DIRECTORATE OF EDUCATION
Govt. of NCT of Delhi

SUPPORT MATERIAL
(2024-2025)

Class : X

MATHEMATICS
(English Medium)

Under the Guidance of

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Secretary (Education)

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सत्यमेव जयते

DE.5/228/Exam/Message/SM/2018/555
Dated:- 01/07/2024

MESSAGE

In the profound words of Dr. Sarvepalli Radhakrishnan, **"The true teachers are those who help us think for ourselves."**

Every year, our teams of subject experts shoulder the responsibility of updating the Support Material to synchronize it with the latest changes introduced by CBSE. This continuous effort is aimed at empowering students with innovative approaches and techniques, thereby fostering their problem-solving skills and critical thinking abilities.

I am confident that this year will be no exception, and the Support Material will greatly contribute to our students' academic success.

The development of the support material is a testament to the unwavering dedication of our team of subject experts. It has been designed with the firm belief that its thoughtful and intelligent utilization will undoubtedly elevate the standards of learning and continue to empower our students to excel in their examinations.

I wish to extend my heartfelt congratulations to the entire team for their invaluable contribution in creating this immensely helpful resource for our students.

Wishing all our students a promising and bright future brimming with success.

(ASHOK KUMAR)

R.N. SHARMA, IAS
Director, Education & Sports

सत्यमेव जयते

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DE.5/228/Exam/Message/SM/2018/576
Dated. 04/07/2024

MESSAGE

It brings me great pleasure to present the support material specifically designed for students of classes IX to XII by our dedicated team of subject experts. The Directorate of Education remains resolute in its commitment to empower educators and students alike, extending these invaluable resources at no cost to students attending Government and Government-Aided schools in Delhi.

The support material epitomizes a commendable endeavour towards harmonizing content with the latest CBSE patterns, serving as a facilitative tool for comprehending, acquiring and honing essential skills and competencies stipulated within the curriculum.

Embedded within this initiative is a structured framework conducive to nurturing an analytical approach to learning and problem-solving. It is intended to prompt educators to reflect upon their pedagogical methodologies, forging an interactive conduit between students and academic content.

In the insightful words of Rabindranath Tagore, **"Don't limit a child to your own learning, for he was born in another time."**

Every child is unique, with their own interests, abilities and potential. By allowing children to learn beyond the scope of our own experiences, we support their individual growth and development, helping them to reach their full potential in their own right.

May every student embrace the joy of learning and be empowered with the tools and confidence to navigate and shape the future.

(R. N. SHARMA)

Dr. RITA SHARMA
Additional Director of Education
(School/Exam)

सत्यमेव जयते

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D.O. No. DE.5/228/Exam/Message/SM/2018/570
Dated: 02/07/2024

MESSAGE

"Children are not things to be molded, but are people to be unfolded." - Jess Lair

In line with this insightful quote, the Directorate of Education, Delhi, has always made persistent efforts to nurture and unfold the inherent potential within each student. This support material is a testimony to this commitment.

The support material serves as a comprehensive tool to facilitate a deeper understanding of the curriculum. It is crafted to help students not only grasp essential concepts but also apply them effectively in their examinations. We believe that the thoughtful and intelligent utilization of these resources will significantly enhance the learning experience and academic performance of our students.

Our expert faculty members have dedicated themselves to the support material to reflect the latest CBSE guidelines and changes. This continuous effort aims to empower students with innovative approaches, fostering their problem-solving skills and critical thinking abilities.

I extend my heartfelt congratulations to the entire team for their invaluable contribution to creating a highly beneficial and practical support material. Their commitment to excellence ensures that our students are well-prepared to meet the challenges of the CBSE examinations and beyond.

Wishing you all success and fulfilment in your educational journey.

