONS-I

18. Two unbiased coins are tossed simultaneously. If the probability of getting no head is $\frac{a}{b}$ then find $(a + b)^2$? **[CBSE 2016]**

19. Two different dice are rolled together. Find the probability

(a) of getting a doublet,

(b) of getting a sum of 10, of the numbers on the two dice. **[CBSE 2018]**

20. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the box. **[CBSE 2018]**

21. An integer is chosen random between 1 and 100. Find the probability that (i) it is divisible by 8, (ii) Not divisible by 8. **[CBSE 2018]**

22. Three different coins are tossed together. Find the probability of getting (i) exactly two heads, (ii) at least two heads.

23. Card from 11 to 30, are put in a box and mixed thoroughly. A card is then drawn from the box at random. Find the probability that the number on the drawn card is a prime number.

24. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag. **(CBSE 2020)**

25. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5. **(CBSE 2020)**

26. Find the probability that 5 sundays occurs in the month of November of a randomly selected year. **(CBSE 2020)**

27. In a family of three children. Find the probability of having at least two boys. **(CBSE 2020)**

28. Two dice are thrown at the same time. Find the probability of getting different numbers on the two dice. **(CBSE 2020)**

29. If a number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$? **(CBSE 2020)**

SHORT ANSWER TYPE QUESTIONS-II

30. A number x is selected at random from the numbers 1, 2, 3. Another number y is selected at random from the numbers 1, 4, 9. Find the probability that the product of x and y is less than 9.

31. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

32. An integer is chosen between 0 and 100. What is the probability that it is

    (i) divisible by 7?

    (ii) not divisible by 7?

33. Two dice are rolled once. Find the probability of getting such numbers on the two dice,

    (a) whose product is 12.

    (b) Sum of numbers on the two dice is atmost 5.

34. Card with number 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has (i) an even number (ii) a square number.

35. In a lottery, there are 10 prizes and 25 are empty. Find the probability of getting a prize. Also verify $P(E) + P(\overline{E}) = 1$ for this event. **[CBSE 2020]**

36. $P(\text{winning}) = \frac{x}{12}$, $P(\text{Losing}) = \frac{1}{3}$. Find x.

LONG ANSWER TYPE QUESTIONS

37. Cards marked with numbers 3, 4, 5, ...,50 are placed in a box and mixed thoroughly. One card is drawn at random from the box, find the probability that the number on the drawn card is

    (i) divisible by 7 (ii) a two digit number.

38. A bag contains 5 white balls, 7 red balls, 4 black balls and 2 blue balls. One ball is drawn at random from the bag. Find the probability that the balls drawn is

(i) White or blue (ii) red or black

(iii) not white (iv) neither white nor black

39. The king, queen and jack of diamonds are removed from a pack of 52 playing cards and the pack is well shuffled. A card is drawn from the remaining cards.

Find the probability of getting a card of

(i) diamond (ii) a jack

40. The probability of a defective egg in a lot of 400 eggs is 0.035. Calculate the number of defective eggs in the lot. Also calculate the probability of taking out a non defective egg from the lot.

41. Slips marked with numbers 3,3,5,7,7,7,9,9,9,11 are placed in a box at a game stall in a fair. A person wins if the mean of numbers are written on the slip. What is the probabilty of his losing the game?

42. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two digit number (ii) a perfect square number

(iii) a number divisible by 5.

43. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a card of spade or an ace (ii) a red king

(iii) neither a king nor a queen (iv) either a king or a queen

44. A card is drawn from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a face card (ii) red colour face card

(iii) black colour face card

45. Ramesh got ₹ 24000 as Bonus. He donated ₹ 5000 to temple. He gave ₹ 12000 to his wife, ₹ 2000 to his servant and gave rest of the amount to his daughter. Calculate the probability of

(i) wife's share (ii) Servant's Share

(iii) daughter's share.

46. 240 students reside in a hostel. Out of which 50% go for the yoga classes early in the morning, 25% go for the Gym club and 15% of them go for the morning walk. Rest of the students have joined the laughing club. What is the probability of students who have joined laughing club?

47. A box contains cards numbered from 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is: **[CBSE 2018]**

(i) A square number (ii) a multiple of 7.

48. A die is thrown twice. Find the probability that:

(i) 5 will come up at least once

(ii) 5 will not come up either time **[CBSE 2019]**

49. Cards marked 1, 3, 5 .... 49 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is : **[CBSE 2017]**

(i) divisible by 3 (ii) a composite number

(iii) not a perfect square (iv) multiple of 3 and 5

50. 50 A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a **[CBSE 2015]**

(i) triangle (ii) square

(iii) square of blue colour (iv) triangle of red colour

51. A box contain 24 balls of which x are red, 2x are white and 3x are blue. A ball is selected at random. What is the probability that it is

(i) not red?

(ii) White?

ANSWERS AND HINTS

1. (a) 0 and 1 (b) 0 (c) 1 and sure event (d) 1

(e) 1 (f) $\frac{5}{9}$ (g) 4 (h) 36

2. (a) False, because $0 \leq P(A) \leq 1$

(b) False, because $0 \leq P(A) \leq 1$

3. (i) (C) (ii) (C)

(iii) (A) (as unlikely to happen) (iv) (B) (prime no. 2, 3, 5, 7)

(v) (A) (vi) (A)

(vii) (B) (Face card = 12, Remaining cards = 40)

4. (i) (C)

(ii) (B) $x = 8$

(iii) (D) (Probability $\frac{1}{15}$)

(iv) (A) (Total weeks 52, Remaining day 1, sample space = {S, M, Tu, W, Th, F, Sat})

(v) (C)

(vi) (D) (vowels A, A, E, I)

(vii) (D)

5. Total = 52

No. of Aces = 4

No. of kings = 4

P (neither ace nor king) = $\frac{44}{52} = \frac{11}{13}$

6. P(not defective) = $1 - \frac{35}{250} = \frac{43}{50}$

7. Total case = 3 + 4 = 7

P(occurrence) = $\frac{4}{7}$

8. P(prime no.) = 0

9. No. of face card = 12

P(face card) = $\frac{12}{52} = \frac{3}{13}$

10. Probability of winning = $\frac{5}{1000} = 0.005$

11. Total black king = 2

P(Black King) = $\frac{2}{52} = \frac{1}{26}$

12. Sample space : 1, 2, 3, 4, 5, 6

Perfect square : 1, 4

P(perfect square) = $\frac{2}{6} = \frac{1}{3}$

13. Total cases = 36

Favourable cases (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)

P(sum of two numbers is $\geq$ 10) = $\frac{6}{36} = \frac{1}{6}$

14. Multiples of 7 are 7, 14, 21, 28, 35

Probability (multiple of 7) = $\frac{5}{35} = \frac{1}{7}$

15. P(sum of 8) = $\frac{5}{36}$

16. P (consonant) = $\frac{21}{26}$

17. P (losing) = 1 – 0.07 = 0.93

18. $(a + b)^2 = 25$

19. (i) Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Required probability = $\frac{6}{36} = \frac{1}{6}$

(ii) Sum 10 cases : (4, 6), (5, 5), (6, 4)

Required probability = $\frac{3}{36} = \frac{1}{12}$

20. $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$

21. Total outcomes between 1 and 100 = 98

(i) Nos. divisible by 8 : 8, 16, 24, ..., 96

favourable cases = 12

Required probability = $\frac{12}{98} = \frac{6}{49}$

(ii) Probability (integer is not divisible by 8) = $1 - \frac{6}{49} = \frac{43}{49}$

22. Sample space : HHH, TTT, HTT, THT, TTH, THH, HTH, HHT

(i) P(exactly 2 heads) = $\frac{3}{8}$

(ii) P(atleast 2 heads) = $\frac{4}{8} = \frac{1}{2}$

23. Total cards = 20

Prime Nos. are 11, 13, 17, 19, 23, 29

Required probability = $\frac{6}{20} = \frac{3}{10}$

24. Let the number of blue balls = x

Total balls = $(5 + x)$

P (Blue ball) = 3 × P (Red ball)

$$\frac{x}{5+x} = 3\times\left(\frac{5}{5+x}\right)$$

$\Rightarrow x = 15$

25. Favourable outcomes : (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)

P (sum less than 5) = $\frac{6}{36} = \frac{1}{6}$

26. Number of total days in the month of November = 30

i.e. 4 complete weeks and 2 days.

$\therefore$ P (5 Sundays) = $\frac{2}{7}$

27. P (atleast two boys) = $\frac{4}{8} = \frac{1}{2}$

28. P (Different numbers) = $\frac{30}{36} = \frac{5}{6}$

29. Favourable outcomes : –2, –1, 0, 1, 2

$P(x^2 \leq 4) = \frac{5}{7}$

30. Sample space = (1, 1), (1, 4), (1, 9)
(2, 1), (2, 4), (2, 9)
(3, 1), (3, 4), (3, 9)
(1, 1), (1, 4), (2, 1), (2, 4), (3, 1)

Favourable cases : $xy < 9$

Required probability = $\frac{5}{9}$

31. Total outcomes = 36

(a) Favourable outcomes = (1, 3), (2, 4), (3, 5), (4, 2), (4, 6), (5, 3), (6, 4), (3, 1)

Required probability $\frac{8}{36} = \frac{2}{9}$

(b) Favourable outcomes
(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)

Required probability = $\frac{6}{36} = \frac{1}{6}$

32. Total number of integers = 101

Favourable out conmes = 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98

Required probability = $\frac{14}{101}$

33. (a) $S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \end{Bmatrix}$

Favourable outcomes: (2, 6), (3, 4), (4, 3), (6, 2)

Required probability = $\frac{4}{36} = \frac{1}{9}$

(b) Favourable outcomes (sum $\leq$ 5)
= (1, 1), (1, 2), (1, 3) (1, 4) (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (4, 1)

Required probability = $\frac{10}{36} = \frac{5}{18}$

34. (i) Total cards = 101 – 2 + 1 = 100, Even numbers = 2, 4, ..., 100 = 50

Required probability = $\frac{50}{100} = \frac{1}{2}$

(ii) Square number = 4, 9, 16, 25, 36 ,49, 64, 81, 100

Required probability = $\frac{9}{100} = 0.09$

35. Total tickets = 35

P(E) = P(getting a prize) = $\frac{10}{35} = \frac{2}{7}$

$P(\bar{E})$ = P(not getting a prize) = $\frac{25}{35} = \frac{5}{7}$

$P(E) + P(\bar{E}) = \frac{2}{7} + \frac{5}{7} = \frac{7}{7} = 1$

36. P(winning) + P(losing) = 1

$\frac{x}{12} + \frac{1}{3} = 1 \Rightarrow x = 8$

37. Total cards = 50 – 3 + 1 = 48

(i) No. divisible by 7 are 7, 14, 21, 28, 35, 42, 49

Required probability = $\frac{7}{48}$

(ii) Two digit no. are 10, 11, 12, ... 50

No. of favourable outcomes = 50 – 10 + 1 = 41

Required probability = $\frac{41}{48}$

38. (i) $\frac{5+2}{18} = \frac{7}{18}$ (ii) $\frac{7+4}{18} = \frac{11}{18}$

(iii) $\frac{7+4+2}{18} = \frac{13}{18}$ (iv) $\frac{7+2}{18} = \frac{9}{18} = \frac{1}{2}$

39. (i) Remaining cards = 52 – 3 = 49

Remaining diamonds = 13 – 3 = 10

Required probability = $\frac{10}{49}$

(ii) P(jack) = $\frac{3}{49}$ (as 1 jack has been removed)

40. Total eggs = 400

P(defective eggs) = 0.035

Let defective eggs = x

$$\frac{x}{400} = 0.035$$

$$x = 400 \times 0.035$$

$$x = 14$$

P(non defective eggs) = 1 – 0.035 = 0.965

41. Mean = $\frac{3+3+5+7+7+7+9+9+9+11}{10} = \frac{70}{10} = 7$

$$\text{P(he loses)} = 1 - \frac{3}{10} = \frac{7}{10}$$

42. Total no. = 90

(i) Two digit no.s 10, 11, 12, ..., 90

No. of favourable cases = 90 – 10 + 1 = 81

Required probability = $\frac{81}{90} = \frac{9}{10}$

(ii) Perfect square no. : 1, 4, 9, 16, 25, 36, 49, 64, 81

Required probability = $\frac{9}{90} = \frac{1}{10}$

(iv) No.s divisible by 5 :

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90

Required probability = $\frac{18}{90} = \frac{1}{5}$

43. (i) P(a card of spade or an ace) = $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

(ii) P(red king) = $\frac{2}{52} = \frac{1}{26}$

(iii) P(neither a king nor a queen) = $1 - \frac{8}{52} = 1 - \frac{2}{13} = \frac{11}{13}$

(iv) P(either a king or a queen) = $\frac{8}{52} = \frac{2}{13}$

44. (i) $\frac{12}{52} = \frac{3}{13}$ (ii) $\frac{6}{52} = \frac{3}{26}$ (iii) $\frac{6}{52} = \frac{3}{26}$

45. (i) P(wife's share) = $\frac{12000}{24000} = \frac{1}{2}$

(ii) P(servant's share) = $\frac{2000}{24000} = \frac{1}{12}$

(iii)P(Daughter's share) = $\frac{5000}{24000} = \frac{5}{24}$

46. 10% students joined laughing club

P(students who have joined laughing clubs) = $\frac{10}{100} = \frac{1}{10}$

47. Total cards = 123 – 11 + 1 = 113

(i) Square numbers : 16, 25, 36, 49, 64, 81, 100, 121

Required probability = $\frac{8}{113}$

(ii) Multiple of 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119.

Required Probality = $\frac{16}{113}$

48. Total outcomes = 36

(i) P(5 will come up at least once) = $\frac{11}{36}$

Favourable cases (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)

(ii) P(5 will not come up either time) = $1 - \frac{11}{36} = \frac{25}{36}$

49. S = 1, 3, 5, ..., 49. Total outcome = 25

(i) No. divisible by 3 are 3, 9, 15, 21, 27, 33, 39, 45

Required probability = $\frac{8}{25}$

(ii) Composite Nos : 9, 15, 21, 25, 27, 33, 35, 39, 45, 49

Required probability = $\frac{10}{25} = \frac{2}{5}$

(iii) P(not a perfect square) = 1 – P(perfect square) {Perfect square no. : 1, 9, 25, 49}

$$= 1 - \frac{4}{25} = \frac{21}{25}$$

(iv) Multiple of 3 and 5

$\Rightarrow$ Multiple of 15 = 15, 45

Required probability = $\frac{2}{25}$

50. (i) $\frac{8}{18} = \frac{4}{9}$ (ii) $\frac{10}{18} = \frac{5}{9}$

(iii) $\frac{6}{18} = \frac{1}{3}$ (iv) $\frac{5}{18}$

51. (i) P(not red) = $\frac{20}{24} = \frac{5}{6}$

(ii) P(white) = $\frac{8}{24} = \frac{1}{3}$

❒❒❒

PRACTICE-TEST

Probabiltiy

Time : 45 Minutes *M.M. : 20*

SECTION-A

1. When a die is thrown once, the probability of getting an odd number less than 3 is: **1**

(a) 1/6 (b) 1/3

(c) 1/2 (d) 0

2. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting neither green ball nor red ball. **1**

3. One card is drawn at random from the well shuffled pack of 52 cards. Find the probability of getting a non face card. **1**

4. Cards are marked with numbers 5, 6, 7,.......50 are placed in the box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting a two digit number? **1**

SECTION-B

5. A letter is chosen at random from 26 alphabets. Find the probability that the letter chosen is from the word 'ASSASSINATION'. **2**

6. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective. **2**

7. Find the probability of getting 53 Fridays or 53 Saturdays in a leap year. **2**

SECTION - C

8. Daksh and Moksh are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? [ignoring a leap year). **3**

9. Two dice are thrown together. Find the probability that sum of two numbers will be a multiple of 4. **3**

SECTION - D

10. Five cards—the ten, jack, queen, king and ace of diamonds, are removed from the well-shuffled 52 playing cards. One card is then picked up at random. Find the probability of getting:

(a) neither a heart nor a king

(b) either a heart or a spade card

(c) neither a red card nor a queen card

(d) a black card or an ace. **4**

CASE STUDY BASED QUESTIONS
REAL NUMBERS

1. During a health check-up camp three types of patients registered themselves. 60 were suffering from joint problem, 84 were suffering from some type of fever and 108 were diabetic. The organisers want to call doctors for this camp.

(i) What is the maximum number of doctors required if each doctor treats same number of patients of each type of problem?

(a) 64 (b) 14

(c) 16 (d) 12

(ii) How many patients each doctor will treat?

(a) 7 (b) 12

(c) 21 (d) 9

(iii) At the end of the day when the total count was done the number of patients with joint problems were 48, suffering from fever were 60 and diabetic patients were 72 only. How many patients each doctor treated?

(iv) If HCF (48,60, 72) = 7m – 2, what is the value of m?

REAL NUMBERS

2. Deepika wants to organize her birthday party. She was happy on her birthday. She is very health conscious, thus she decided to serve fruits only. She has 36 apples and 60 bananas at home and decided to serve them. She wants to distribute fruits among guests. She does not want to discriminate among guests so she decided to distribute equally among all.

(i) How many maximum guests Deepika can invite?

(a) 6 (b) 12

(c) 18 (d) 24

(ii) How many apples and bananas will each guest get?

(a) 3 apples and 5 bananas (b) 5 apples and 3 bananas

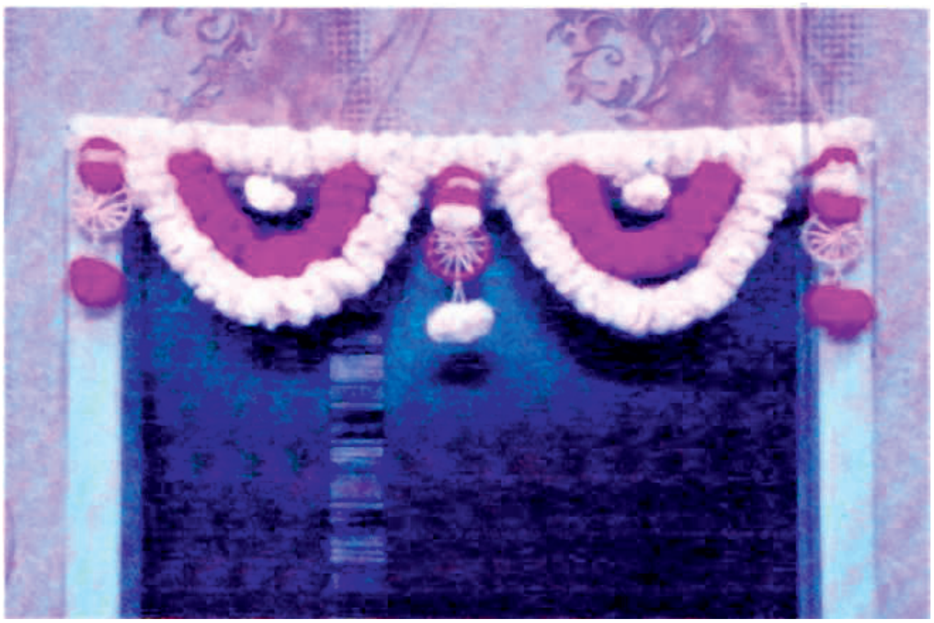
(c) 2 apples and 4 bananas (d) 4 apples and 2 bananas

(iii) Deepika decides to add 42 mangoes also. In this case how many maximum guests Deepika can invite?

(iv) How many total fruits will each guest get now?

POLYNOMIALS

3. Radha decorated the door of her house with garlands on the occasion of Diwali. Each garland forms the shape of a parabola.