(Dr. Rita Sharma)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2024-2025)

MATHEMATICS
Class : X

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1]**[संपूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य]** बनाने के लिए, तथा उसके समस्त नागरिकों को :

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की **समता**

प्राप्त कराने के लिए,

तथा उन **सब में**

व्यक्ति की गरिमा और [2]**[राष्ट्र की एकता**

और अखंडता] सुनिश्चित करने वाली **बंधुता**

बढ़ाने के लिए

दृढ़संकल्प होकर **अपनी इस संविधान सभा में** आज तारीख 26 नवंबर, 1949 ई. को **एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।**

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य'' के स्थान पर प्रतिस्थापित।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''राष्ट्र की एकता'' के स्थान पर प्रतिस्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51 क

मूल कर्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की संप्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण बनाए रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभावों से परे हो, ऐसी प्रथाओं का त्याग करे जो महिलाओं के सम्मान के विरुद्ध हों;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की, जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणिमात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे, जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई ऊँचाइयों को छू सके; और

(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य को शिक्षा के अवसर प्रदान करे।

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

Team Members for Review of Support Material

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5.	Mr. Abadhesh Kumar Singh Lecturer	Sarvodaya Co-Ed Vidyalaya, Mukhmel Pur, Delhi
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SESSION-(2024-2025)
CLASS-X
Subject: Mathematics (Code: 041 & 241)

Course Structure

Units	Unit Name	Marks
I	Number Systems	06
II	Algebra	20
III	Coordinate Geometry	06
IV	Geometry	15
V	Trigonometry	12
VI	Mensuration	10
VII	Statistics and Probability	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

I. REAL NUMBER

Fundamental Theorem of Arithmetic - statement after reviewing work done earlier and after illustrating and motivating through examples. Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (Internal division).

UNIT IV: GEOMETRY

1. TRIANGLES

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY

Trigonometic ratios of an actue angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation /depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES

Area of sectors and segments of a circle. Problems based on areas and perimeter/circumference of the above said plane figures. In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY

Classical definition of probability. Simple problems on finding the probability of an event.

CLASS-X (2024-25)

Time: 3 Hours | ***Max. Marks: 80***

S.No.	Typology of Questions	Total Marks	% Weightage (approx)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	60	75
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	12	15
3	**Analysing:** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	8	10
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5 + 5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Standard

QUESTION PAPER DESIGN
CLASS-X (2024-25)

Time: 3 Hours — ***Max. Marks: 80***

S.No.	Typology of Questions	Total Marks	% Weightage (approx)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analysing:** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	18	22
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5 + 5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

Content

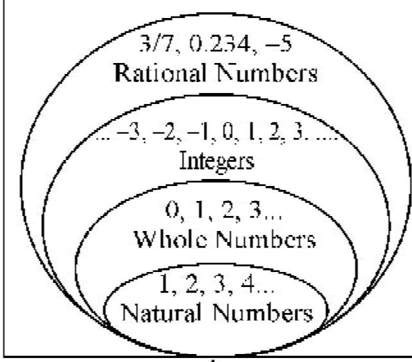
CHAPTER

1 Real Numbers

Decimal form of Real Numbers

RATIONAL NUMBERS

A number that can be expressed in the form of p/q where 'p' and 'q' are co-prime integers and $q \neq 0$, is called a rational number. Their decimal representation is either terminating or non-terminating and recurring (repeating).



IRRATIONAL NUMBERS

A number that cannot be expressed in the form p/q where 'p' and 'q' are co-prime integers, is called an irrational number. Their decimal representation is non-terminating and non-recurring (non-repeating).

(e.g. $\sqrt{3}, \sqrt{2} + \sqrt{5}, \pi, 0.102003102...$)

REAL NUMBERS

FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite number can be written as a product of primes in one and only one way, apart from the order in which the primes are written.

HCF USING PRIME FACTORIZATION

HCF is the product of the smallest power of each common prime factor of the given numbers.

LCM USING PRIME FACTORIZATION

LCM is the product of the greatest power of each prime factor of the given numbers.

RELATIONSHIP BETWEEN HCF AND LCM OF TWO NUMBERS

HCF is always a factor of the LCM of two numbers.