(i) Suppose the quadratic polynomial for the given curve is $ax^2 + bx + c$, then 'a' is always

(a) >0 (b) <0

(c) ≥ 0 (d) ≤ 0

(ii) A quadratic polynomial with the sum and product of its zeroes as -1 and -2 respectively, is:

(a) $x^2 + x + 2$ (b) $x^2 - x - 2$

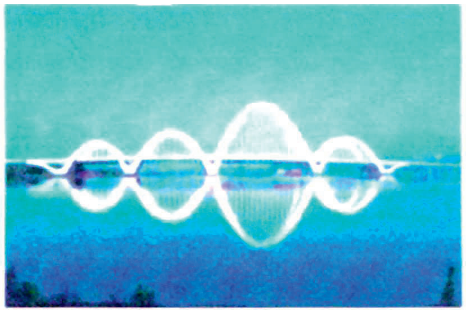
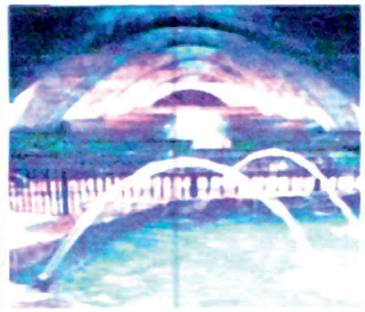
(c) $x^2 + x - 2$ (d) $x^2 - x + 2$

(iii) For what value of 'k', -1 is one of the zeroes of the quadratic polynomial $(k - 2)x^2 - 2x - 5$.

(iv) If α, β are the zeroes of the polynomial $f(x) = x^2 - 7x + 12$, then find the value of: $1/\alpha + 1/\beta$.

POLYNOMIALS

4. The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.

(i) In the standard form of quadratic polynomial $ax^2 + bx + c$, a, b and c are

(a) All are real numbers.

(b) All are rational numbers.

(c) 'a' is a non-zero real number and b and c are any real numbers.

(d) All are integers

(ii) The quadratic polynomial whose zeroes are - 4 and - 5 is

(a) $x^2 - 9x - 20$ (c) $x^2 - 9x - 20$

(b) $x^2 + 9x - 20$ (d) $x^2 + 9x + 20$

(iii) If α and $1/\alpha$ are the zeroes of the quadratic polynomial $2x^2 - 8x + k$, then find 'k'.

(iv) Form a quadratic polynomial whose sum of zeroes is '$-p$' and product of zeroes is '$-1/p$'.

5. The consumption of water in a family is divided into two parts; one is fixed consumption used for various activities like cleaning, washing etc. and the other is individual consumption. For a family of 6 persons the water required is 445 cubic units and for a family of 8 members the total requirement of the water is 575 cubic units. Form a pair of linear equations for the above situation.

(i) What is the individual and fixed consumption?

(a) 55, 65 (b) 65, 55

(c) 55, 55 (d) 65, 65

(ii) If we draw the graph of the pair of equations then what is the distance of the point of intersection from y-axis?

(a) 55 (b) 65

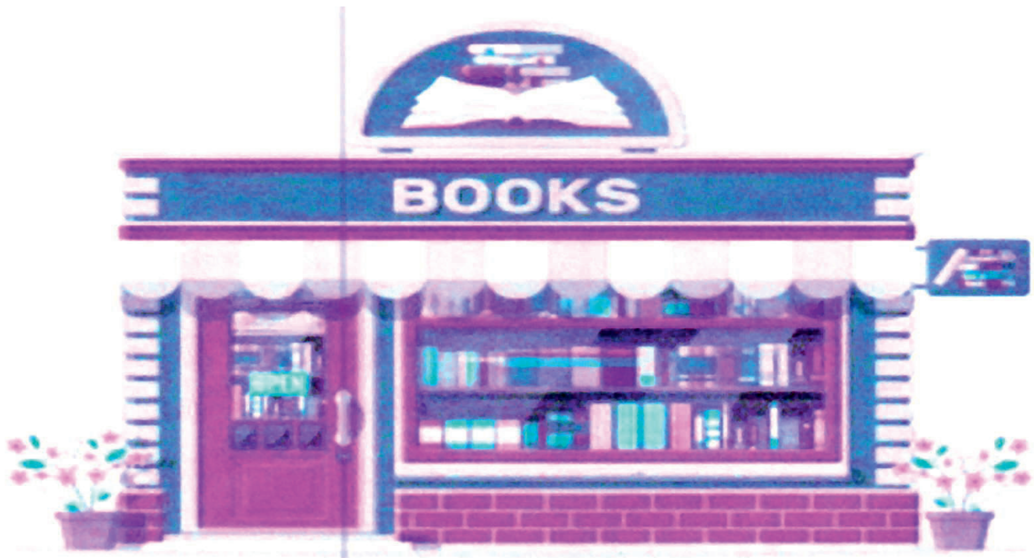
(d) 45 (d) 50

(iii) Form a linear equation for the total consumption of a family of four members.

(iv) How much water is required for a family of 5 members?

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

6. A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, story books, quiz books etc. He takes a fixed charge for the first two days and an additional charge for each day thereafter. Ayush paid ₹ 22 for a book and kept for six days, while Sonali paid ₹ 16 when she kept a book for four days. Let the fixed charges be represented by ₹ x and charges for each additional day be represented by ₹ y.



(i) The situation of amount paid by Sonali is algebraically represented as:

(a) $x - 4y = 16$ (b) $x + 4y = 16$

(c) $x - 2y = 16$ (d) $x + 2y = 16$

(ii) The situation of amount paid by Ayush is algebraically represented as:

(a) $x - 2y = 11$ (b) $x - 2y = 22$

(c) $x + 4y = 22$ (d) $x - 4y = 22$

(iii) What are the fixed charges and additional charges for each day?

(iv) What is the total amount paid by both Ayush and Sonali, if both of them kept the books for two more extra days?

7. Nikhil and Niharika are very close friends. Both the famihes decide to go for a picnic to Palampur in their own cars, Niharika's car travels 5 km/h more than Nikhil's car. Nikhil's car took 4 hours more than Niharika's car in covering 400 km. Assume that Nikhil's carwas travelling at a speed of 'y' km/h.

(i) What will be the distance covered by Niharika's car in two hours?

(a) $2(y+5)$ km (b) $(y-5)$ km

(c) $2(y+10)$ km (d) $(2y+5)$ km

(ii) Which of the following quadratic equations describes the speed of Nikhil's car?

(a) $y^2 - 5y - 500 = 0$ (b) $y^2 + 4y - 400 = 0$

(c) $y^2 + 5y - 500 = 0$ (d) $y^2 - 4y + 400 = 0$

(iii) What is the speed of Nikhil's car?

(iv) How much time it took for Niharika's family to complete the journey?

8. A farmer wants to make a rectangular pen for his sheep in the garden near his house. To make the pen the farmer planned to make it with wooden fencing to cover the three sides. He has 60m fencing material to cover three sides and the other side being a brick wall.

(i) If the width be x, then the length of the pen

(a) $60 - 2x$ (b) $2x + 6$

(c) $6x + 20$ (d) $20 - 6x$

(ii) According to the given conditions area of the pen using length as calculated in (i) is

(a) $60x^2 - 2x$ (b) $60x + 2x^2$

(c) $6x - 20x^2$ (d) $60x - 2x^2$

(iii) Form a quadratic equation if the area of the pen is $250m^2$.

(iv) What could be the possible width if area of the pen is $400m^2$?

9. With the increasing demand and supply pressure worldwide, India has emerged as a competitive manufacturing location due to the low cost of manpower and strong engineering capabilities. The production in a factory increased uniformly by a fixed number every year. If the production in the factory was 4100 units in the fifth year which was then increased to 7600 units in the 10th year. With the given data answer the following questions:

(i) Find the production during 1st year.

(a) 500 units

(b) 400 units

(c) 1300 units

(d) 700 units

(ii) Find the difference in production during 9th year and 7th year.

(a) 700 units

(b) 1400 units

(c) 350 units

(d) 2100 units

(iii) Find the general term representing the number of units produced during a particular year.

(iv) Calculate the total number of units produced from 4th year to 10th year.

10. As we know a tree or a plant needs both soil and water along with sunlight to grow. It will have the necessary nourishment from both water and Sun to make its leaves green and fruit to grow. A group of people planted 20 trees at equal distances of 10 m in a line with a water tank placed at a distance of 15m from the tree at one end. Everyday a member of the group waters all the trees separately starting from the water tank and returns to the tank after watering each tree to get water for the next tree from the tank.

(i) Distance travelled by the member to water nearest tree and back to the tank is;

(a) 15m

(b) 30m

(c) 7.5m

(d) 40m

(ii) Progression so formed in the above condition is :

(a) 15. 25, 35, 45 ....

(b) 30, 40, 50, 60 ....

(c) 30, 50, 70. 90 ....

(d) 15, 35, 55, 75 ....

(iii) Calculate the distance travelled by the member to water the last tree.

(iv) Calculate the total distance travelled by the member in a day in order to water all the trees.

TRIANGLES

11. Burj Khalifa is the tallest tower in the world which is located in Dubai, United Arab Emirates. The height of Burj Khalifa is about 828 m. It has the highest observation deck open to the public in the world. A person walking on the deck observed the shadows of Burj Khalifa and the buildings in the proximity. At an instance, he found the length of shadow of Burj Khalifa was 207 m and the length of shadow of a building A was 46 m. He thought whether the height of the building can be calculated with the given data. If you think it can be calculated, then answer the following questions;

(i) Name the property which can be used to find out the length of the building A.

(ii) At the same instance when the length of the shadow of Burj Khalifa was 207 m, what will be the length of the shadow of building B of height 108 m?

a) 108m b) 54m

c) 216m d) 27m

(iii) Caiculate the height of building A.

(iv) What is the length of shadow of Burj Khalifa when the length of shadow of building B is 81 metres?

12. Waiking regularly is a good habit to keep us healthy and stress free.After dinner, some people were walking in the society park. A person noticed the dynamic shadows of walking people formed due to light from the lamp posts and started observing them. He observed that as people were moving away from the lamp post, the length of the shadow gradually increases. In the same group there was Neha of height 180 cm, who was talking to Yamini and moving away from a 5.4m high lamp post at a speed of 0.6 m per second.

(i) How far Neha was from the lamppost after 4 seconds?

(a) 240 cm

(b) 2.4 cm

(c) 120 cm

(d) 60 cm

(ii) What would be the length of Neha's shadow after 3 seconds?

(a) 0.6 m

(b) 0.9 m

(c) 1.08 m

(d) 1.8 m

(iii) After how much time the length of Neha shadow will be 1.8 m.

(iv) At an instance the shadow of Yamini was 1.5 times his height. How far were they from the lamp post?

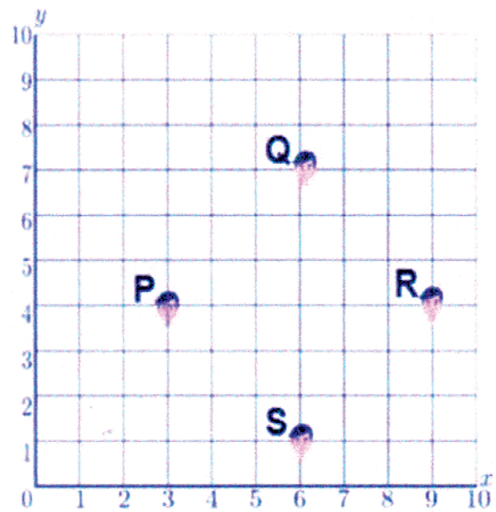
CO-ORDINATE GEOMETRY

13. Biria Science Museum is the first Science and Technology Museum of the country, established in 1954. It houses exhibits and displays on science and technology where visitors can interact with the exhibits to make the understanding of science and technology easy and entertaining.

BirIa Science Museum has set aside a children's room having planets and stars painted on the ceiling. Suppose an imaginary coordinate system is placed on the ceiling in the room with the centre of the ceiling at (0, 0). Three particular stars are located space (–8, 3), T(5, –10) and R (–5, – 7), where the coordinates represent the distance in metre from the centre of the room.

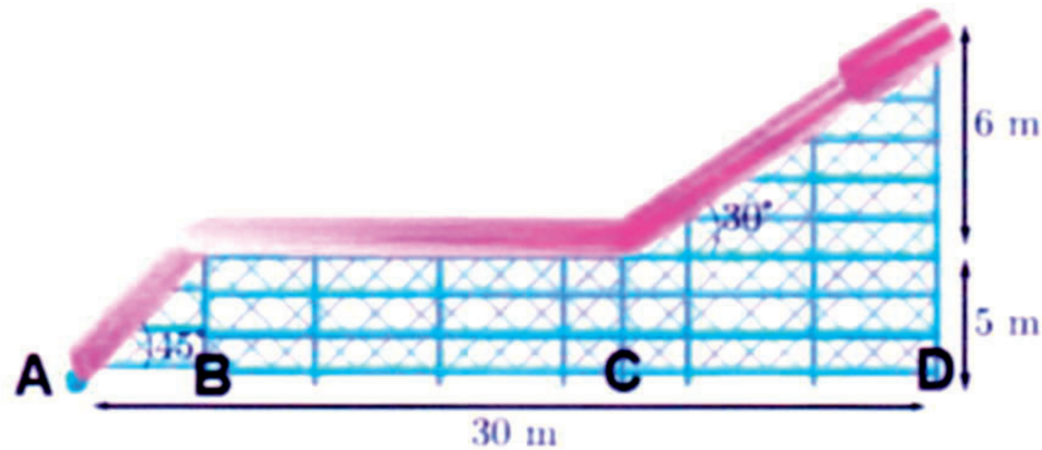
(i) What is the distance between S (–8, 3) and T(5, –10)?	(ii) What is the distance between S (–8, 3) and R(–5, – 7)?
(a) $4\sqrt{29}$ m	(a) $4\sqrt{15}$ m
(b) $2\sqrt{29}$ m	(b) $\sqrt{109}$ m
(c) $13\sqrt{2}$ m	(c) $8\sqrt{15}$ m
(d) $16\sqrt{3}$ m	(d) $16\sqrt{3}$ m
(iii) Which star is farthest from the centre of the room?	(iv) What is the distance between R (–5, –7) and 7(5, –10)?

14. Morning assembly is an integral part of the school's schedule. Almost all the schools conduct morning assemblies which include prayers, information of latest happenings, inspiring thoughts, speech, national anthem, etc. A good school is always particular about their morning assembly schedule. Morning assembly is important for a child's development. It is essential to understand that morning assembly is not just about standing in long queues and singing prayers or national anthem, but it's something beyond just prayers. All the activities carried out in morning assembly by the school staff and students have a great influence in every point of life. The positive effects of attending school assemblies can be felt throughout life.



(i) What is the distance between P and Q?	(ii) What is the distance between S and Q?
(a) 8 units	(a) 8 units
(b) 6 units	(b) 6 units
(c) $3\sqrt{3}$ units	(c) $3\sqrt{3}$ units
(d) $2\sqrt{3}$ units	(d) $2\sqrt{3}$ units
(iii) What is the distance between P and R?	(iv) What is the distance between R and S?

15. Water Slide Design: Slide shown in the figure is part of a design for a water slide.

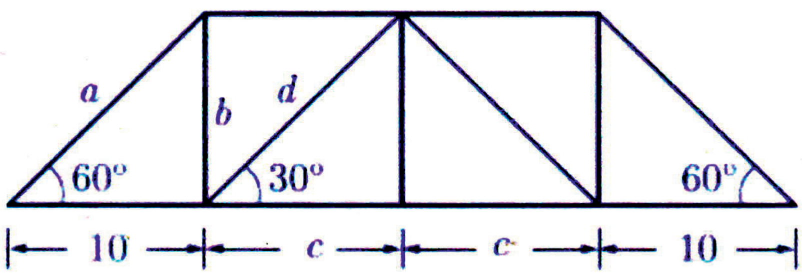


(i) What is the length of flat part of slide?	(ii) What is the total length of the slide?
(a) 44.69 m	(a) 5.4 m
(b) 22.16 m	(b) 21.6 m
(c) 16.34 m	(c) 33.7 m
(d) 34.18 m	(d) 42.2 m
(iii) Find the total slant height of the slide.	(iv) Find the distance of CD.

TRIGONOMETRY

16. A truss is a structure that consists of members organised into connected triangles so that the overall assembly behaves as a single object. Trusses are most commonly used in bridges, roofs and towers.

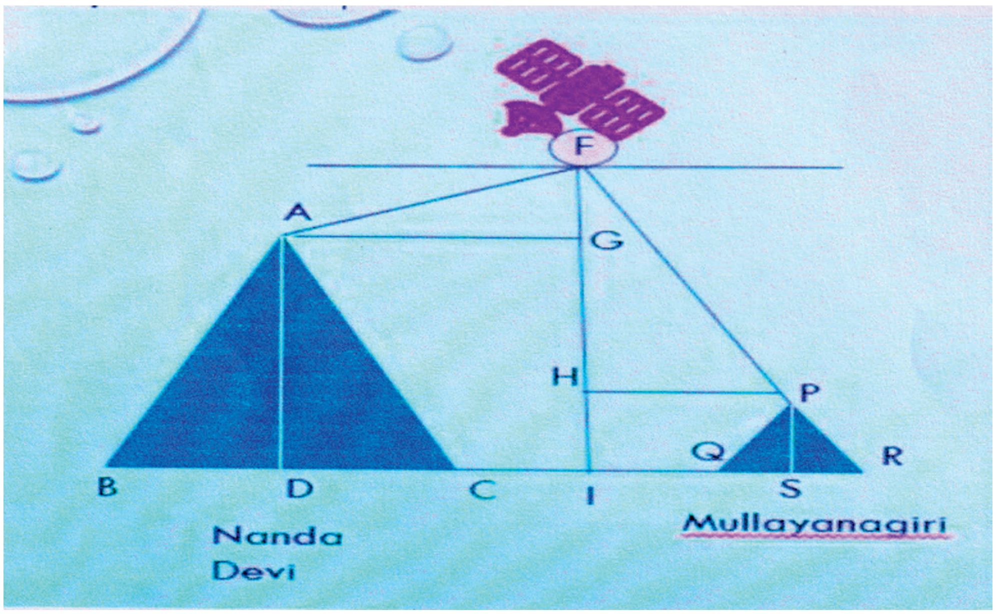
Consider the line diagram of truss shown below and find the following length:



(i) What is the length a?	(ii) What is the length b?
(a) 30 m	(a) 30 m
(b) 20 m	(b) 20 m
(c) 34.6 m	(c) 34.6 m
(d) 17.32 m	(d) 17.32 m
(iii) Find the length c.	(iv) Find the length d.

HEIGHT AND DISTANCE

17. A Satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Kamataka, them being Nanda Devi (height 7, 816m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the distance between two mountains is 1937 km and the satellite is vertically above the midpoint of the distance between the two mountains.



(i) The distance of the satellite from the top of Nanda Devi is	(ii) The distance of the satellite from the top of Millayanagiri is
(a) 1139.4 km	(a) 1139.4 km
(b) 577.52 km	(b) 577.52 km
(c) 1937 km	(c) 1937 km
(d) 1025.36 km	(d) 1025.36 km
(iii) Find the height of the satellite from the ground.	(iv) Find the angle of elevation of the Nanda Devi, if Rahul is standing at a distance of 7816 m from the base of Nanda Devi.

CASE STUDY- HEIGHT AND DISTANCE

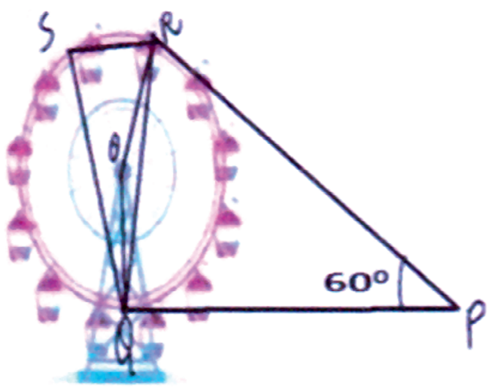
18. Statue of Unity. It is a colossal statue of Indian statesman an independent activist Sardar Vallabh Bhai Patel, who was the first Deputy Prime Mnister and first Home Minister of Independent India. Patel was highly respected for a leadership in uniting the 562 princely states of India form a single Union of India. It is located in the state of Gujarat and it is the world tallest statue.