If 'a' and 'b' are two numbers, then

HCF (a, b) × LCM (a, b) = Product of 'a' and 'b'

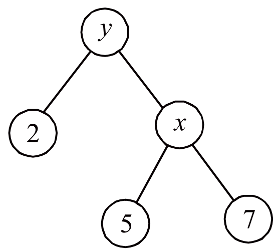
VERY SHORT ANSWER TYPE QUESTIONS

1. A number N when divided by 16 gives the remainder 5. ______ is the remainder when the same number is divided by 8.

2. HCF of $3^3 \times 5^4$ and $3^4 \times 5^2$ is ________ .

3. If $a = xy^2$ and $b = x^3y^5$ where x and y are prime numbers then LCM of (a, b) is ______ .

4. In the given factor tree, find x and y



5. If n is a natural number, then $25^{2n} - 9^{2n}$ is always divisible by :

(a) 16 (b) 34

(c) both 16 or 34 (d) None of these

6. Given HCF (2520, 6600) = 120 and LCM (2520, 6600) is 252k, then value of 'k' is

(a) 165 (b) 550

(c) 990 (d) 1650

7. The product of HCF and LCM of the smallest prime number and the smallest composite number is

(a) 2 (b) 4

(c) 6 (d) 8

8. If the LCM of two numbers is 3600, then which of the following cannot be their HCF?

(a) 600 (b) 500

(c) 400 (d) 150

9. The ratio of HCF and LCM of the least prime number and the least composite number is:

(a) 1:2 (b) 2:1

(c) 1:3 (d) 1:1

10. The greatest number which divides both 30 and 80, leaving. remainder 2 and 3 respectively is:

(a) 10 (b) 7

(c) 14 (d) 11

11. All decimal numbers are

(a) rational numbers (b) irrational numbers

(c) real numbers (d) integers

12. Which of these numbers always end with the digits 6.

(a) 4^n (b) 2^n (c) 6^n (d) 8^n

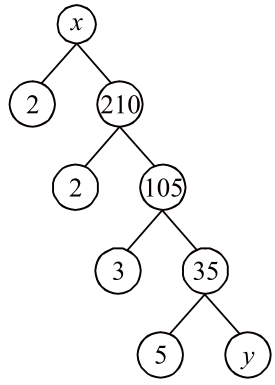
13. Write the prime factor of $2 \times 7 \times 11 \times 13 \times 17 + 21$

14. Write the form in which every odd integer can be written taking t as variable.

15. Find the least number which is divisible by all numbers from 1 to 10 (both inclusive).

16. The numbers 525 and 3000 are divisible by 3, 5, 15, 25 and 75. What is the HCF of 525 and 3000?

17. What is $x : y$ in the given factor-tree?



SHORT ANSWER TYPE QUESTIONS-I

18. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

19. What is the smallest number by which $\sqrt{5}-\sqrt{3}$ is to be multiplied to make it a rational number? Also find the number so obtained?

20. Find one rational number and one irrational number between $\sqrt{2}$ and $\sqrt{5}$.

21. If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of m.

(CBSE 2014)

22. Find the value of : $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$, where n is any positive odd integer. **(CBSE 2016)**

23. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times. **(CBSE 2016)**

SHORT ANSWER TYPE QUESTIONS-II

24. Express 2658 as a product of its prime factors.

25. If $7560 = 2^3 \times 3^p \times q \times 7$, find p and q.

26. Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.

27. Prove that $5-\frac{3}{7}\sqrt{3}$ is an irrational number.

28. Prove that $\frac{1}{2-\sqrt{5}}$ is an irrational number.

29. Find HCF and LCM of 56 and 112 by prime factorization method.

30. Explain why:

(i) $7 \times 11 \times 13 \times 15 + 15$ is a composite number

(ii) $11 \times 13 \times 17 + 17$ is a composite number.

(iii) $1 \times 2 \times 3 \times 5 \times 7 + 3 \times 7$ is a composite number.

31. On a morning walk, three persons steps off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

32. During a sale, colour pencils were being sold in the pack of 24 each and crayons in the pack of 32 each. If you want full packs of both and the same number of pencils and crayons, how many packets of each would you need to buy? **(CBSE : 2017)**

33. Find the largest number that divides 31 and 99 leaving remainder 5 and 8 respectively. **(CBSE 2017)**

34. The HCF of 65 and 117 is expressible in the form 65 m – 117. Find the value of m. Also find the LCM of 65 and 117 using prime factorisation method.

35. Find HCF and LCM of 26, 65 and 117 using prime factorisation.

36. Find the HCF of 180, 252 and 324.

37. Find the greatest number of six digits exactly divisible by 18, 24 and 36.

38. Three bells ring at intervals of 9, 12, 15 minutes respectively. If they start ringing together at a time, after how much time will they next ring together?

39. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm respectively. Find the length of the longest rod that can measure the three dimensions of the room exactly.

40. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of two given number. **(CBSE 2018)**

LONG ANSWER TYPE QUESTIONS

41. Find the HCF of 56, 96, 324 by prime factorization.

42. What will be the least possible number of the planks, if three pieces of timber 42 m. 49 m, and 63 m long have to be divided into planks of the same length?

43. Amit, Sunita and Sumit start preparing cards for all the persons in an old age home. In order to complete one card, they take 10, 16 and 20 minutes respectively. If they all started together, after what time will they begin preparing a new card together?

44. Aakriti decided to distribute milk in an orphanage on her birthday. The supplier brought two milk containers which contain 398 *l* and 436 *l* of milk. The milk is to be transferred to another containers so that 7 *l* and 11 *l* of milk is left in both the containers respectively. What will be the maximum capacity of the drum?

45. Find the smallest number, which when increased by 17, is exactly divisible by both 520 and 468.

46. A street shopkeeper prepares 396 Gulab jamuns and 342 ras-gullas. He packs them, in combination. Each containter consists of either gulab jamuns or ras-gulla but have equal number of pieces. Find the number of pieces he should put in each box so that number of boxes are least. How many boxes will be packed in all. **(CBSE 2016)**

47. Find the number nearest to 110000 but greater than 1 lakh, which is exactly divisible by 8, 15, 21.

48. In a seminar, the number. of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of the them being of the same subject.

49. State Fundamental Theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

50. Find the smallest number which when increased by 20 is exactly divisible by 90 and 144. Is LCM, a multiple of 144?

51. If the HCF of 1032 and 408 is expressible in the form $1032\,p - 408 \times 5$, find p.

52. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one of the number is 280. Find the other number.

ANSWERS AND HINTS

1. 5

2. $3^3 \times 5^2$

3. $x^3 \times y^5$

4. $x = 35, y = 70$

5. (*c*) $25^{2n} - 9^{2n}$ is of the form $a^{2n} - b^{2n}$ which is divisible by both $a - b$ and $a + b$ so, by both $25 + 9 = 34$ and $25 - 9 = 16$.

6. (b) 550

7. (d) 8

8. (b) 500

9. (a) 1:2

10. (b) 7

11. (c) real numbers

12. (c) 6^n

13. 7

14. $2t + 1$ or $2t - 1$

15. 2520

16. 75

17. 60 : 1

18. As 12 has factors 2, 2, 3. It doesnot has 5 as its factor so 12^n will never end with 0 or 5.

19. $\sqrt{5} + \sqrt{3}$, 2

21. HCF of 180 and 144 is 36.

$$13m - 3 = 36$$
$$13\ m = 39$$
$$m = 3$$

22. Given that n is a positive odd integer

$\Rightarrow$ $2n$ and $4n + 2$ are even positive integers and n and $2n + 1$ are odd positive integers.

$\therefore$ $(-1)^n = -1, (-1)^{2n} = +1, (-1)^{2n+1} = -1, (-1)^{2n+2} = +1$

$\therefore$ $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2} = -1 + 1 - 1 + 1 = 0$

23. HCF of 850 and 680 is $2 \times 5 \times 17 = 170$ litres.

24. $2658 = 2 \times 3 \times 443$

25. $p = 3$ and $q = 5$

26. Prove that $\sqrt{3}$ and $\sqrt{5}$ is irrational number separately. Sum of two irrational number is an irrational number.