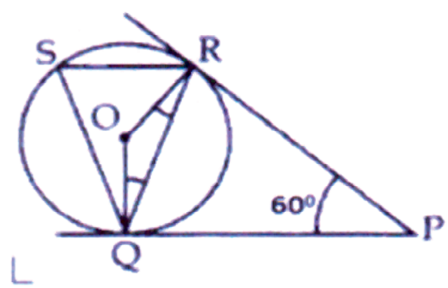
(i) For a person, standing 120 m from the centre of the base of the statue. The angle of elevation from the base of statue is 45°. Find the height of the statue.	(ii) For a person, standing x m from the centre of the base of the statue. The angle of elevation from the base of statue is 30°. Find the value of x if the height of the statue is 182 metre.
(a) 110 m	(a) $182\sqrt{3}$ m
(b) 240 m	(b) $364\sqrt{3}$ m
(c) $120\sqrt{3}$ m	(c) $91\sqrt{3}$ m
(d) 120 m	(d) $\frac{182}{\sqrt{3}}$ m
(iii) A cop in the helicopter near the top of the statue (height of statue is 182 metre) notices a car at some distance from the statue. If the angle of the depression from the cop's eyes to the car is 60°. How far is the car from the centre of the base of the statue?	(iv) A cop in the helicopter near the top of the statue (height of statue is 182 metre) notices a car at some distance from the statue. If the angle of the depression from the cop's eyes to the car is 60°. Find the distance between car and helicopter?

TANGENTS TO CIRCLE

19. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride . She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





(i) In the given figure find $\angle ROQ$.

(a) 60°

(b) 120°

(c) 150°

(d) 90°

(ii) Find $\angle RQP$

(a) 75°

(b) 60°

(c)30°

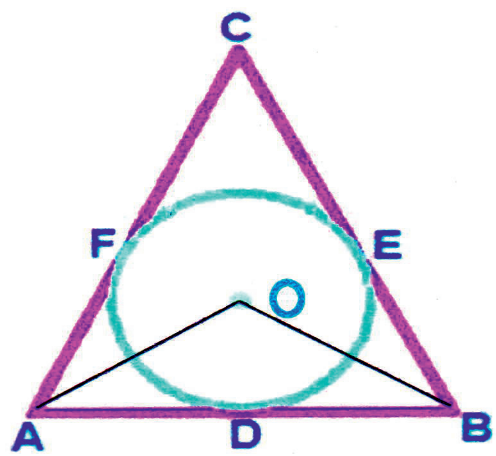
(d) 90°

(iii) Find $\angle RSQ$

(iv) Find $\angle ORP$

TANGENTS TO CIRCLE

20. Varun has been selected by his school to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB. BC and CA are 12 cm, 8 cm and 10 cm respectively.



(i) Find the length AF.

(a) 7

(b) 8

(c) 5

(d) 9

(ii) Find the Length BD.

(a) 8

(b) 5

(c) 2

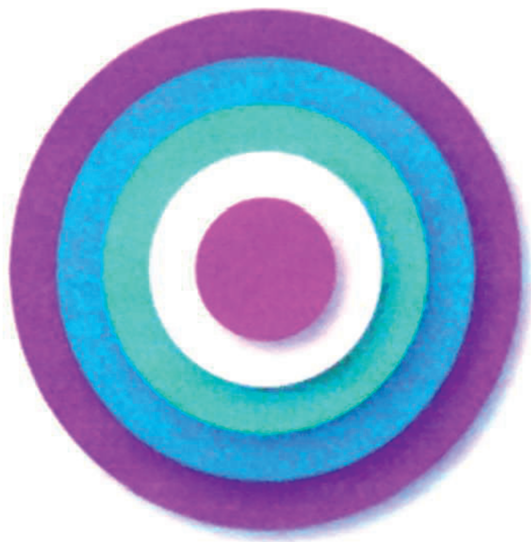
(d) 9

(iii) Find the length CE.

(iv) If radius of the circle is 4cm, Find the area of ΔOAB.

AREA RELATED TO CIRCLES

21. Honouring students is an efficient way to motivate all the students to work hard and achieve more in their life. So a function was to be organised in a school. In order to properly organise the function, the managing committee of the school decided to tag circular badges to the students to be honoured. The badges were made using laces of different colour as shown in the figure, if diameter of red portion is 5 cm and every adjacent colour has a width of 3 cm, answer the following questions.



(i) The distance of blue color from the center is;

   a) 15 cm

   b) 8.5 cm

   c) 11 cm

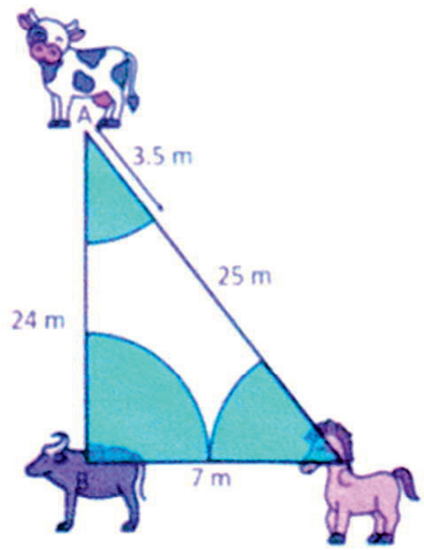
   d) 9.5 cm

(ii) Circles as shown in the figure are also known as....

(iii) Calculate the area of green colour in a badge.

(iv) If 20 such badges are to be made, how much length of yellow lace is required considering 0.12 cm of lace is wasted in making of one badge?

22. A buffalo, a cow and a horse are tied to pegs at the corners of a right triangular field of sides 24 m, 7 m and 25 m by means of a 3.5 m long rope as shown in the figure. Use TT = 22/7 to answer the following questions



(i) What is the area of right triangular grass field?
   (a) 84 sq.m
   (b) 168 sq.m
   (c) 175 sq.m
   (d) 87.5 sq.m

(ii) The combined angle made by the grazing area of horse and cow is;
   (a) 45°
   (b) 90°
   (c) 60°
   (d) Cannot be determined

(iii) The area of that part of field in which buffalo can graze.

(iv) Calculate the decrease in the grazing area, if the ropes were 3 m instead of 3.5 m.

SURFACE AREA AND VOLUMES

23. A committee has decided to celebrate Durga Puja in a circular Park of radius 35m. The committee has given the contract to a tent house to set up the tent. The architect has designed a Canvas tent in the shape of a semi cylindrical roof surmounted on an open cuboidal shape as shown in the figure. The dimensions of the rectangular base is 50 m × 21 m and the total height of the tent is 19 m.

On the basis of the above information answer the following questions:

(i) The height of the cuboidal part of the tent is :

(a) 19m (b) 8.5m

(c) 11.5m (d) 15m

(ii) Area of the park outside the tent is:

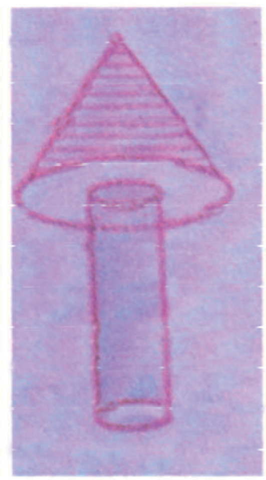
(a) 2800 sq.m (b) 3850 sq.m

(c) 1050 sq.m (d) 1570 sq.m

(iii) Find the total cost of Canvas if it is purchased at the rate of ₹ 4 per square metre.

(iv) Find the volume of air present in the tent.

24. In a toys store wooden parts are assembled and painted to prepare a toy full stop. One such specific toy is in the shape of a cone mounted on a cylinder.



For the wood processing activity centre, the wood is taken out of storage to be saved, after which it undergoes rough polishing, then it is cut, drilled and has holes punched in it. it is then fine polished using sandpaper and then decorated using paint.

The total height of the toy is 26 cm and the height of the conical part is 6 cm. The diameters of the base of the conical part is 5 cm and that of the cylindrical part is 4 cm. On the basis of the above information, answer the following questions:

(i) If the cylindrical part is to be painted yellow the surface area need to be painted is:

(a) $80\ \pi$ sq.cm (b) $82\ \pi$ sq.cm

(c) $84\ \pi$ sq.cm (d) $88\ \pi$ sq cm

(ii) The volume of the wood used in making this toy is:

(a) $92.5\ \pi$ cu.cm (b) $89.5\ \pi$ cu.cm

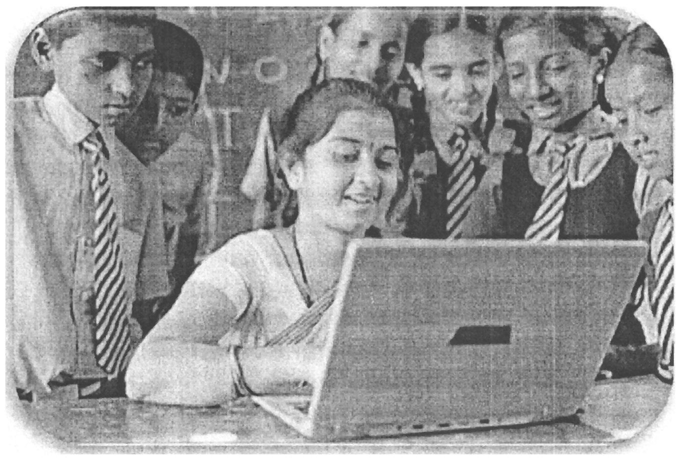
(c) $85.5\ \pi$ cu.cm (d) $72.5\ \pi$ cu.cm

(iii) find the cost of painting the toy at 3 Paisa per square cm.

(iv) Find the cost of painting 200 toys, if the paint company gives the discount of 5%.

STATISTICS

25. Student-Teacher Ratio: Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number of teachers in that school. It is important for a number of reasons. For example, it can be an indicator of the amount of individual attention any child is likely to receive, keeping in mind that not all calss size are going to be the same. The following distribution gives the state-wise student-teacher ratio in higher secondary schools of India (28 states and 7 UTs only).



Number of Students	Number of States/UTs
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

(i) In order to find the mean by direct method, we use the formula.

(a) $\frac{\Sigma_{i=1}^{n} f_i x_i}{n}$ (b) $\frac{n}{\Sigma_{i=1}^{n} f_i x_i}$

(c) $n \times \Sigma_{i=1}^{n} f_i x_i$ (d) $n + \Sigma_{i=1}^{n} f_i x_i$

(ii) The formula to find the median is

(a) $l + \frac{\frac{n}{2} - f}{c.f.} \times h$ (b) $h + \frac{\frac{n}{2} - c.f.}{f} \times l$

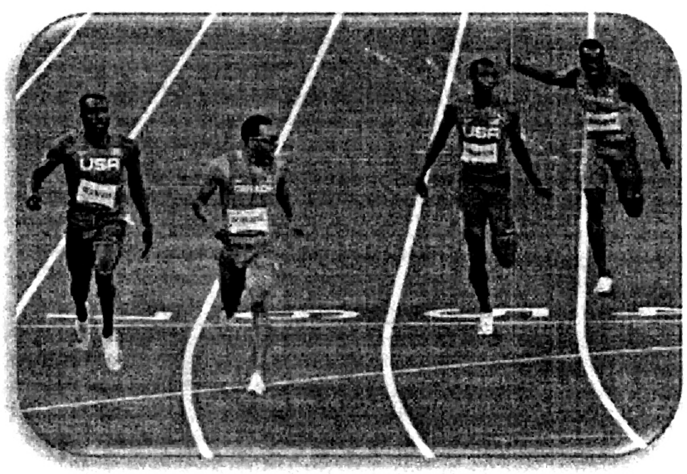
(c) $l + \frac{\frac{n}{2} - c.f.}{f} \times h$ (d) $\frac{l + \frac{n}{2} - c.f.}{f} \times h$

(iii) Find the average of class marks of 25-30 and 45-50?

(iv) Find the mean of above data.

STATISTICS

26. The men's 200 m race event at the 2020 Tokyo Olympic took place 3 and 4 August. A stopwatch was used to find the time that it took a group of Athletes to run 200 m.



Time (in seconds)	0-20	20-40	40-60	60-80	80-100
No. of Students	8	10	13	6	3

(i) No. of students who finished the race within 1 min:

(a) 10 (b) 8

(c) 31 (d) 13

(ii) Average of lower limits of median class and modal class is :

(a) 30 (b) 50

(c) 60 (d) 40

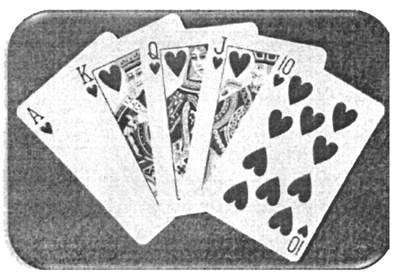
(iii) Find the mean time taken by a student to finish the race.

(iv) Find the mode of the above data.

PROBABILITY

27. Read the following and answer four questions from (i) to (iv).

Aisha took a pack of 52 cards. She kept aside all the face cards and shuffled the remaining cards well.



(i) The number of favourable outcomes for the event a club card or a '4' is

(a) 13 (b) 17

(c) 14 (d) 12

(ii) She drew a card from the well-shuffled pack of remaining cards. The probability that the card drawn is a red card is

(a) 1/4 (b) 1/2

(c) 4/13 (d) 2/13

(iii) Find the probability of drawing a black queen.

(iv) Find the probability of getting neither a black card nor an ace card.

PROBABILITY

28. Akriti and Sukriti have to start the game of ludo. They are fighting for who will start the game. They found three coins and decided to toss them simultaneously to know who will start the game.

(i) The possible number of outcomes:

(a) 8 (b) 6

(c) 2 (d) 4

(ii) The probability of getting 3 tails on tossing three coins simultaneously:

(a) 1/4 (b) 1/8

(c) 7/8 (d) 1/6

(iii) Akriti says if I get atleast one head, I will win and start the game. Find the probability that Akriti will start the game.

(iv) Sukriti says if I get atmost one tail, I will start the game. Find the probability that Sukriti will start the game.

ANSWERS

1. (i) (d) 12

   (ii) (c) 21

   (iii) 15 patients

   (iv) m = 2

2. (i) (b) HCF (36, 60) = 12. Thus fruits will be equally distributed among 12 guests.

   (ii) (a) each guest will get (36 ÷ 12) = 3 apples and (60 ÷ 12) = 5 bananas.

   (iii) HCF (36, 42, 60) = 6. Thus fruits will be equally distributed among 6 guests.

   (iv) Each guest will get (36 ÷ 6) = 6 apples, (42 ÷ 6) = 7 mangoes, and (60 ÷ 6) = 10 bananas. Thus each guest will get 6 + 7 + 10 = 23 fruits.

3. (i) (*a*) > 0

   (ii) (c) $x^2 + x - 2$

   (iii) Put $x = -1$ to get 'k' = 5

   (iv) $\alpha + \beta = 7$ and $\alpha\beta = 12$

   $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = 7/12$

4. (i) (*c*) 'a' is a non-zero number and b and c are any real numbers.

   (ii) (*d*) $x^2 + 9x + 20$

   (iii) 'k' = 1/4

   (iv) $k(x^2 + px - 1/p)$

5. (i) (*b*) 65, 55

   (ii) (*a*) 55

   (iii) $x + 4y = 315$

   (iv) 380 cubic units

6. (i) (d) $x + 2y = 16$

   (ii) (c) $x + 4y = 22$

   (iii) Put x = ₹ 10 and y = ₹ 3

(iv) for Ayush x + 6y = 28 and for Sonali $x + 4y = 22$

Total amount paid is ₹ 50

7. (i) (*a*) $2(y + 5)$ km

   (ii) (*c*) $y^2 + 5y - 500 = 0$

   (iii) speed = 20 km/h

   (iv) time = 16 hours

8. (i) (*a*) $60 - 2x$

   (ii) (*d*) $60x - 2x^2$

   (iii) $x^2 - 30x + 125 = 0$

   (iv) width could be 10 m or 20 m

9. (i) (c) 1300 units

   (ii) (b) 1400 units

   (iii) $a_n = 600 + 700\,n$

   (iv) 38500 units

10. (i) (b) 30 m

    (ii) (c) 30, 50, 70, 90, ...

    (iii) 410 m

    (iv) 4400 m

11. (i) Similarily of triangles

    (ii) (d) 27 m

    (iii) 184 m

    (iv) 621 m

12. (i) (a) 240 cm

    (ii) (b) 10.9 m

    (iii) 3.6 m

    (iv) 5.4 m

13. (i) (c) $13\sqrt{2}$ m

(ii) (b) $\sqrt{109}$ m

(iii) T

(iv) $\sqrt{109}$ m

14. (i) (c) $3\sqrt{3}$ units

(ii) (b) 6 units

(iii) 6 units

(iv) $3\sqrt{2}$ units

15. (i) (c) 14.34 m

(ii) (c) 33.7 m

(iii) 19 m

(iv) 10.2 m

16. (i) (b) 20 m

(ii) (d) 17.32 m

(iii) 30m approx

(iv) 34.64 m

17. (i) (c) 1136.4 km

(ii) (c) 1937 km

(iii) 8385.7 km

(iv) 45°

18. (i) (d) 120 m

(ii) (d) 107 m approx

(iii) 107 m approx

(iv) 214 m approx

19. (i) (b) 120°

    (ii) (b) 60°

    (iii) 60°

    (iv) 60°

20. (i) (a) 7m

    (ii) (b) 5m

    (iii) 3m

    (iv) $20m^2$

21. (i) (b) 8.5 cm

    (ii) concentric circles

    (iii) 132 cm^2

    (iv) 505.20 cm

22. (i) (a) 84 m^2

    (ii) (b) 90°

    (iii) 9.625 m^2

    (iv) 5.11 m^2

23. (i) (b) 8.5m

    (ii) (a) 2800 m^2

    (iii) Rs. 11407

    (iv) 17587.5 m^3

24. (i) (c) 84π square cm

    (ii) (a) 92.5 π cu. cm.

    (iii) Rs. 9.65 approx

    (iv) Rs. 1833.50

25. (i) (a) $\frac{\Sigma_{i=1}^{n} f_i x_i}{n}$

(ii) (c) $l + \frac{\frac{n}{2} - cf}{f} \times h$

(iii) 37.5

(iv) 28.21 approx

26. (i) (c) 31

(ii) (d) 40

(iii) 43 second

(iv) 40 seconds

27. (i) (a) 13

(ii) (a) 1/4

(iii) 0

(iv) $^{18}/_{40}$ or $^{9}/_{20}$

28. (i) (a) 8

(ii) (b) 1/8

(iii) 7/8

(iv) $\frac{4}{8}$ or $\frac{1}{2}$

Practice Paper-I

Class- X Session- 2022-23

Subject- Mathematics (Standard)

Time : 3 Hours **Max. Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 5 marks, 2 Questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION A

Section A consists of 20 questions of 1 mark each.

1. Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF(a,b) = p^mq^n and LCM(a, b) = p^rq^s, then $(m + n)(r + s) =$

   (a) 15 (b) 30 (c) 35 (d) 72

2. Let p be a prime number. The quadratic equation having its roots as factors of p is

(a) $x^2 - px + p = 0$ (b) $x^2 - (p + 1)x + p = 0$

(c) $x^2 + (p + 1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$

3. If α and β are the zeroes of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

4. If the system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k =

(a) -1 (b) 0 (c) 1 (d) 2

5. If the vertices of a parallelogram PQRS taken in order arc P(3, 4), Q(-2, 3) and R(-3,-2), then the coordinates of its fourth vertex S are

(a) (-2,-1) (b) (-2, -3) (c) (2,-1) (d) (1,2)

6. $\Delta ABC \sim \Delta PQR$. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2 : PQ^2 = 4 : 9$, then AM: PN =

(a) 3:2 (b) 16:81 (c) 4:9 (d) 2:3

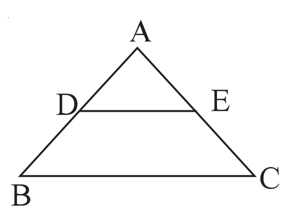
7. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then x =

(a) cos30° (b) tan30° (c) sin30° (d) cot30°

8. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

(a) 1 (b) 2 (c) 3 (d) 4

9. In the given figure, DE || BC, AE = a units, EC = b units, DE = x units and BC = y units. Which of the following is true?