27. 5 is rational no. and $\frac{3}{7}\sqrt{3}$ is an irrational number. Difference of a rational number and irrational number is an irrational number.

29. HCF = 56, LCM = 112

30. (i) $15 \times (7 \times 11 \times 13 + 1)$ as it has more than two factors so it is composite no. Similarly for part (ii) and (iii)

31. LCM of 40, 42, 45 = 2520

Minimum distance each should walk 2520 cm.

32. LCM of 24 and 32 is 96

96 crayons or $\frac{96}{32}$ = 3 packs of crayons

96 pencils or $\frac{96}{24}$ = 4 packs of pencils.

33. Given number = 31 and 99

$31 - 5 = 26$ and $99 - 8 = 91$

Prime factors of $26 = 2 \times 13$

$91 = 7 \times 13$

HCF of (26, 91) = 13.

$\therefore$ 13 is the largest number which divides 31 and 99 leaving remainder 5 and 8 respectively.

34. HCF (117, 52) = 13.

Given that 65 m – 117 = 13 $\Rightarrow$ 65 m = 130 $\Rightarrow$ $m = 2$.

LCM (65, 117) = $13 \times 3^2 \times 5 = 585$

35. HCF = 13

LCM = 1170

36. HCF (324, 252, 180) = 36

37. LCM of (18, 24, 36) = 72.

Greatest six digit number = 999999

72) 999999 (13888
– 72
279
– 216
639
– 576
639
– 576
639
– 576
63

Require six digit number

999999
– 63
999936

38. LCM of (9, 12, 15) = 180 minutes.

39. HCF of 8m 25 cm, 6m 75 cm and 4 m 50 cm = 75 cm

40. HCF (404, 96) = 4

LCM (404, 96) = 9696

$$\text{HCF} \times \text{LCM} = 38,784$$

Also, $404 \times 96 = 38,784$

41. HCF (56, 96, 324) = 4

42. HCF of 42m, 49m and 63 m = 7 m

Number of planks = $\frac{42}{7} + \frac{49}{7} + \frac{63}{7} = 6 + 7 + 9 = 22$

43. LCM of 10, 16 and 20 minutes = 80 minutes

44. 17

45. 4663

LCM of (468, 520) = 4680

$\therefore$ Required no. = 4680 – 17 = 4663

46. HCF (396, 342) = 18

No. of boxes = $\frac{396+342}{18} = 41$

47. 109200

48. HCF of 60, 84 and 108 is $2^2 \times 3 = 12$ = No. of participants in each row.

$$\text{No. of rooms required} = \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12} = 21 \text{ rooms}$$

49. HCF =24, LCM = 540

$$\frac{\text{LCM}}{\text{HCF}} = \frac{540}{24} = 22.5, \text{ not an integer.}$$

Hence two numbers cannot have HCF and LCM as 24 and 540 respectively.

50. [The LCM of (90, 144) – 20] = Required No.

$\Rightarrow$ Required No. = 700

51. $p = 2$

52. HCF = 40, LCM = 560

$\therefore$ Other No. = 80.

PRACTICE-TEST

Real Number

Time : 45 Minutes *M.M. : 20*

SECTION A

1. Check whether $17 \times 19 \times 21 \times 23 + 17$ is a composite number. 1

2. What is the LCM of the smallest 2 digit number and the smallest composite number? 1

3. Find the HCF of x^4y^5 and x^8y^3. 1

4. Find the LCM of 14 and 122 . 1

SECTION B

5. Show that 9^n can never ends with unit digit zero. 2

6. Find the pairs of the natural numbers whose least common multiple is 78 and the greatest divisor is 13.

7. Find prime factors of 7650 using factor tree.

2

SECTION C

8. Prove that $3 - 2\sqrt{5}$ is an irrational number. 3

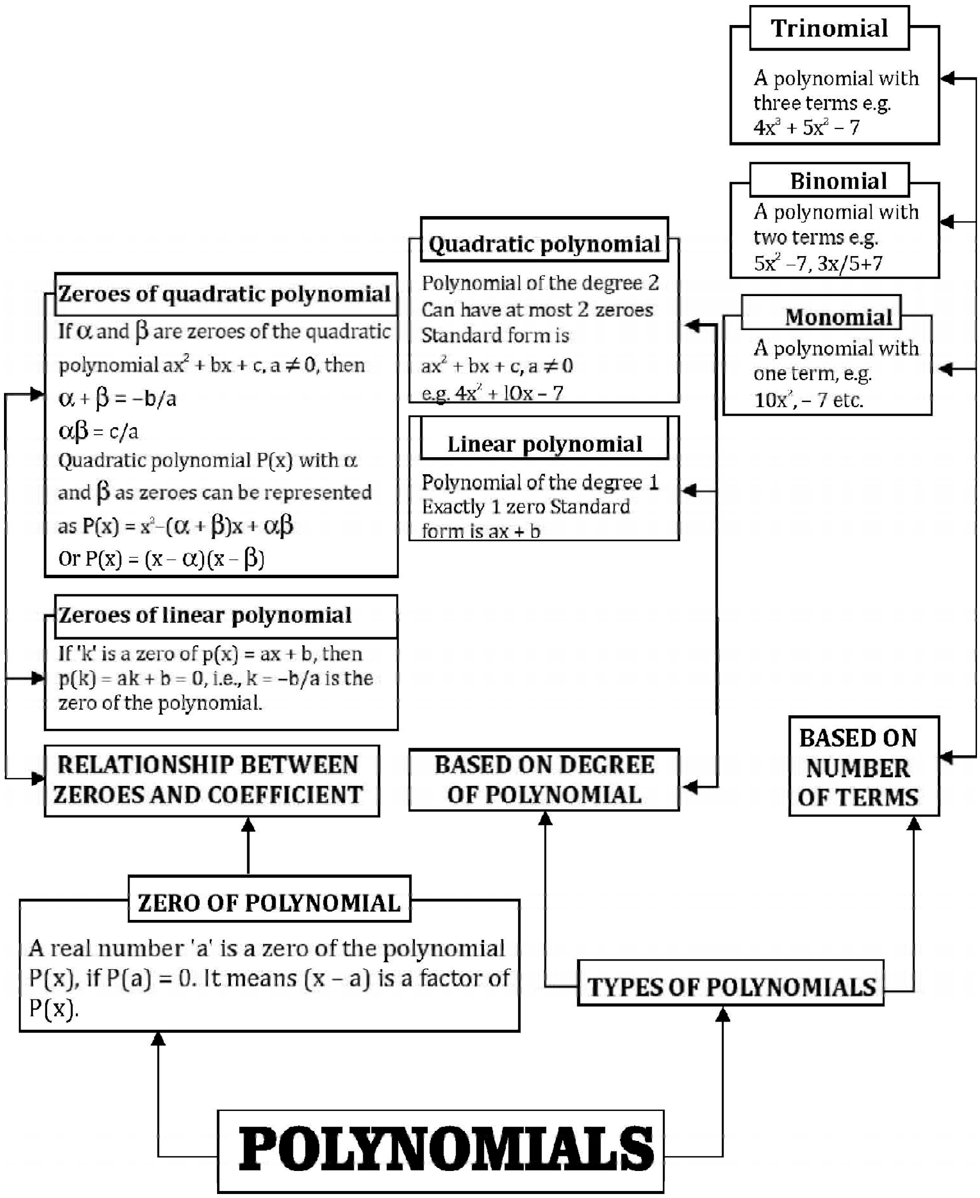
9. Find the HCF of 36, 96 and 120 by prime factorization. 3

SECTION D

10. Once a sports goods retailer organized a campaign "Run to remember" to spread awareness about benefits of walking. In that Soham and Baani participated. There was a circular path around a sports field. Soham took 12 minutes to drive one round of the field, while Baani took 18 minutes for the same. Suppose they started at the same point and at the same time and went in the same direction. After how many minutes have they met again at the starting point? 4