(a) $x = \frac{a+b}{ay}$ (b) $y = \frac{ax}{a+b}$ (c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$

10. ABCD is a trapezium with AD || BC and AD = 4cm. If the diagonals AC and BD intersect each other at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then BC =

(a) 6cm (b) 7cm (c) 8cm (d) 9cm

11. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(a) $\frac{3\sqrt{3}}{2}$ cm (b) 3cm (c) 6 cm (d) $3\sqrt{3}$ cm

12. The area of the circle that can be inscribed in a square of 6cm is

(a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2

13. The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is

(a) 48 cm^2 (b) 72 cm^2 (c) 96 cm^2 (d) 108 cm^2

14. If the difference of Mode and Median of a data is 24, then the difference of median and mean is

(a) 8 (b) 12 (c) 24 (d) 36

15. The number of revolutions made by a circular wheel of radius 0.25m in rolling a distance of 11 km is

(a) 2800 (b) 4000 (c) 5500 (d) 7000

16. For the following distribution,

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of the lower limits of the median and modal class is

(a) 15 (b) 25 (c) 30 (d) 35

17. Two dice are rolled simultaneously. What is the probability that 6 will come up at least once?

(a) $\frac{1}{6}$ (b) $\frac{7}{36}$ (c) $\frac{11}{36}$ (d) $\frac{13}{36}$

18. If 5 tan $\beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) 6

DIRECTION: In the question number 19 and 20, a statement of **assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option

19. ***Statement A*** (Assertion): If product of two numbers is 5780 and their HCF is 17, then their LCM is 340.

Statement R(Reason) : HCF is always a factor of LCM

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is tmc.

20. ***Statement A (Assertion):*** If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3,5) and E(-3,-3) respectively, then BC = 20 units

Statement R(Reason) : The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is tme but reason(R) is false.

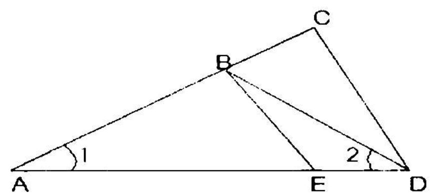
(d) Assertion (A) is false but reason(R) is true.

SECTION B

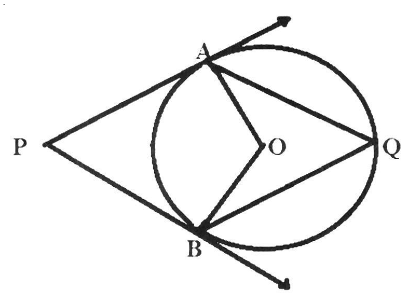
Section B consists of 5 questions of 2 marks each

21. If $49x + 51y = 499$, $51x + 49y = 501$, then find the value of x and y

22. In the given figure below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$. Show that $\Delta BAE \sim \Delta CAD$.



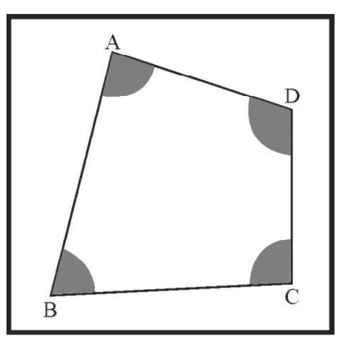
23. In the given figure, O is the centre of circle. Find $\angle AQB$, given that PA and PB are tangents to the circle and $\angle APB = 75°$.



24. The length of the minute hand of a clock is 6cm. Find the area swept by it when it moves from 7:05 p.m. to 7:40 p.m.

OR

In the given figure, arcs have been drawn of radius 7cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.



25. If $\sin(A+B)=1$ and $\cos(A-B)=\dfrac{\sqrt{3}}{2}$, $0° < A+B \le 90°$ and $A > B$, then find the measures of angles A and B.

OR

Find an acute angle θ when $\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

SECTION C

Section C consists of 6 questions of 3 marks each.

26. Given that $\sqrt{3}$ is irrational, prove that $5+2\sqrt{3}$ is irrational.

27. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of p and q.

28. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr it would have taken 6 hours more than the scheduled time. Find the length of the journey.

OR

Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹2 for 3 chocolates and the second lot at the rate of ₹ 1 per chocolate, and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per chocolate, and the second lot at the rate of ₹ 4 for 5 chocolates, his total collection would have been ₹ 460. Find the total number of chocolates he had.

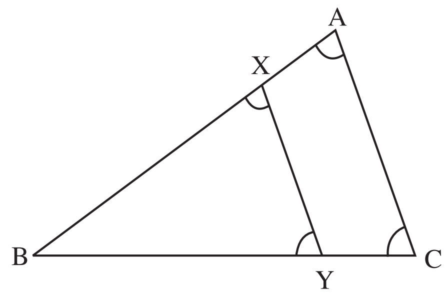
29. Prove the following that-

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\,\text{cosec}\theta-2\sin\theta\cos\theta$$

30. Prove that a parallelogram circumscribing a circle is a rhombus

OR

In the figure XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and $X'Y'$ at B, what is the measure of $\angle AOB$.



31. Two coins are tossed simultaneously. What is the probability of getting

(i) At least one head?

(ii) At most one tail?

(iii) A head and a tail?

SECTION D

Section D consists of 4 questions of 5 marks each.

32. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?

OR

In a flight of 600km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight.

33. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.

Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non parallel sides in the same ratio.

34. Due to heavy floods in a state, thousands were rendered homeless. 50 sehools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹120 per m^2, find the amount shared by each school to set up the tents.

OR

There are two identical solid cubical boxes of side 7cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to from a dome. Find

(i) the ratio of the total surface area of the two new solids formed

(ii) volume of each new sohd formed.

35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100

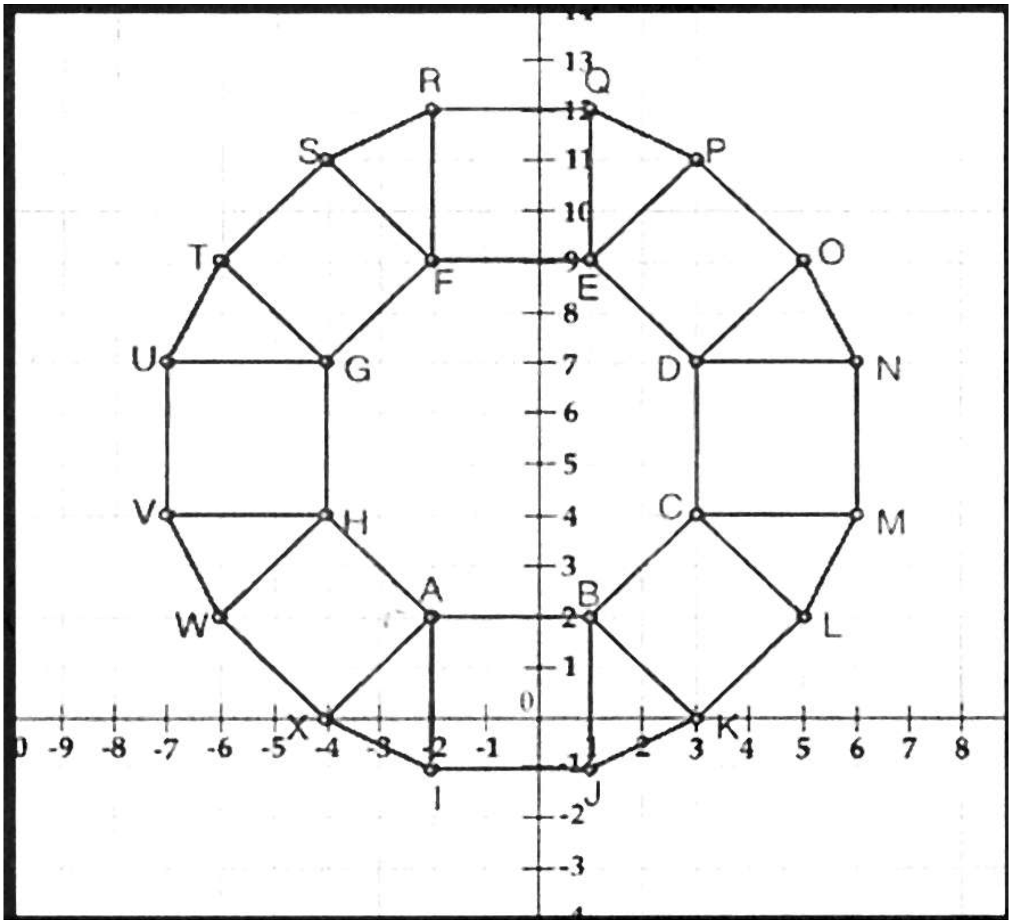
Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

SECTION E

Case study based questions are compulsory.

36. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern



Use the above figure to answer the questions that follow:

(i) What is the length of the line segment joining points B and F?

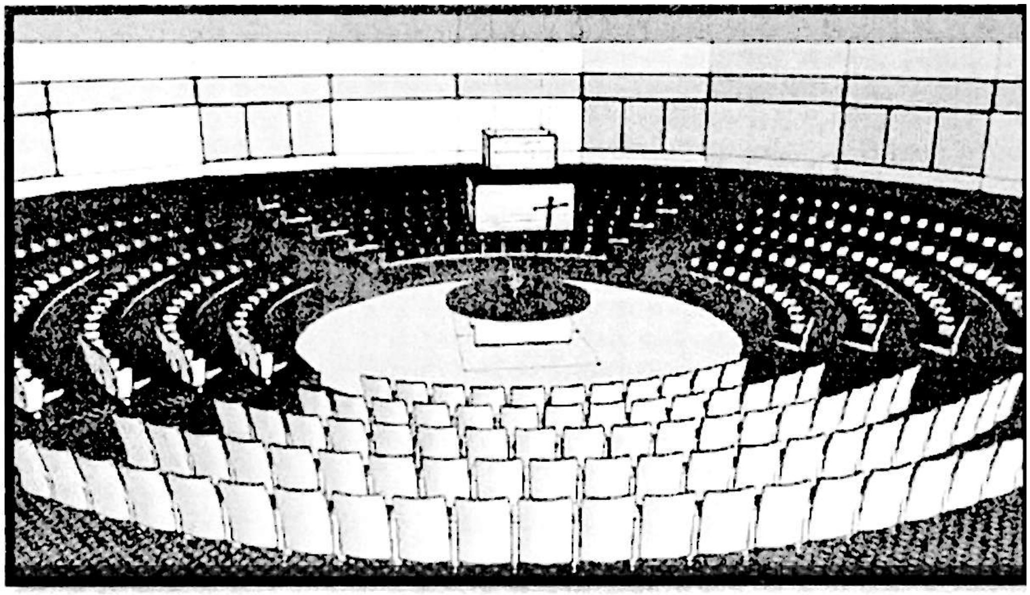
(ii) The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z?

(iii) What are the coordinates of the point on y axis equidistant from A and G?

OR

What is the area of Trapezium AFGH?

37. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.



(i) If the first circular row has 30 seats, how many seats will be there in the 10th row?

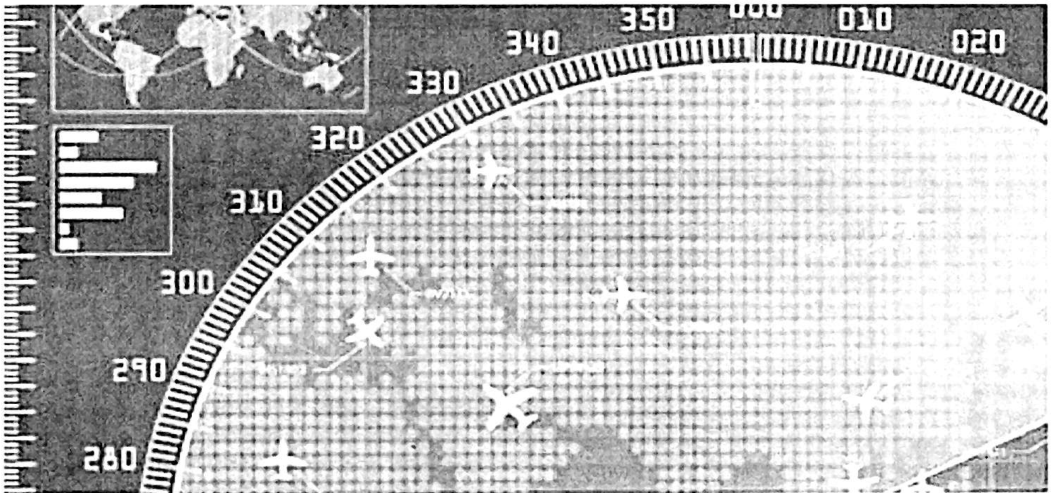
(ii) For 1500 seats in the auditorium, how many rows need to be there?

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10^{th} row?

(iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row?

38. We all have seen the airplanes flying in the sky but might have not thought of how they actually reach the correct destination. Air Traffic Control (ATC) is a service provided by ground-based air traffic controllers who direct aircraft on the ground and through a given section of controlled airspace, and can provide advisoiy services to aircraft in non-controlled airspace. Actually, all this air traffic is managed and regulated by using various concepts based on coordinate geometry and trigonometry.



At a given instance, ATC finds that the angle of elevation of an airplane from a point on the ground is 60°. After a flight of 30 seconds, it is observed that the angle of elevation changes to 30°. The height of the plane remains constantly as $3000\sqrt{3}$ m. Use the above information to answer the questions that follow-

(i) Draw a neat labelled figure to show the above situation diagrammatically.

(ii) What is the distance travelled by the plane in 30 seconds?

OR

Keeping the height constant, during the above flight, it was observed that after $15(\sqrt{3}-1)$ seconds, the angle of elevation changed to 45°. How much is the distance travelled in that duration.

(iii) What is the speed of the plane in km/hr.?

SOLUTION PRACTICE PAPER - I

SUBJECT: MATHEMATICS-STANDARD

CLASS X

SECTION - A

1 (c) 35

2 (b)$x^2 - (p + 1)x + p = 0$

3 (b) $\frac{2}{3}$

4 (d) 2

5 (c) (2, -l)

6 (d) 2 : 3

7 (b) tan 30°

8 (b) 2

9 (c) $x = \frac{ay}{a+b}$

10 (c) 8 cm

11 (d) $3\sqrt{3}$ cm

12 (d) 9π cm^2

13 (c) 96 cm^2

14 (b) 12

15 (d) 7000

16 (b) 25

17 (c) $\frac{11}{36}$

18 (a) $\frac{1}{3}$

19 (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

20. (a) Both assertion (A) and reason (R) arc true and reason (R) is the correct explanation of assertion (A)

SECTION-B

21. Adding the two equations and dividing by 10, wc get: x + y = 10

Subtracting the two equations and dividing by -2, wc get : x-y = 1

Solving those two new equations, wc get, $x = \mathbf{\frac{11}{2}}$

$y = \mathbf{\frac{9}{2}}$

22. In ΔABC,

$\angle 1 = \angle 2$

$\therefore$ AB = BD ..... (i)

Given,

$$\frac{AD}{AE} = \frac{AC}{BD}$$

Using equation (i), we get

$$\frac{AD}{AE} = \frac{AC}{AB} \quad .....(ii)$$

In ΔBAE and ΔCAD, by equation (ii),

$$\frac{AC}{AB} = \frac{AD}{AE}$$

$\angle A = \angle A$ (common)

$\therefore$ $\Delta BAE \sim \Delta CAD$ [By SAS similarity criterion]

23. $\angle PAO = \angle PBO = 90°$ (angle b/w radius and tangent)

$\angle AOB = 105°$ (By angle sum property of a triangle)

$\angle AQB = \frac{1}{2} \times 105° = 52.5°$ (Angle at the remaining part of the circle is half the angle subtended by the arc at the centre)

24. We know that, in 60 minutes, the tip of minute hand moves 360°

In 1 minute, it will move $\frac{360°}{60} = 6°$

$\therefore$ From 7 : 05 pm to 7: 40 pm i.e. 35 min, it will move through = 35 × 6° = 210°

$\therefore$ Area of swept by the minute hand in 35 min = Area of sector with sectorial angle θ of 210° and radius of 6 cm

$$= \frac{210°}{360°} \times \pi \times 6^2$$

$$= \frac{7}{12} \times \frac{22}{7} \times 6 \times 6$$

$= 66\ cm^2$

OR

Let the measure of ∠A, ∠B, ∠C and ∠D be θ_1, θ_2, θ_3 and θ_4 respectively

Required area = Area of sector with centre A + Area of sector with centre B + Area of sector with centre C + Area of sector with centre D

$$= \frac{\theta_1}{360°} \times \pi \times 7^2 + \frac{\theta_2}{360°} \times \pi \times 7^2 + \frac{\theta_3}{360°} \times \pi \times 7^2 + \frac{\theta_4}{360°} \times \pi \times 7^2$$

$$= \frac{(\theta_1 + \theta_2 + \theta_3 + \theta_4)}{360°} \times \pi \times 7^2$$

$$= \frac{(360°)}{360°} \times \frac{22}{7} \times 7 \times 7 \text{ (By angle sum property of a triangle)}$$

$= 154\ cm^2$

25. sin (A + B) =1 = sin 90°, so A + B = 90° ...(i)

$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30°$, so A-B = 30° ...(ii)

From (i) & (ii) $\angle A = 60°$

And $\angle B = 30°$

OR

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing the numerator and denominator of LHS by $\cos\theta$, we get

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Which on simplification (or comparison) gives $\tan\theta = \sqrt{3}$

Or $\theta = 60°$

SECTION-C

26. Let us assume $5+2\sqrt{3}$ is rational, then it must be in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$

i.e $5+2\sqrt{3} = \frac{p}{q}$

So $\sqrt{3} = \frac{p-5q}{2q}$ ...(i)

Since p, q, 5 and 2 arc integers and $q \neq 0$, RHS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational. This is not possible.

This contradiction has arisen due to our wrong assumption that $5+2\sqrt{3}$ is rational. So, $5+2\sqrt{3}$ is irrational.

27. Let α and β be the zeroes of the polynomial $2x^2 - 5x - 3$

Then $\alpha + \beta = \frac{5}{2}$

And $\alpha\beta = \frac{-3}{2}$

Let 2α and 2β be the zeroes of $x^2 + px + q$

Then $2\alpha + 2\beta = -p$

$2(\alpha + \beta) = -p$

$2 \times \frac{5}{2} = -p$

So p = -5

And $2\alpha \times 2\beta = q$

$4\,\alpha\beta = q$

So $q = 4 \times \frac{-3}{2}$

$= -6$

28. Let the actual speed of the train be x km/hr and let the actual time taken be y hours. Distance covered is xy km

If the speed is increased by 6 km/lir, then time of journey is reduced by 4 hours i.e., when speal is (x + 6) km/hr, time of journey is (y-4) hours.

$\therefore$ Distance covered = (x + 6)(y - 4)

$\Rightarrow xy = (x + 6)(y - 4)$

$\Rightarrow -4x + 6y - 24 = 0$

$\Rightarrow -2x + 3y-12 = 0 \quad ...(i)$

Similaily $xy = (x-6)(y+6)$

$\Rightarrow 6x-6y-36 = 0$

$\Rightarrow x-y-6 = 0 \quad ...(ii)$

Solving (i) and (ii) we get x = 30 and y = 24

Putting the values of x and y in equation (i), we obtain

Distance (30×24)km = 720km.

Hence, the length of the journey is 720 km.

OR

Let the number of chocolates in lot A be x

And let the number of chocolates in lot B be y

$\therefore$ total number of chocolates = x + y

Price of 1 chocolate = ₹ 2/3, so for x chocolates = $\frac{2}{3}x$

and price of y chocolates at the rate of ₹ 1 per chocolate = y

$\therefore$ by the given condition $\frac{2}{3}x + y = 400$

$\Rightarrow$ 2x + 3y = 1200 ...(i)

Similarly $x + \frac{4}{5}y = 460$

$\Rightarrow$ 5x + 4y = 2300 ...(ii)

Solving (i) and (ii) we get

x = 300 and y = 200

$\therefore$ x + y = 300 + 200 = 500

So, Anuj had 500 chocolates.

29. LHS : $\frac{\sin^3\theta/\cos^3\theta}{1+\sin^2\theta/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{1+\cos^2\theta/\sin^2\theta}$

$$= \frac{\sin^3\theta/\cos^3\theta}{(\cos^2\theta+\sin^2\theta)/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{(\sin^2\theta+\cos^2\theta)/\sin^2\theta}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

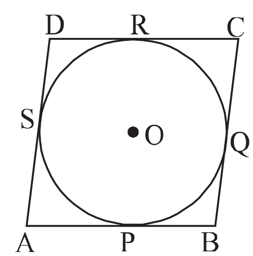
$$= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$= \sec\theta\operatorname{cosec}\theta - 2\sin\theta\cos\theta$

= RHS

30. Let ABCD be the rhombus circumscribing the circles with centre O, such that AB, BC, CD and DA touch the circle at points P, Q, R and S respectively.

We know that the tangents drawn to a circle from an exterior point are equal in length.



$\therefore$ AP = AS ......(1)

BP = BQ ....(2)

CR = CQ ......(3)

DR = DS .....(4)

Adding (1), (2), (3) and (4) we get

AP+BP+CR+DR = AS+BQ+CQ+DS

(AP+BP) + (CR+DR) = (AS+DS) + (BQ + CQ)

$\therefore$ AB+CD = AD + BC ......(5)

Since AB = DC and AD = BC (opposite sides of parallelogram ABCD)

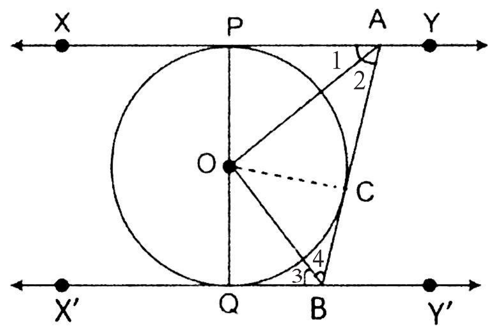
putting in (5) we get, 2AB = 2AD

or AB = AD.

$\therefore$ AB = BC = DC = AD

Since a parallelogram with equal adjacent sides is a rhombus, so ABCD is a rhombus

OR



Join OC

In ΔOPA and ΔOCA

OP = OC (radii of same circle)

PA = CA (length of two tangents from an external point)

AO = AO (Common)

Therefore, $\Delta OPA \cong \Delta OCA$ (By SSS congruency criterion)

Hence, $\angle 1 = \angle 2$ (CPCT)

Similarly $\angle 3 = \angle 4$

$\angle PAB + \angle QBA = 180°$(co interior angles are supplementary as XY||X'Y')

$2\angle 2 + 2\angle 4 = 180°$

$\angle 2 + \angle 4 = 90°$ ...(1)

$\angle 2 + \angle 4 + \angle AOB = 180°$ (Angle sum property)

Using (1), we get, $\angle AOB = 90°$

31. (i) P (At least one head) = $\frac{3}{4}$

(ii) P(At most one tail) = $\frac{3}{4}$

(iii)P(A head and a tail) = $\frac{2}{4} = \frac{1}{2}$

SECTION-D

32. Let the time taken by larger pipe alone to fill the tank = x hours

Therefore, the time taken by the smaller pipe = (x + 10) hours

Water filled by larger pipe running for 4 hours = $\frac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours = $\frac{9}{x+10}$ litres

We know that

$$\frac{4}{x}+\frac{9}{x+10}=\frac{1}{2}$$

Which on simplification gives:

$x^2-16x-80 = 0$

$x^2-20x + 4x -80 = 0$

$x^2-20x + 4x-80 = 0$

$x (x-20) + 4(x - 20) = 0$

$(x + 4) (x - 20) = 0$

$x = -4, 20$

x cannot be negative.

Thus, $x = 20$

$x + 10 = 30$

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours.

OR

Let the usual speed of plane be x km/hr and the reduced speed of the plane be (x-200) km/hr

Distance = 600 km [Given]

According to the question,

(time taken at reduced speed) - (Schedule time) = 30 minutes = 0.5 hours.

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$

Which on simplification gives:

$x^2 - 200x - 240000 = 0$

$x^2 - 600x + 400x - 240000 = 0$

$x(x-600) + 400(x-600) = 0$

$(x-600)(x+400)=0$

$x = 600$ or $x = 400$

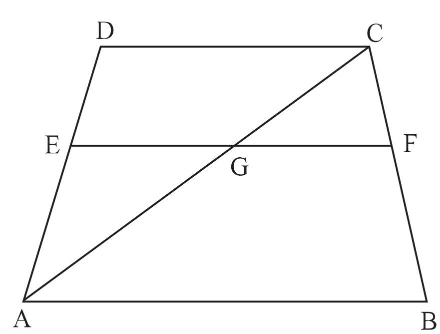
But speed cannot be negative.

$\therefore$ The usual speed is 600 km/hr and the scheduled duration of the flight is $\frac{600}{600}$ = 1 hour

33. For the theorem :

Given, To prove. Construction and figure

Proof



Let ABCD be a trapezium DC||AB and EF is a line parallel to AB and hence to DC.

To prove : $\frac{DE}{EA} = \frac{CF}{FB}$

Construction : Join AC, meeting EF in G.

Proof:

In ΔABC, we have

GF||AB

$\frac{CG}{GA} = \frac{CF}{FB}$ [By BPT] ...(1)

In ΔADC, we have

EG||DC (EF||AB & AB||DC)

$\frac{DE}{EA} = \frac{CG}{GA}$ [By BPT] ...(2)

From (l) & (2), we get,

$\frac{DE}{EA} = \frac{CF}{FB}$

34. Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m.

Slant height of conical part $(l) = \sqrt{r^2 + H^2}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41}$

$= \sqrt{12.25} = 3.5\text{m}$

Area of canvas used to make tent = CSA of cylinder + CSA of cone

$= 2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$

$= 61.6 + 30.8$

$= 92.4\ m^2$

Cost of 1500 tents at ₹120 per sq.m

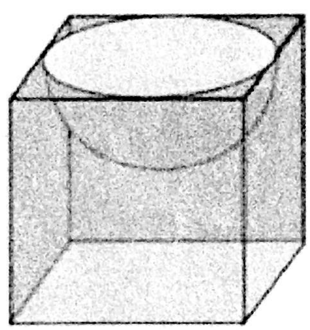
$= 1500 \times 120 \times 92.4$

$= 16,632,000$

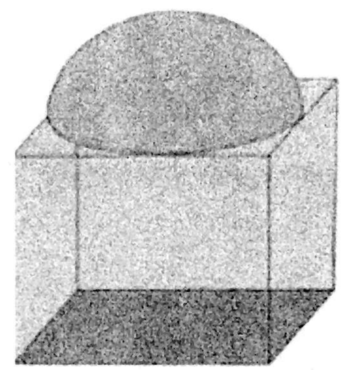
Share of each school to set up the tents = 16632000/50 ₹332,640

OR

First Solid



Second Solid



(i) SA for first new solid (S_1):

$6 \times 7 \times 7 + 2\ \pi \times 3.5^2 - \pi \times 3.5^2$

$= 294 + 77 - 38.5$

$= 332.5 cm^2$

SA for second new solid (S_2):

$6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2$

$= 294 + 77 - 38.5$

$= 332.5\ cm^2$

So $S_1 : S_2 = 1:1$

(ii) Volume for first new solid $(V_1) = 7 \times 7 \times 7 - \frac{2}{3}\pi \times 3.5^3$

$$= 343 - \frac{539}{6} = \frac{1519}{6}\text{cm}^3$$

Volume for second new solid $(V_2) = 7 \times 7 \times 7 + \frac{2}{3}\pi \times 3.5^3$

$$= 343 + \frac{539}{6} = \frac{2597}{6}\text{cm}^3$$

35. Median = 525, so Median Class = 500 600

Class interval	Frequency	Cumulative Frequency
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500 600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y

$76 + x + y = 100 \Rightarrow x + y = 24$ ....(i)

Median $= l + \dfrac{\frac{n}{2} - cf}{f} \times h$

Since, $l = 500$, h = 100, f = 20, cf = 36 + x and n = 100

Therefore, putting the value in the Median formula, we get;

$$525 = 500 + \frac{50-(36+x)}{20} \times 100$$

so x = 9

y = 24 - x (from eq.i)

y = 24-9 =15

Therefore, the value of x = 9 and y = 15.

36. (i) B(l,2), F(-2,9)

$BF^2 = (-2-1)^2 + (9-2)^2$

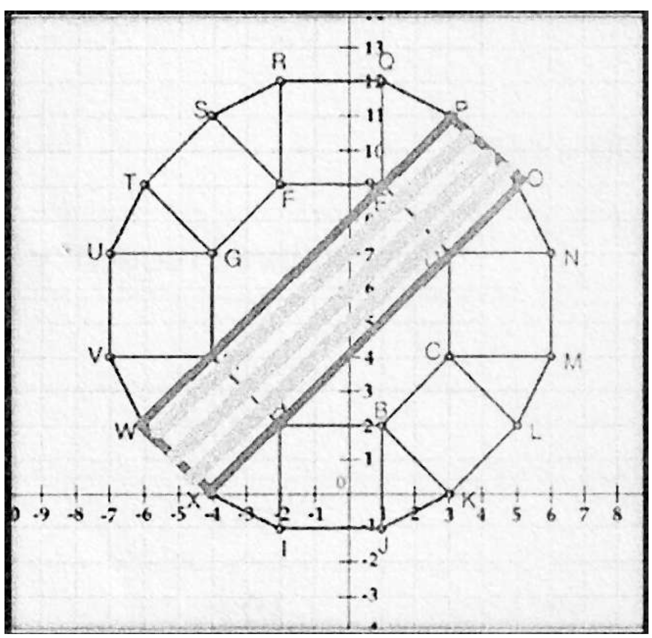
$= (-3)^2 + (7)^2$

$= 9 + 49$

$= 58$

So, BF = $\sqrt{58}$ units

(ii)



W(-6, 2), X(-4, 0), O(5, 9), P(3, 11)

Clearly WXOP is a rectangle

Point of intersection of diagonals of a rectangle is the mid point of the diagonals.

So the required point is mid point of WO or XP

$$= \left(\frac{-6+5}{2}, \frac{2+9}{2} \right)$$

$$= \left(\frac{-1}{2}, \frac{11}{2} \right)$$

(iii) A(-2, 2), G(-4, 7)

Let the point on y-axis be Z(0, y)

$AZ^2 = GZ^2$

$(0+2)^2 + (y-2)^2 = (0 + 4)^2 = (y-7)^2$

$(2)^2 + y^2 + 4 - 4y = (4)^2 + y^2 - 49 -14y$

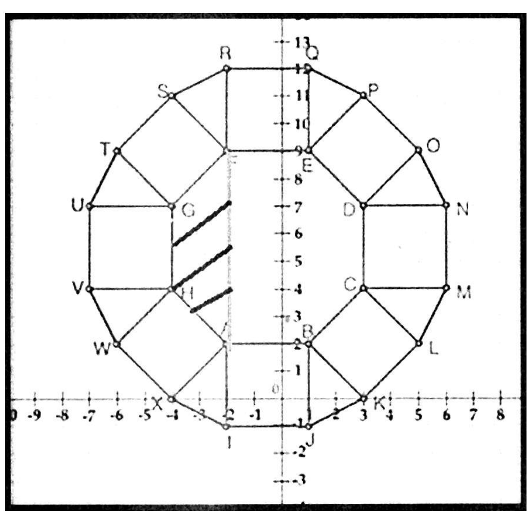
$8-4y = 65-14y$

$10y = 57$

So, $y = 5.7$

i.e. the required point is (0, 5.7)

OR



A(-2, 2), F(-2, 9), G(-4,7), ri{-4,4)

Clearly GH = 7-4 = 3units

AF = 9-2 = 7 units

So, height of the trapezium AFGH = 2 units

So, area of AFGH = $\frac{1}{2}(AF + GH) \times \text{height}$

$= \frac{1}{2}(7+3) \times 2$

= 10 sq. units

37. (i) Since each row is increasing by 10 seats, so it is an AP with first term a = 30, and common difference d = 10.

So number of seats in 10th row = a_{10} = a + 9d

= 30 + 9 × 10 = 120

(ii) $S_n = \frac{n}{2}(2a + (n\text{-}l)d)$

$1500 = \frac{n}{2}(2 \times 30 + (n\text{-}l)10)$

3000 = 50n+10n^2

n^2+5n-300 = 0

n^2 + 20n-15n -300 = 0

(n+20) (n-15) = 0

Rejecting the negative value, n = 15

OR

No. of seats already put up to the 10th row = S_{10}

$S_{10} = \frac{10}{2}\{2 \times 30 + (10-1)10)\}$

$= 5(60+ 90) = 750$

So, the number of seats still required to be put are 1500 -750 = 750

(iii) If no. of rows =17

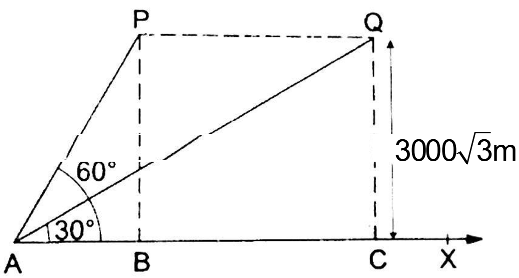
then the middle row is the 9th row

$a_8 = a + 8d$

$= 30 + 80$

$= 110$ seats

38. (i)



P and Q are the two positions of the plane flying at a height of $3000\sqrt{3}m$. A is the point of observation.

(ii) In ΔPAB, $\tan 60° = \frac{PB}{AB}$

Or $\sqrt{3} = \frac{3000\sqrt{3}}{AB}$

So AB = 3000m

$\tan 30° = \frac{QC}{AC}$

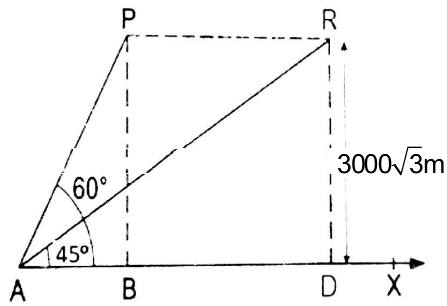
$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{AC}$

AC = 9000m

distance covered = 9000-3000

= 6000 m.

OR



In ΔPAB, $\tan 60° = \dfrac{PB}{AB}$

Or $\sqrt{3} = \dfrac{3000\sqrt{3}}{AB}$

So AB = 3000m

$\tan 45° = \dfrac{RD}{AD}$

$l = \dfrac{3000\sqrt{3}}{AD}$

AD = $3000\sqrt{3}m$

distance covered = $3000\sqrt{3} - 3000$

= $3000(\sqrt{3}-1)m$

(iii) speed = $\dfrac{6000}{30}$

= 200 m/s

= $\dfrac{200 \times 3600}{1000}$

= 720km/hr

Alternatively: speed = $\dfrac{3000(\sqrt{3}-1)}{15(\sqrt{3}-1)}$

= 200 m/s

= $\dfrac{200 \times 3600}{1000}$

= 720km/hr

Practice Paper-II

Class - X Session 2022-23

Subject - Mathematics (Basic)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section **A** has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section **B** has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section **C** has 6 Short Answer-ll (SA-II) type questions carrying 3 marks each.
5. Section **D** has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section **E** has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Questions of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section-A

Section A consists of 20 questions of 1 mark each.

1. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^2b$; a, b being prime numbers, then LCM (p, q) is

   (a) ab (b) a^2b^2 (c) a^3b^2 (d) a^2b^2

2. What is the greatest possible speed at which a man can walk 52 km and 91 km in an exact number of hours?

   (a) 17 km/hours (b) 7 km/hours

   (c) 13 km/hours (d) 26 km/hours

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10 (c) 5 (d) -5

4. Graphically, the pair of equations given by

$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

represents two lines which are

(a) intersecting at exactly one point

(b) parallel

(c) coincident

(d) intersecting at exactly two points.

5. If the quadratic equation $x^2 + 4x + k = 0$ has real and equal roots, then

(a) $k < 4$ (b) $k > 4$

(c) $k = 4$ (d) $k \geq 4$

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) 5 units (b) 12 units

(c) 11 units (d) $(7+\sqrt{5})$ units

7. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$

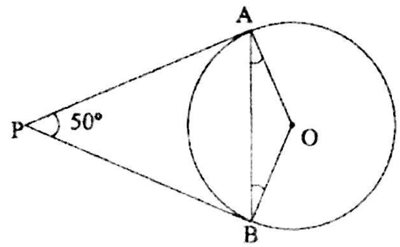
(c) $\angle B = \angle D$ (d) $\angle A = \angle F$

8. In which ratio the y-axis divides the line segment joining the points (5, - 6) and (-1, - 4)?

(a) 1 : 5 (b) 5 : 1

(c) 1 : 1 (d) 1 : 2

9. In the figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to



(a) 25° (b) 30° (c) 40° (d) 50°

10. If $\sin A = \frac{1}{2}$, then the value of sec A is :

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

11. $\sqrt{3}\cos^2 A + \sqrt{3}\sin^2 A$ is equal to

(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0

12. The value of cos1° cos2° cos3° cos4° .... cos90° is

(a) 1 (b) 0 (c) -1 (d) 2

13. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(a) 22: 7 (b) 14:11 (c) 7 : 22 (d) 11:14

14. If the radii of two circles are in the ratio of 4 : 3, then their areas are in the ratio of:

(a) 4:3 (b) 8 : 3 (c) 16:9 (d) 9:16

15. The total surface area of a solid hemisphere of radius 7 cm is :

(a) 447π cm^2 (b) 239π cm^2 (c) 174π cm^2 (d) 147π cm^2

16. For the following distribution :

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the upper limit of the modal class is

(a) 10 (b) 15 (c) 20 (d) 25

17. If the mean of the following distribution is 2.6, then the value of y is

Variable (x)	1	2	3	4	5
Frequency	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

18. A card is selected at random from a well shuffled deck of 52 cards. The probability of its being a red face card is

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$ (c) $\frac{2}{13}$ (d) $\frac{1}{2}$

Direction for questions 19 & 20: In question numbers 19 and 20, a statement of Assertion (A) Is followed by a statement of Reason (R). Choose the correct option.

19. **Assertion:** If HCF of 510 and 92 is 2, then the LCM of 510 & 92 is 32460
Reason: as HCF(a,b) X LCM(a,b) = a × b

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

20. **Assertion (A):** The ratio in which the line segment joining (2, -3) and (5, 6) internally divided by x-axis is 1:2.

Reason (R): as formula for the internal division is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Section B

Section B consists of 5 questions of 2 marks each

21. For what values of k will the following pair of linear equations have infinitely many solutions?

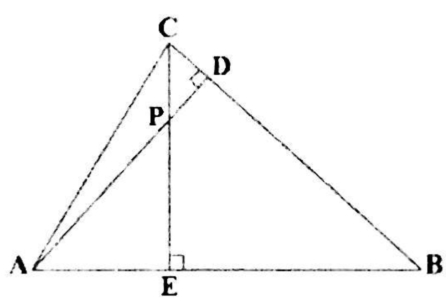
$kx + 3y-(k-3) = 0$

$12x + ky - k = 0$

22. In the figure, altitudes AD and CE of Δ ABC intersect each other at the point P. Show that:

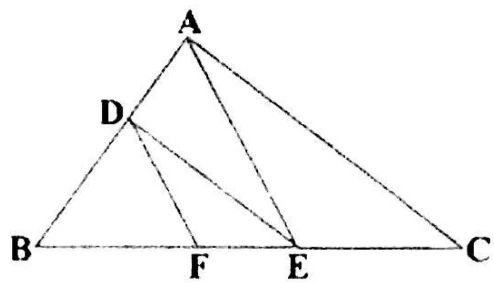
(i) $\Delta ABD \sim \Delta CBE$

(ii) $\Delta PDC \sim \Delta BEC$



[OR]

In the figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$



23. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

24. If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

25. Find the perimeter of a quadrant of a circle of radius 14 cm.

[OR]

Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm.

Section C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{5}$ is an irrational number.

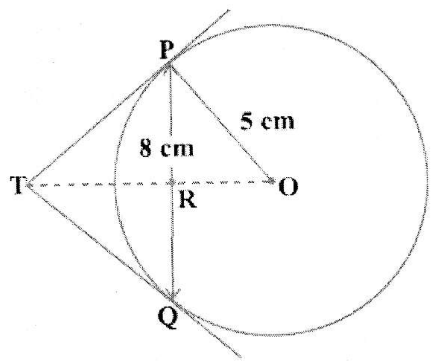
27. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

28. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

[OR]

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

29. In the figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.



30. Prove that

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$$

[OR]

If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$

31. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is

(i) 8?

(ii) 13?

(iii)less than or equal to 12?

Section D

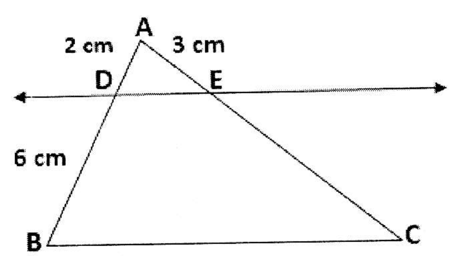
Section D consists of 4 questions of 5 marks each.

32. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

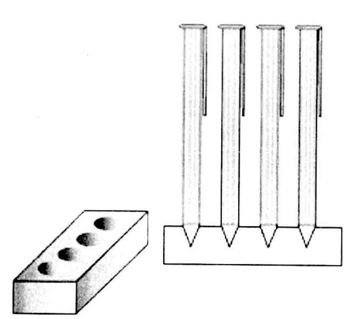
[OR]

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

33. Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. In the figure, find EC if $\frac{AD}{DB} = \frac{AE}{EC}$ using the above theorem.

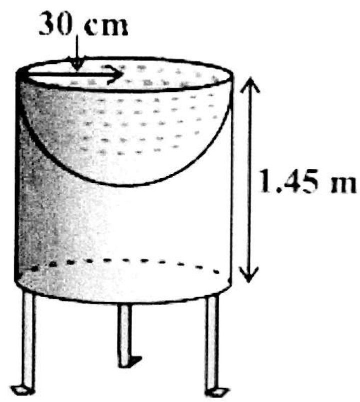


34. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.



[OR]

Ramesh made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.



35. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2
20-25	4
25-30	18
30-35	21
35-40	33
40-45	11
45-50	3
50-55	6
55-60	2

Section E

Case study based questions are compulsory

36. **Case Study - 1**

In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021-22, as per a report. A car manufacturing company planned to produce 1800 cars in 4^{th} year and 2600 cars in 8^{th} year. Assuming that the production increases uniformly by a fixed number every year.

Based on the above information answer the following questions.

I. Find the production in the 1^{st} year.

II. Find the production in the 12^{st} year.

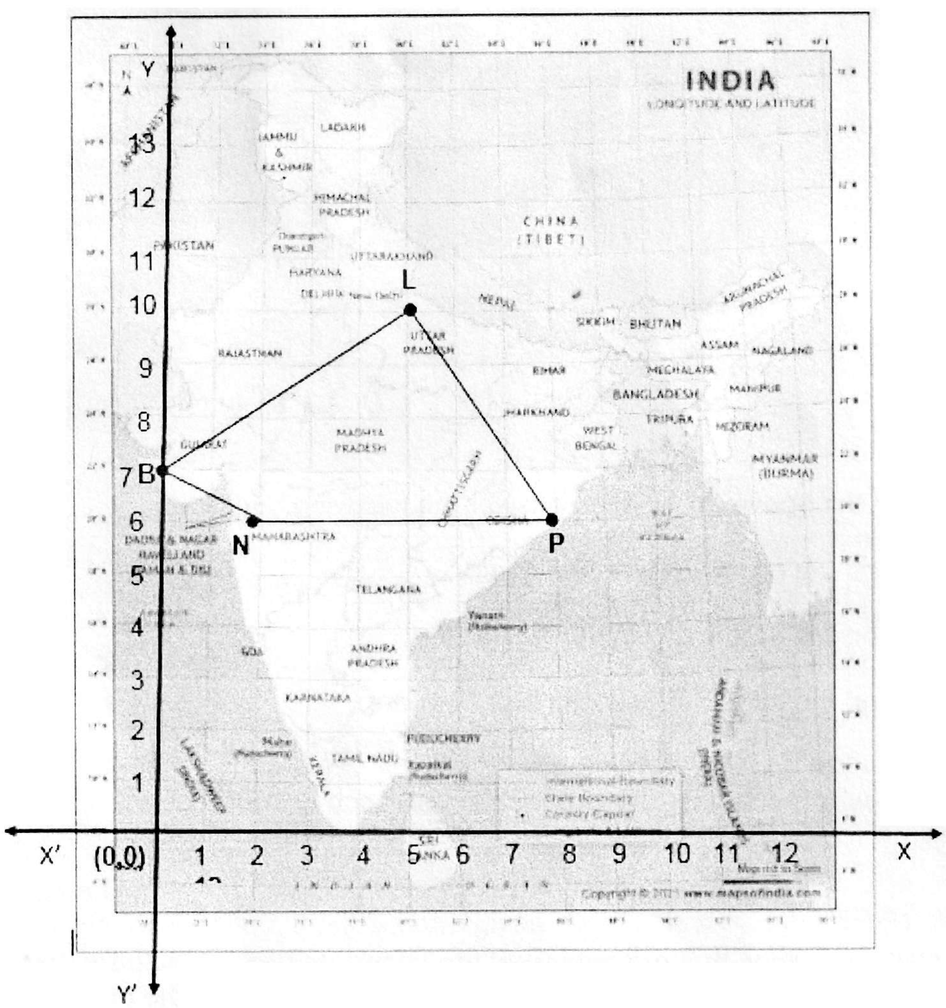
III. Find the total production in first 10 years.

[OR]

In which year the total production will reach to 15000 cars?

37. **Case Study-2**

In a GPS, The lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.



Based on the above information answer the following questions using the coordinate geometry.

I. Find the distance between Lucknow (L) to Bhuj(B).

II. If Kota (K), internally divide the line segment joining Lucknow (L) to Bhuj (B) into 3 : 2 then find the coordinate of Kota (K).

III. Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P)

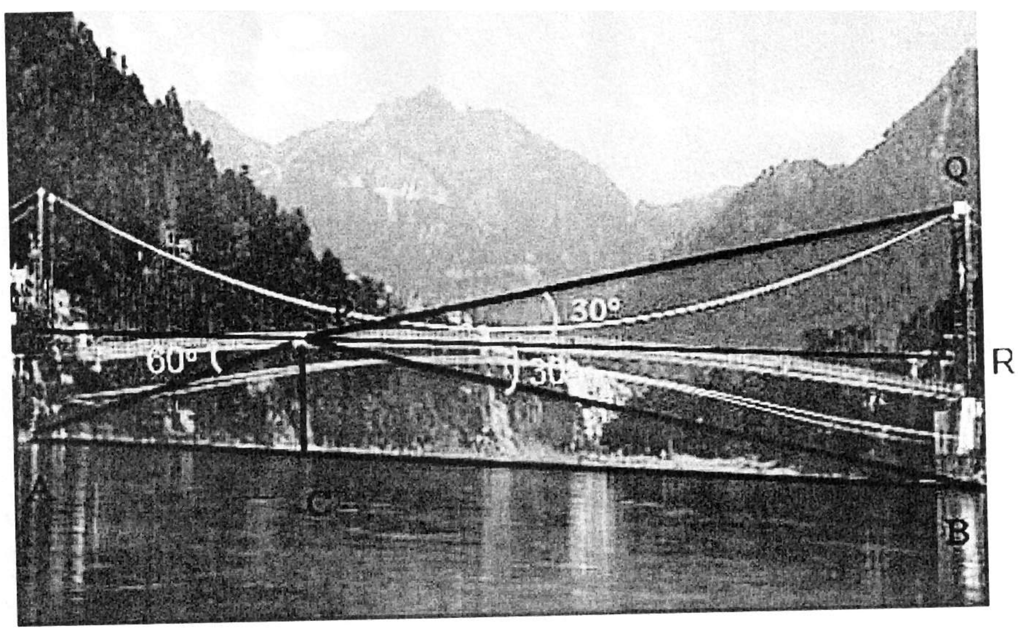
[OR]

Find a place (point) on the longitude (y-axis) which is equidistant from the points Lucknow (L) and Puri (P).

38. **Case Study - 3**

Lakshaman Jhula is located 5 kilometers north-east of the city of Rishikesh in the Indian state of Uttarakhand. The bridge connects the villages of Tapovan to Jonk. Tapovan is in Tehri Garhwal district, on the west bank of the river, while Jonk is in Pauri Garhwal district, on the east bank. Lakshman Jhula is a pedestrian bridge also used by motorbikes. It is a landmark of Rishikesh.

A group of Class X students visited Rishikesh in Uttarakhand on a trip. They observed from a point (P) on a river bridge that the angles of depression of



opposite banks of the river are 60° and 30° respectively. The height of the bridge is about 18 meters from the river.

Based on the above information answer the following questions.

I. Find the distance PA.

II. Find the distance PB

III. Find the width AB of the river.

[OR]

Find the height BQ if the angle of the elevation from P to Q be 30°.

SOLUTION PRACTICE PAPER-II

Mathematics Basic (241)

Class- X

Section A

1 (c) a^2b^2

2 (c) 13 km/hours

3 (b) -10

4 (b) Parallel.

5 (c) $k = 4$

6 (b) 12

7 (c) $\angle B = \angle D$

8 (b) 5 : 1

9 (a) $25°$

10. (a) $\dfrac{\sqrt{3}}{2}$

11. (c) $\sqrt{3}$

12 (b) 0

13 (b) 14:11

14 (c) 16:9

15 (d) 147π cm^2

16 (c) 20

17 (b) 8

18. (a) $\frac{3}{26}$

19. (d) Assertion (A) is false but Reason (R) is true.

20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION B

21. For a pair of linear equations to have infinitely many solutions :

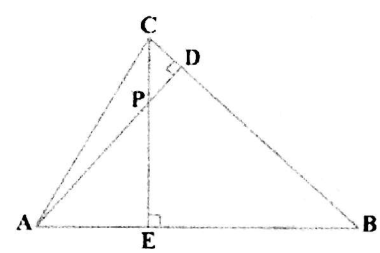
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} = \frac{5-3}{k}$$

$$\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k \pm 6$$

Also, $\frac{3}{k} = \frac{k-3}{k} \Rightarrow k^2 - 6k = 0 \Rightarrow k = 0, 6$

Therefore, the value of k, that satisfies both the conditions, is k = 6.

22. (i) In ΔABD and ΔCBE



$\angle ADB = \angle CEB = 90°$

$\angle ABD = \angle CBE$ (Common angle)

$\Rightarrow \Delta ABD \sim \Delta CBE$ (AA criterion)

(ii) In ΔPDC and ΔBEC

$\angle PDC = \angle BEC = 90°$

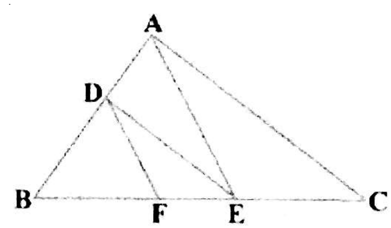
$\angle PCD = \angle BCE$ (Common angle)

$\Rightarrow \Delta PDC \sim \Delta BEC$ (AA criterion)

[OR]

In ΔABC, $DE \parallel AC$

$$\frac{BD}{AD} = \frac{BE}{EC} \text{ ....(i) (Using BPT)}$$



In ΔABE, $DF \parallel AE$

$$\frac{BD}{AD} = \frac{BF}{FE} \text{ ..... (ii) (Using BPT)}$$

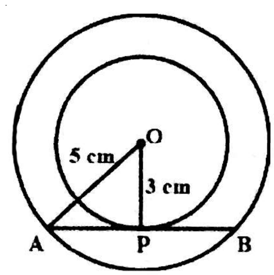
From (i) and (ii)

$$\frac{BD}{AD} = \frac{BE}{EC} = \frac{BF}{FE}$$

Thus, $\dfrac{BF}{FE} = \dfrac{BE}{EC}$

23. Let O be the centre of the concentric circle of radii 5 cm and 3 cm respectively. Let AB be a chord of the larger circle touching the smaller circle at P

Then $AP = PB$ and $OP \perp AB$



Applying Pythagoras theorem in ΔOPA, we have

$OA^2 = OP^2 + AP^2 \Rightarrow 25 = 9 + AP^2$

$\Rightarrow AP^2 = 16 \Rightarrow AP = 4\text{cm}$

$\therefore AB = 2AP = 8 \text{ cm}$

24. Now, $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$$

$= \cot^2\theta$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

25. Perimeter of quadrant $= 2r + \frac{1}{4} \times 2\pi r$

$\Rightarrow$ Perimeter $= 2 \times 14 + \frac{1}{2} \times \frac{22}{7} \times 14$

$\Rightarrow$ Perimeter = 28 + 22 = 28 + 22 = 50 cm

[OR]

Area of tine circle = Area of first circle + Area of second circle

$\Rightarrow \pi R^2 = \pi(r_1)^2 + \pi(r_1)^2$

$\Rightarrow \pi R^2 = \pi(24)^2 + \pi(7)^2 \Rightarrow \pi R^2 = 276 + 49\pi$

$\Rightarrow \pi R^2 = 625\pi \Rightarrow R^2 = 625 = R = 25$ Thus, diameter of the circle = 2R = 50 cm

Section C

26. Let us assume to the contrary, that $\sqrt{5}$ is rational. Then we can find a and b ($\neq 0$) such that $\sqrt{5} = \frac{a}{b}$ (assuming that a and b are co-primes).

So, $a = \sqrt{5}b \Rightarrow a^2 = 5b^2$

Here 5 is a prime number that divides a^2 then 5 divides a also

(Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer)

Thus 5 is a factor of a

Since 5 is a factor of a, we can write a = 5c (where c is a constant).

Substituting a = 5c

We get $(5c)^2 = 5b^2 \Rightarrow 5c^2 = b^2$

This means 5 divides b^2 so 5 divides b also (Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer).

Hence a and b have at least 5 as a common factor.

But this contradicts the fact that a and b are coprime. This is the contradiction to our assumption that p and q are co-primes.

So, $\sqrt{5}$ is not a rational number. Therefore, $\sqrt{5}$ is irrational.

27. $6x^2 - 7x - 3 = 0 \Rightarrow 6x^2 - 9x + 2x - 3 = 0$

$\Rightarrow 3x(2x-3) + 1(2x-3) = 0 \Rightarrow (2x - 3)(3x + 1) = 0$

$\Rightarrow 2x-3 = 0$ & $3x+1=0$

$x = \frac{3}{2}$ & $x = \frac{-1}{3}$ Hence, the zeroes of the quadratic polynomials are $\frac{3}{2}$ and $\frac{-1}{3}$.

For verification

Sum of zeroes $= \frac{coefficient\ of\ x}{cofficient\ of\ x^2} \Rightarrow \frac{3}{2}+\left(\frac{-1}{3}\right)=\frac{-(-7)}{6} \Rightarrow \frac{7}{6}=\frac{7}{6}$

Product of zeroes $= \frac{coefficient\ of\ x}{cofficient\ of\ x^2} \Rightarrow \frac{3}{2}\times\left(\frac{-1}{3}\right)=\frac{-3}{6} \Rightarrow \frac{-1}{2}=\frac{-1}{2}$

Therefore, the relationship between zeroes and their coefficients is verified.

28. Let the fixed charge by ₹ x and additional charge by ₹ y per day

Number of days for Latika = 6 = 2 + 4

Hence, Charge x + 4y = 22

x = 22-4y .....(1)

Number of days for Anand =4 = 2 + 2

Hence, Charge x + 2y = 16

x = 16 - 2y ....(2)

On comparing equation (1) and (2), we get.

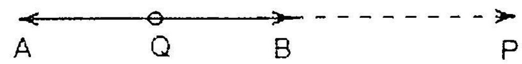
$22 - 4y = 16 - 2y \Rightarrow 2y = 6 \Rightarrow y = 3$

Substituting y = 3 in equation (1), we get,

$x = 22 - 4(3) \Rightarrow x = 22\text{-}12 \Rightarrow x = 10$

Therefore, fixed charge = ₹ 10 and additional charge = ₹ 3 per day

[OR]



AB = 100 knn. We know that, Distance = Speed × Tinne.

AP - BP = 100 $\Rightarrow$ 5x - 5y = 100 $\Rightarrow$ x-y = 20 ...(i)

AO + BQ = 100 $\Rightarrow$ x+ y = 100 ....(ii)

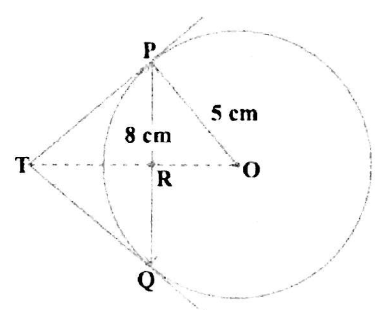
Adding equations (i) and (ii), we get,

x - y + x + y = 20 +100 $\Rightarrow$ 2x = 120 $\Rightarrow$ x = 60

Substituting x = 60 in equation (ii), we get, 60 + y = 100 $\Rightarrow$ y = 40

Therefore, the speed of the first car is 60 km/hr and the speed of the second car is 40 km/hr.

29. Since OT is perpendicular bisector of PQ.



Therefore, PR = RQ = 4 cm

Now, OR = $\sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} = 3\text{cm}$

Now, $\angle TPR + \angle PRO = 90°(\because \angle TPO = 90°)$

$\& \angle TPR + \angle PTR = 90°(\because \angle TRP = 90°)$

So, $\angle RPO = \angle PTR$

So, $\Delta TRP \sim \Delta PRO$[By A – A Rule of similar triangles]

So, $\dfrac{TP}{PO} = \dfrac{RP}{RG}$

$\Rightarrow \dfrac{TP}{5} = \dfrac{4}{3} \Rightarrow TP = \dfrac{20}{3}\text{cm}$

30. LHS $= \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = \dfrac{\tan\theta}{1-\dfrac{1}{\tan\theta}} + \dfrac{\dfrac{1}{\tan\theta}}{1-\tan\theta}$

$$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$$

$$= \frac{(\tan^2\theta+\tan\theta+1)}{\tan\theta}$$

$$= \tan\theta+1+\sec = 1+\tan\theta+\sec\theta$$

$$= 1+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$= 1+\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$= 1+\frac{1}{\sin\theta\cos\theta} = 1+\sec\theta\ \mathrm{cosec}\ \theta = \text{RHS.}$$

[OR]

$\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow \sin\theta\cos\theta = 1$

Now $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

31. (i) $P(8) = \dfrac{5}{36}$

(ii) $P(13) = \dfrac{0}{36} = 0$

(iii) P(less than or equal to 12) = 1

SECTION D

32. Let the average speed of passenger train = x km/h and the average speed of express train = (x + 11) km/h

As per given data, time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance. Therefore,

$$\frac{132}{x} - \frac{132}{x+11} = 1$$

$$\Rightarrow \frac{132(x+11-x)}{x(x+11)} = 1 \Rightarrow \frac{132 \times 11}{x(x+11)} = 1$$

$\Rightarrow 132 \times 11 = x(x + 11) \Rightarrow x^2 + 11x - 1452 = 0$

$\Rightarrow x^2 + 44x - 33x - 1452 = 0$

$\Rightarrow$ x(x + 44) -33 (x + 44) = 0 $\Rightarrow$ (x + 44) (x - 33) = 0

$\Rightarrow$ x = -44, 33

As the speed cannot be negative, the speed of the passenger train will be 33 km/h and the speed of the express train will be 33 + 11 =44 km/h.

[OR]

Let the speed of the stream be x km/hr

So, the speed of the boat in upstream = (18 - x) km/hr

& the speed of the boat in downstream = (18 + x) km/hr

ATQ, $\frac{\text{distance}}{upstream\ speed} - \frac{\text{distance}}{downstream\ speed} = 1$

$$\Rightarrow \frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow 24\left[\frac{1}{18-x} - \frac{1}{18+x}\right] = 1 \Rightarrow 24\left[\frac{18+x-(18-x)}{(18-x).(18+x)}\right] = 1$$

$$\Rightarrow 24\left[\frac{2x}{(18-x).(18+x)}\right] = 1 \Rightarrow 24\left[\frac{2x}{(18-x).(18+x)}\right] = 1$$

$48x = 324 - x^2 \Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow (x + 54)\ (x - 6) = 0 \Rightarrow x = -54$ or 6

As speed to stream can never be negative, the speed of the stream is 6 km/hr.

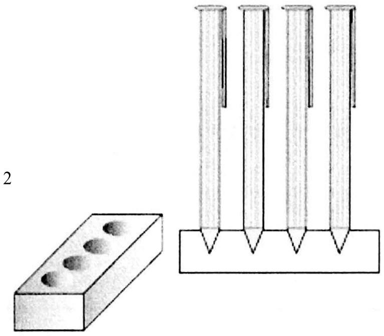
33. Figure

Given, To prove, constructions

Proof

Application

34. Volume of one conical depression = $\frac{1}{3} \times \pi r^2 h$



$= \frac{1}{3} \times \frac{22}{7} \times 0.5^2 \times 1.4 cm^3 = 0.366 \ cm^3$

Volume of 4 conical depression = 4 × 0.366 cm^2

= 1.464 cm^2

Volume of cuboidal box = L × B × H

= 15 × 10 × 3.5 cm^3 = 525 cm^3

Remaining volume of box = Volume of cuboidal box- Volume of 4 conical depressions

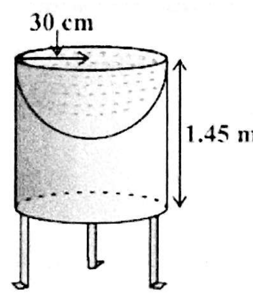
= 525 cm^3 - 1.464 cm^3 = 523.5 cm^3

[OR]

Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere.

Then, the total surface area = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi \ r(h + r)$



$= 2 \times \frac{22}{7} \times 30 \ (145 + 30) cm^2$

$= 2 \times \frac{22}{7} \times 30 \times 175 \ cm^2$

= 33000 cm^2 = 3.3 m^2

35.

Class Interval	Number of policy holders (f)	Cumulative Frequency (cf)
Below 20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

n = 100 ⇒ n/2 = 50, Therefore, median class = 35 -40,

Class size, h = 5, Lower limit of median class, l = 35,

frequency f = 33, cumulative frequency cf = 45

$$\Rightarrow \text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$\Rightarrow \text{Median} = 35 + \left[\frac{50 - 45}{33}\right] \times 5$$

$$= 35 + \frac{25}{33} = 33 + 0.76$$

= 35.76

Therefore, median age is 35.76 years

SECTION E

36. I. Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, ...years will form an AP.

So, $a + 3d = 1800$ & $a + 7d = 2600$

So, $d = 200$ & $a = 1200$

II. $t_{12} = a + 11\,d \Rightarrow t_{30} = 1200 + 11 \times 200$

$\Rightarrow t_{12} = 3400$

III. $S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2 \times 1200 + (10-1) \times 200]$

$\Rightarrow S_{10} = \frac{13}{2}[2x1200 + 9 \times 200]$

$\Rightarrow S_{10} = 5 \times [2400 + 1800]$

$\Rightarrow S_{10} = 5 \times 4200 = 21000$

[OR]

Let in n years the production will reach to 31200

$S_n = \frac{n}{2}[2a + (n - 1)d] = 31200 \Rightarrow \frac{n}{2}[2x\ 1200 + (n - 1)200] = 31200$

$\Rightarrow \frac{n}{2}[2 \times 1200 + (n - 1)200] = 31200 \Rightarrow n\,[12 + (n - 1)] = 312$

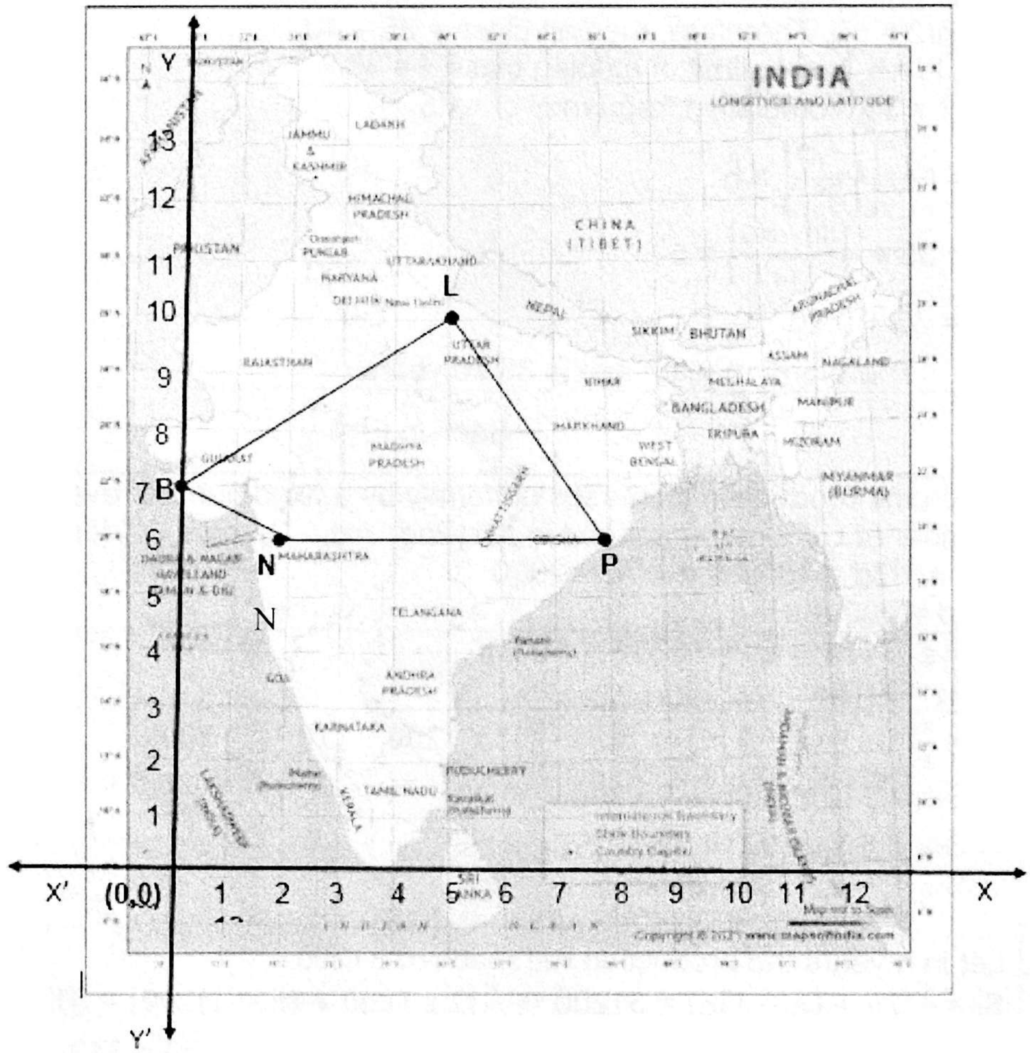
$\Rightarrow n^2 + 11n - 312 = 0$

$\Rightarrow n^2 + 24n - 13n - 312 = 0$

$\Rightarrow (n + 24)(n - 13) = 0$

$\Rightarrow n = 13$ or $- 24$. As n can't be negative. So $n = 13$

37. **Case Study - 2**



I. LB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow \text{LB} = \sqrt{(0-5)^2 + (7-10)^2}$

LB = $\sqrt{(5)^2 + (3)^2} \Rightarrow \text{LB} = \sqrt{25+9} \Rightarrow \text{LB} = \sqrt{34}$

Hence the distance is $150\sqrt{34}$ km

II. Coordinate of Kota (K) is $\left(\frac{3\times5+2\times0}{3+2}, \frac{3\times7+2\times10}{3+2}\right)$

$= \left(\frac{15+0}{5}, \frac{21+20}{5}\right) = \left(3, \frac{41}{5}\right)$

III. L(5, 10), N(2,6), P(8,6)

LN = $\sqrt{(2-5)^2+(6-10)^2} = \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

NP = $\sqrt{(8-2)^2+(6-6)^2} = \sqrt{(4)^2+(0)^2} = 4$

PL = $\sqrt{(8-5)^2+(6-10)^2} = \sqrt{(3)^2+(4)^2} \Rightarrow LB = \sqrt{9+16} = \sqrt{25} = 5$

as LN = PL ≠ NP, so ΔLNP is an isosceles triangle.

[OR]

Let A (0, b) be a point on the y - axis then AL = AP

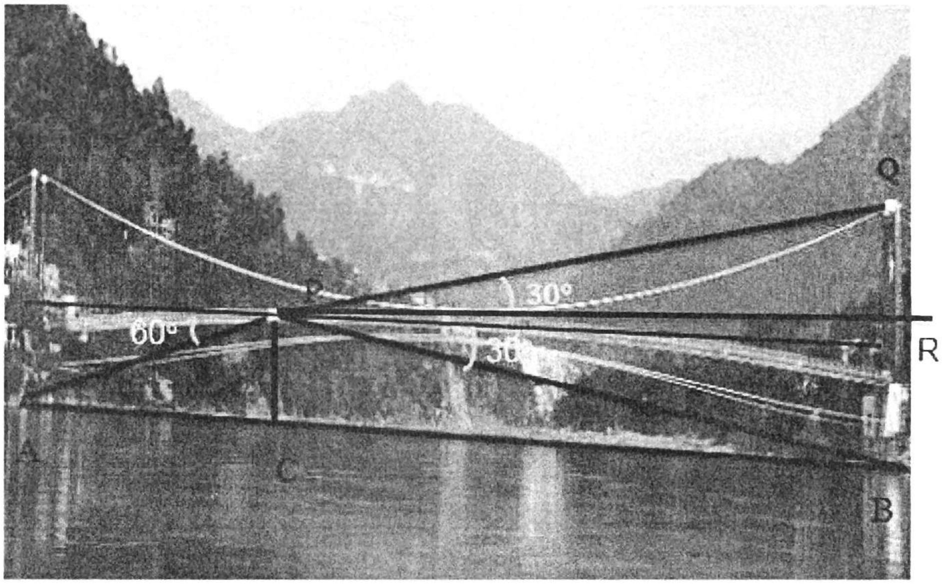
$\Rightarrow \sqrt{(5-0)^2+(10-b)^2} = \sqrt{(8-0)^2+(6-b)^2}$

$\Rightarrow (5)^2+(10-b)^2 = (8)^2+(6-b)^2$

$\Rightarrow 25 + 100 - 20b + b^2 = 64 + 36 - 12b + b^2 \Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$

So, the coordinate on y axis is $\left(0, \frac{25}{8}\right)$

38. **Case Study - 3**



I. $\sin 60° = \frac{PC}{PA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = 12\sqrt{3}\text{m}$

II. $\sin 30° = \frac{PC}{PB}$

$\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36\text{m}$

III. $\tan 60° = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC} \Rightarrow AC = 6\sqrt{3}\text{m}$

$\tan 30° = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB} \Rightarrow CB = 18\sqrt{3}\text{m}$

Width AB = AC + CB = $6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}$ m

[OR]

RB = PC = 18 m & PR = CB = 18 $\sqrt{3}$ m

$$\tan 30^{o} = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}} \Rightarrow QR = 18m$$

QB = QR + RB = 18 + 18 = 36m. Hence height BQ is 36m

PRACTICE PAPER - III

Class - X Session 2022-23

Subject - Mathematics

Time : 3 Hours	**Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section **A** has 20 Muhiple Choice Questions (MCQs) carrying 1 mark each.
3. Section **B** has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section **C** has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section **D** has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section **E** has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Questions of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each

1. If 'z' is a prime number, then the LCM of z, z^2, z^3

   (a) z^2 (b) z^3 (c) z^4 (d) z^5

2. The LCM of two numbers is 1200. Which of the following cannot be their HCF?

   (a) 200 (b) 400

   (c) 500 (d) 600

3. If 2 is a zero of the polynomial $3x^2 + 4x + 2k$, then the value of 'k' is:

(a) 10 (b) 20 (c) -20 (d) -10

4. Graphically, the pair of equations given by

$3x - 2y + 10 = 0$ and $9x - 6y + 30 = 0$

represents two lines which are

(a) intersecting at exactly one point

(b) parallel

(c) coincident

(d) intersecting at exactly two points

5. If the quadratic equation $3x^2 + 4x + 2k = 0$ has equal roots, then

(a) $k = \frac{2}{3}$ (b) $k = -4$ (c) $k = 4$ (d) $k = \frac{3}{2}$

6. In two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then:

(a) $\Delta PQR \sim \Delta CAB$ (b) $\Delta PQR \sim \Delta ABC$

(c) $\Delta PQR \sim \Delta CBA$ (d) $\Delta PQR \sim \Delta BCA$

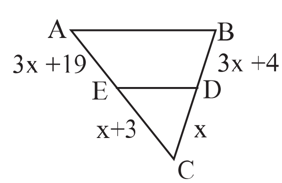
7. The perimeter of a triangle with vertices (0, - 4), (0, 0) and (- 3, 0) is

(a) 5 units (b) 12 units

(c) 11 units (d) $(7 + \sqrt{5})$ units

8. In the given figure, the value of 'x' for which DE||AB is:

(a) 1 (b) 2 (c) 3 (d) 4



9. PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is:

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$

10. If ΔABC is right angled at C, then the value of $\cos(A + B)$ is:

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

11. $\frac{1+\tan^2 A}{1+\cot^2 A}$ is equal to:

(a) $\sec^2 A$ (b) $\cot^2 A$ (c) $\text{cosec}^2 A$ (d) $\tan^2 A$

12. If $\sqrt{3}\tan\theta = 3\sin\theta$, $(\theta \neq 0)$, then the value of $\sin^2\theta - \cos^2\theta$ is:

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

13. The perimeter of the sector of a circle of radius 14 cm is 68 cm. The area of this sector is:

(a) 140 cm^2 (b) 280 cm^2 (c) 340cm^2 (d) 680cm^2

14. The median and mode of a frequency distribution are 26 and 29 respectively. The mean of this data is:

(a) 24.5 (b) 25.5 (c) 26.5 (d) 27.7

15. The area of three adjacent faces of a cube is 'x', 'y' and 'z' respectively. Its volume 'V is:

(a) xyz (b) $\sqrt{xyz}$

(c) $x\sqrt{yz}$ (d) $y\sqrt{zx}$

16. R and r ($R > r$) are radii of two concentric circles. The area of the region between the bigger and smaller circles is:

(a) $\pi(R-r)$ (b) $\pi(R^3 - r^3)$

(c) $\pi(R^2 + r^2)$ (d) $\pi(R^2 - r^2)$

17. For the following frequency distribution,the upper limit of median class is:

Class	1-6	6-11	11-16	16-21	21-26
Frequency	13	10	15	8	11

(a) 11 (b) 16 (c) 21 (d) 26

18. In a family of 3 children, the probability of having atleast one boy is:

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{8}$

Direction for questions 19 & 20: In question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

19. **Assertion:** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. is 162.

Reason: If a and b are two positive integers, then H.C.F. × L.C.M. = a × b

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

20. **Assertion:** Centroid of a triangle formed by the points (a, b), (b, c) and (c, a) is at origin, Then $a + b + c = 0$.

Reason: Centroid of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

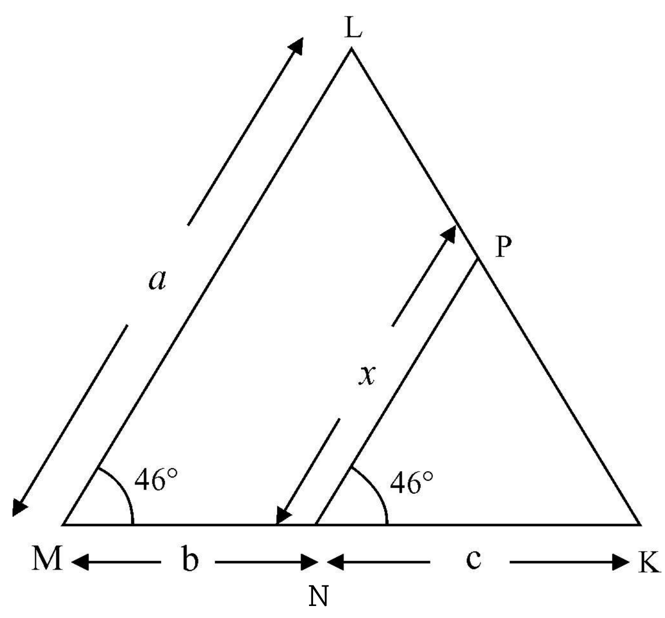
(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Section B

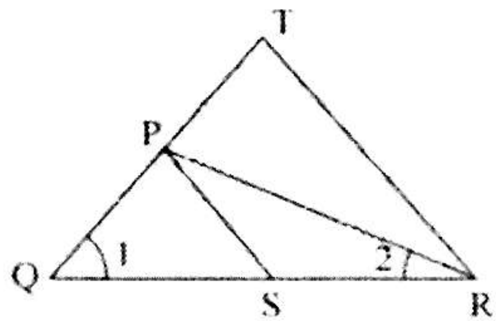
Section B consists of 5 questions of 2 marks each.

21. In the given figure, $\angle M = \angle N = 46°$, Express 'x' in terms of a, b and c.



OR

In the given figure, $\frac{QR}{QT} = \frac{QS}{PR}$ and $\angle 1 = \angle 2$ then prove that $\Delta PQS \sim \Delta TQR$

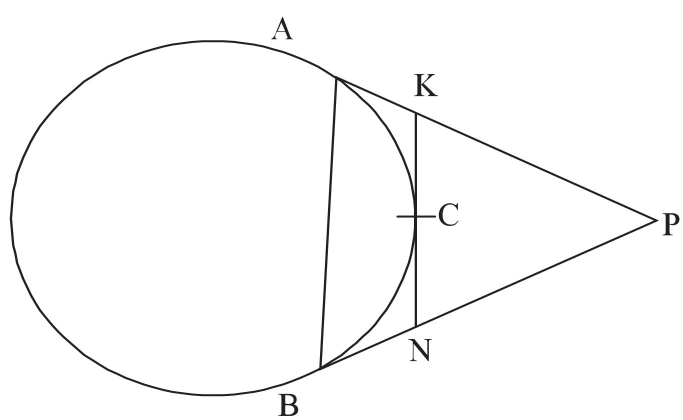


22. For what value of k, the following system of equations will be inconsistent
$kx + 3y = k - 3$

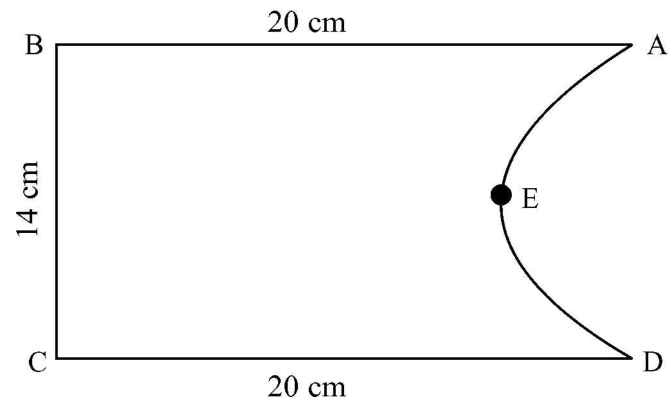
$12x + ky = k$

23. If $\sin (A - B) = \frac{1}{2}$, $\cos (A + B) = \frac{1}{2}$ then find the value of A and B.

24. In the given figure KN, PA and PB are tangents to the circle. Prove that: KN = AK + BN.

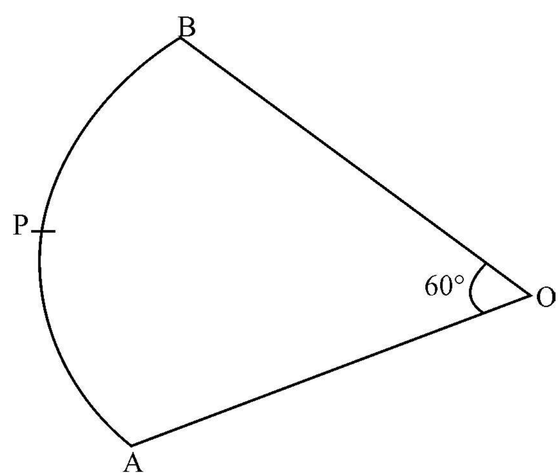


25. Find the perimeter of the given figure, where AED is a semicircle and ABCD is a rectangle.



OR

In figure, OAPBO is a sector of a circle of radius 10.5 cm. Find the perimeter of the sector.



Section C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{3}$ is an irrational number.

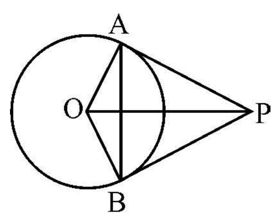
27. Obtain zeroes of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

28. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹400. If he had sold the first lot at the rate of ₹1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹460. Find the total number of bananas he had.

OR

A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket costs ₹2530. One reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket.

29. In the given figure, OP is equal to the diameter of the circle with centre O. Prove that ΔABP is an equilateral triangle.



30. If tan θ + sin θ = m, tan θ - sin θ = n, then prove that $m^2 - n^2 = 4\sqrt{mn}$.

OR

Prove $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \sec^2 A + \tan^2 A - 2\sec A \tan A$

31. There are hundred cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card is:

(i) divisible by 9 and is a perfect square.

(ii) a prime number greater than 80.

(iii) a two digit number.

SECTION-D

Section D consists of 4 questions of 5 marks each.

32. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

OR

A piece of cloth costs ₹200. If the piece was 5 m longer and each metre of cloth costs ₹2 less, then the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

33. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

A circle is drawn inside a right angled triangle whose sides are a, b, c where c is the hypotenuse, which touches all the sides of the triangle. Using the above theorem, prove that $= \dfrac{a+b-c}{2}$, where r is the radius of the circle.

34. The difference between outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long is 88 cm^2. If the volume of the metal used in making the cylinder is 176 cm^2. Find the outer and inner diameters of the cylinder.

OR

A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of 80 per m^2.

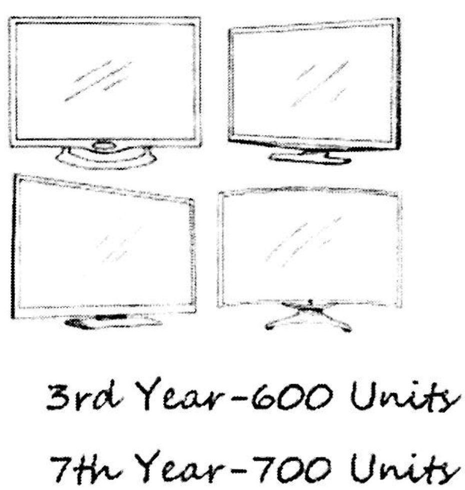
35. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

SECTION-E

Case study based questions are compulsory

36. **Case study -1**



During the summers of 200, Tanya thought of starting some business of her own and lent some money from her father and started a TV manufacturing company. After some years, she was known as one of the leading manufacturers in her area and kept expanding her limit year by year.

Assuming that the production increases uniformly year by year, the number of TV sets produced by her in the third year was 600 units and in the seventh year it was 700.

Based on the above information answer the following questions:

(i) What was the gradual increase in manufacture per year?

(ii) What was the production in the first year?

(iii) What is the total production till seven years?

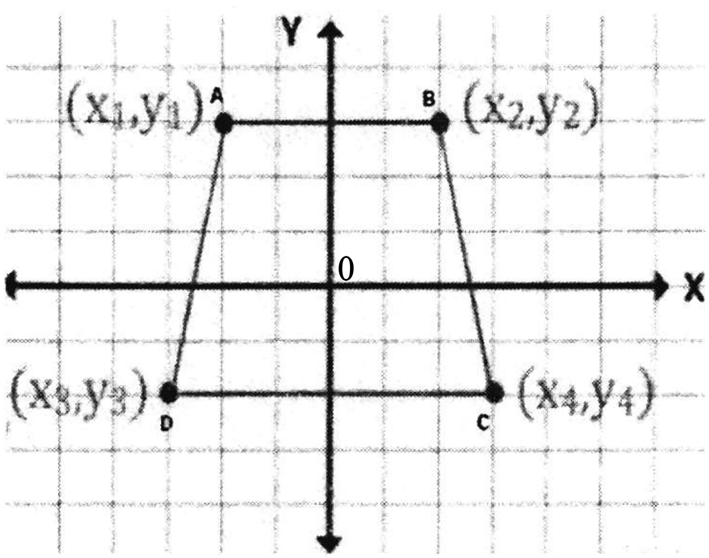
OR

What is the difference in production in fourth year and tenth year?

37. **Case study - 2**

A music album is kept on the table, assuming its top in the Cartesian system as shown in the figure.

Based on the above information answer the following questions:



(i) Find the ratio in which the x-axis divides the side AD.

(ii) Find the area of the top of music album.

(iii) Are the points A and C collinear with the origin? Justify.

OR

Is the point C equidistant from B and D? Justify.

38. **Case study - 3**

Sita and Rita were running on a race track. The finish line was marked with a tall rod. Sita and Rita were standing 'd' metres apart from each other. The angle of elevation of the top of the tower was 30° and 60° respectively for Sita and Rita. The height of the rod was 10m.

Based on the above information answer the following questions:

(i) Draw a neat labelled figure to show the above situation diagrammatically.

(ii) What is the distance 'd'?

(iii) What is the distance of Rita from the rod?

OR

What is the distance of Sita from the rod?

ANSWERS

1. (b) z^3
2. (c) 500
3. (d) -10
4. (c) coincident
5. (a) $k = \frac{2}{3}$
6. (a) $\Delta PQR \sim \Delta CAB$
7. (b) 12 units
8. (b) 2
9. (b) 45°
10. (a) 0
11. (a) $\tan^2 A$
12. (a) $\frac{1}{3}$
13. (b) 280 cm^2
14. (a) 24.5
15. (b) $\sqrt{xyz}$
16. (d) $\pi(R^2-r^2)$
17. (b) 16
18. (d) $\frac{7}{8}$
19. (d) Assertion (A) is false but Reason (R) is true.
20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B

21. $x = \frac{ac}{b+c}$

OR

Correct proof

22. k = -6

23. A = 45°, B = 15°

24. Correct proof

25. Perimeter of ΔAED =76 cm

OR

Perimeter of sector = 32 cm

SECTION-C

26. Correct Proof

27. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

28. Total bananas = 500.

OR

Full first class fare is ₹ 2500 and reservation charge is ₹30.

29. Correct Proof

30. Correct Proof

31. (i) $\frac{3}{100} = 0.03$ (ii) $\frac{3}{100} = 0.03$ (iii) $\frac{90}{100} = 0.9$

SECTION-D

32. Duration of flight = 1 hr.

OR

Length of piece = 20 m, Rate per meter = ₹ 10

33. Correct proof

34. Outer and inner diameter is 5 cm and 3 cm respectively.

OR

Cost of cloth = ₹82720

35. 35.76

SECTION-E

36. (i) 25 units (ii) 550 units (iii) 4375 units OR 150 units

37. (i) 3:2 (ii) 25 sq units

(iii) A and C are not collinear with the origin

OR

B and D are not equidistant from C

38. (i) Correct diagram,

(ii) $\frac{20}{\sqrt{3}}$ m or $\frac{20\sqrt{3}}{3}$ m

(iii) $\frac{10}{\sqrt{3}}$ m or $\frac{10\sqrt{3}}{3}$ m

OR

$10\sqrt{3}$ m

Practice Paper-IV

Class- X Session- 2022-23

Subject- Mathematics

Time : 3 Hours **Max. Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section **A** has 20 Muhiple Choice Questions (MCQs) carrying 1 mark each.
3. Section **B** has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section **C** has 6 Short Answer-Il (SA-II) type questions carrying 3 marks each.
5. Section **D** has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section **E** has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Questions of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION-A

Section A consists of 20 questions of 1 mark each

1. How many maximum zeroes can a polynomial of degree n have?

   (a) n + 1 (b) n - 1 (c) n (d) 2n

2. 9^{th} term of an A.P. is 499 and 499^{th} term is 9. The term which is equal to zero is:

   (a) 507^{th} (b) 508^{th} (c) 509^{th} (d) 510^{th}

3 LCM of the given number 'x' and 'y' where 'y' is a multiple of 'x' is given by:

(a) x (b) y (c) xy (d) $\frac{x}{y}$

4 The point which divides the line segment joining the points (5,-8) and (3,4) in the ratio 2:3 internally lies in :

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

5. The quadratic polynomial whose sum of zeroes is 3 and product of zeroes is -2 is :

(a) $x^2 + 3x-2$ (b) $x^2-3x + 2$ (c) x^2-3x-2 (d) x^2-2x+3

6. In triangles ABC and PQR, $\angle B = \angle Q$, $\angle C = \angle R$ and AB = 2 PQ. Then the two triangles are :

(a) Congruent but not similar (b) Neither congruent nor similar

(c) Similar but not congruent (d) Congruent as well as similar

7. Which of the following is not a measure of central tendency?

(a) Mean (b) Median (c) Mode (d) Ogive

8. The value of m for which the pair of equations mx - y = 2 and 6x - 2y - 4 = 0 will have infinitely many solutions is :

(a) -5 (b) 3 (c) -3 (d) 6

9. LCM of two numbers is 2400, which of the following cannot be their HCF?

(a) 800 (b) 300 (c) 600 (d) 700

10. The quadratic equation $kx^2 - 2kx + 2 = 0$ has equal roots if k =

(a) 1 (b) 2 (c) 4 (d) 0

11. If the angle between two radii of the circle is 150°, the angle between the tangents at the ends of the radii is:

(a) 60° (b) 30° (c) 210° (d) 50°

12. If sec x + tan x = p , then sec x - tan x is:

(a) p^2 (b) $\frac{1}{p}$ (c) p^3 (d) $\frac{p}{2}$

13. The coordinates of the point where the line 2x - 4y = 8 cuts x-axis are:

(a) (0, 4) (b) (-4, 0) (c) (0,0) (d) (4,0)

14. Two equilateral triangles can be proved similar by using:

(a) AAA similarity (b) SSS similarity

(c) SAS similarity (d) All of these

15. The mean of 17 numbers is 23. The new mean if each observation is decreased by 3, is:

(a) 14 (b) 51 (c) 20 (d) 69

16. A point P is 15cm away from the centre of a circle. The length of the tangent drawn from P to the circle is 12cm. The radius of the circle is:

(a) 3cm (b) 5cm (c) 9cm (d) 17cm

17. In triangle ABC, if DE is parallel to BC, AD = x, DB = x - 2, AE = x + 2 and EC = x - 1, then the value of x is:

(a) 4 (b) 3 (c) 5 (d) 3.5

18. What is the probability that two friends have birthday on 29"'February?

(a) $\frac{1}{366}$ (b) $\frac{1}{365}$ (c) $\frac{2}{365}$ (d) $\frac{1}{183}$

Direction for questions 19 & 20: In question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

19. **Assertion:** If HCF of 510 and 92 is 2, then the LCM of 510 & 92 is 32460.

Reason: as H.C.F. x L.C.M. = a × b

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

20. **Statement Assertion (A):** If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D (3, 5) and E (- 3, - 3) respectively, then BC = 20 units.

Statement Reason (R): The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

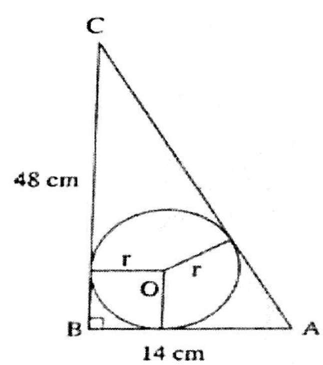
(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

SECTION-B

Section B consists of 5 questions of 2 marks each.

21. In the given figure, ABC is a triangle in which $\angle B = 90°$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius r of in-circle. (In cm)



Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

22. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$, prove that $x^2 + y^2 = 25$.

23. If -3 is one of the zeroes of the polynomial $(k-l)x^2 + kx + l$, then find the value of k.

OR

If α, β are zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, find lind the value of p.

24. Two cubes have their volumes in the ratio 1:27. Find the ratio of their surface areas.

25. Solve the following pair of linear equations:

    $y - 4x = 1$; $6x - 5y = 9$

SECTION-C

Section C consists of 6 questions of 3 marks each.

26. In an A.P., the sum of first ten terms is -150 and the sum of its next ten terms is -550. Find the A.P.

27. If 4 tan x = 3, evaluate $\dfrac{(4 \sin x - \cos x + 1)}{(4 \sin x + \cos x - 1)}$

OR

Prove that: $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.

28. A chord PQ of a circle of radius 10cm subtends an angle of 60° at the centre of the circle. Find the area of major segment and major sector of the circle.

29. A box contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:

    (i) An odd number

    (ii) A perfect square

    (iii)An even prime number

30. Determine the value of 'm' and 'n' so that the following pair of linear equations has infinite number of solutions.

    $(2m - 1)x + 3y = 5$; $3x + (n - l)y = 2$

OR

The area of rectangle reduces by 160 m^2 if its length is increased by 5m and breadth is reduced by 4m. However, if length is decreased by 10m and breadth is increased by 2m, then its area is decreased by $100m^2$. Find the dimensions of the rectangle.

31. A triangle ABC is drawn to circumscribe a circle of radius 2 m such that the segments BD and DC into which BC is divided by the point of contact D are the lengths 4 cm and 3 cm respectively. If the area of triangle ABC is 21 cm^2, then find the lengths of sides AB and AC.

SECTION-D

Section D consists of 4 questions of 5 marks each.

32. Literacy rates of 40 cities are given in the following table. If it is given that mean literacy rate is 63.5, then find the missing frequencies x and y.

Literacy rate (in%)	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
No. of cities	1	2	3	x	y	6	8	4	2	3	2

33. A bird is sitting on the top of a 80m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird.

OR

At a point A, 20m above the level of water in a pond, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the pond, at A is 60°. Find the distance of the cloud from A.

34. State and prove Basic Proportionality Theorem. Using the result prove that in triangle ABC, if DE is drawn parallel to BC then $\frac{DB}{AB} = \frac{CE}{AC}$.

35. To fill a pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find how long it would take for each pipe to fill the pool separately, if the pipe of larger diameter takes 10 hours less than the pipe of smaller diameter to fill the pool.

OR

Solve : $2\left\{\frac{(2x-3)}{(x+3)}\right\}-3\left\{\frac{(x+3)}{(2n-1)}\right\} = 5; x \neq -3, ½$

SECTION-E

Case study based questions are compulsory.

36. Metro pillars are being constructed in cuboidal and cylindrical shapes. In case of cuboidal pillars the base constructed is a square of side 4m and in case of cylindrical pillars the base constructed is a circle of radius 4m. The height of the pillars is 28m at a particular place.

On the basis of above given information answer the following questions:

(i) Find the volume of concrete used in building cuboidal pillar.

(ii) Find the volume concrete used in building cylindrical pillar.

(iii) An advertising agency decides to use these pillars for advertisement. If the rate of advertising is ₹ 4 per m^2, then find the difference in amount for the advertisement on these pillars?

OR

Find the cost of concrete used in cylindrical pillar if the cost of concrete is ₹ 100 per m^3.

37. To boost the reading skills of students, school select two students to set up a reading comer in a library. There are two classes for which the facility is started having 28 students and 36 students.

(i) Find the minimum number of books required for each class, so that they can be distributed equally among students of both classes?

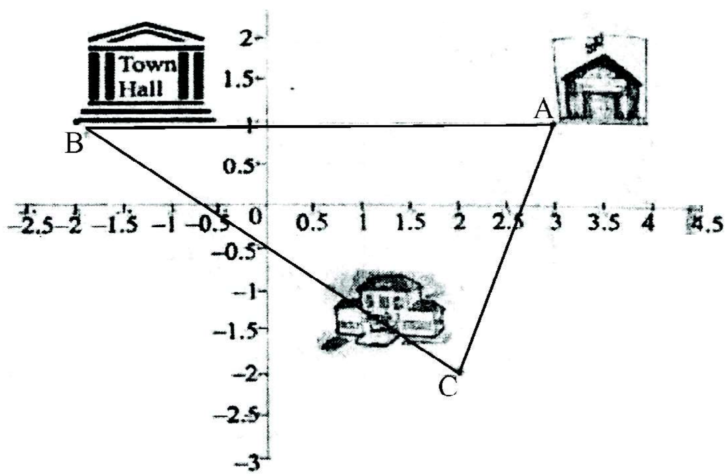
(ii) Write the prime factorisation of 28×36

(iii) Taking this as an example show that LCM × HCF = Product of two numbers.

OR

How many books do each student of both classes get?

38. The diagram shows the map where the library is location at point A, city hall building at B and the school at C.



(i) What are the coordinates of city hall building?

(ii) What are the coordinates of the mid-point of the segment joining school and library?

(iii) What is the distance between library and city hall building?

OR

What is the area of ΔABC?

ANSWERS

1. (C) n
2. (B) 508^{th}
3. (B) y
4. (D) IV quadrant
5. (C) $x^2 - 3x - 2$
6. (C) Similar but not congruent
7. (D) Ogive
8. (B) 3
9. (D) 700
10. (B) 2
11. (B) $30°$
12. (B) $\frac{1}{p}$
13. (D) (4,0)
14. (D) All
15. (C) 20
16. (C) 9 cm
17. (A) 4
18. (A) $\frac{1}{366}$
19. (D) Assertion (A) is false but Reason (R) is true.
20. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
21. r = 6cm

    OR

    Correct proof
22. Correct proof
23. $k = \frac{4}{3}$

    OR

    $P = \frac{2}{3}$
24. 1:9
25. x = 1 ; y = 5
26. $S_{10} = -150$

    $S_{20} - S_{10} = -550$

    a = 3, d= -4

    A.P.: 3, -1, -5, -9
27. $\frac{13}{11}$

    OR

    Correct proof

28. Major segment = 305.20 cm^2; Major sector = 261.90 cm^2 or 261.60 cm^2

29. (i) $\frac{25}{49}$ (ii) $\frac{1}{7}$ (iii) $\frac{1}{49}$

30. $m = \frac{17}{4}$; $n = \frac{11}{5}$

OR

Length = 60m and Breadth = 20m

31. AB = 7.5 cm ; AC = 6.5 cm

32. x = 5 and y = 4

33. Speed of bird = 29.28 m/s

OR

Distance of cloud = 40m

34. Correct proof

35. Larger pipe takes 20 hours and smaller pipe takes 30 hours to fill the pool.

OR

$x = \frac{-1}{5}, -10$

36. (i) 448m^3 (ii) 1408 m^3

(iii) ₹ 1024

OR

₹ 140800

37. (i) 252 (ii) $2^4 \times 3^2 \times 7$

(iii) correct proof

OR

9 books for 28 and 7 books for 36 students

38. (i) (-2, 1) (ii) $\left(\frac{5}{2}, \frac{-1}{2}\right)$

(iii) 5 units

OR

7.5 sq units

Notes

Notes