CHAPTER 2

Polynomials



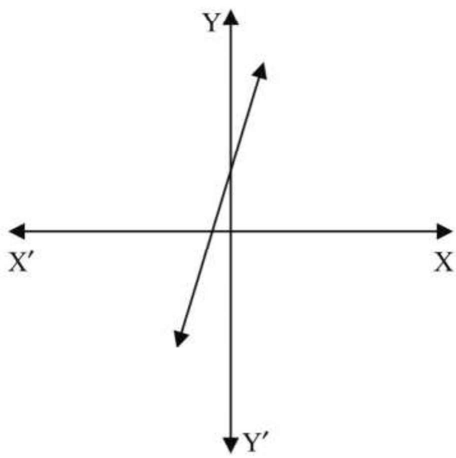
POLYNOMIALS

If 'x' is a variable, 'n' is a natural number, $a_0, a_1, a_2, a_3, \ldots$ are real numbers then, $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots\ldots\ldots\ldots a_1 x + a_0 (n \neq 0)$ is called a polynomial in 'x'.

GRAPH OF A POLYNOMIAL

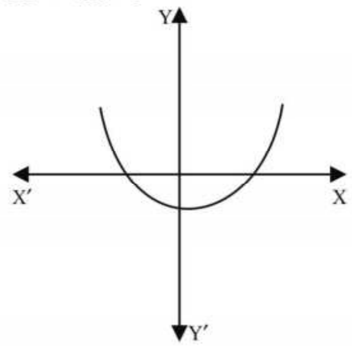
GEOMETRICAL REPRESENTATION OF A LINEAR POLYNOMIAL

Graph of a linear polynomial $P(x) = ax + b, a \neq 0$ is a straight line cutting x– axis exactly at one point.

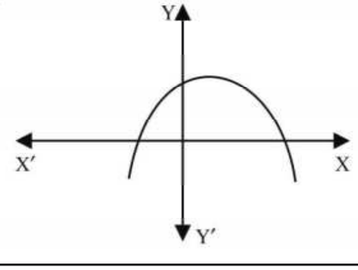


GEOMETRICAL REPRESENTATION OF A QUADRATIC POLYNOMIAL

Graph of a quadratic polynomial $P(x) = ax^2 + bx + c, a \neq 0$, is a parabola open upwards, if $a > 0$.
e.g. $5x^2 + 4x + 1$



Graph of a quadratic polynomial $P(x) = ax^2 + bx + c, a \neq 0$, is a parabola open downwards, if $a < 0$.
e.g. $-x^2 + 7x + 1$



In general a polynomial P(x) of degree 'n' crosses the x-axis at most 'n' points.

VERY SHORT ANSWER TYPE QUESTIONS

1. If one zero of the polynomial $P(x) = 5x^2 + 13x + k$ is reciprocal of the other, then value of k is

(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

2. If α and β are then zeroes of the polynomial $p(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, then $c =$ _______ .

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10 (c) 5 (d) -5

4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

5. What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial.

(a) 1 (b) 2 (c) 4 (d) 5

6. If α and β are the zeroes of the polynomial $f(x) = x^2 + x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$ __.

7. The number of polynomials having zeroes -3 and 5 is

(a) Only one (b) Infinite (c) Exactly two (d) at most two

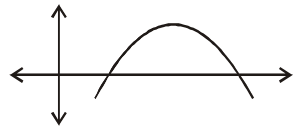
8. If α and β are the zeroes of the polynomial $x^2 - 1$, then the value of $(\alpha + \beta)$ is:

(a) 2 (b) 1 (c) -1 (d) 0

9. Which of the following is a quadratic polynomial having zeroes $\frac{-2}{3}$ and $\frac{2}{3}$?

(a) $4x^2 - 9$ (b) $\frac{4}{9}(9x^2 + 4)$ (c) $x^2 + \frac{9}{4}$ (d) $5(9x^2 - 4)$

10. The quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ is represented by this graph then a is



(a) Natural no. (b) Whole no. (c) Negative Integer (d) Irrational no.

11. If 1 is one zero of the polynomial $p(x)=ax^2-3(a-1)x-1$, then find the value of 'a'.

12. Find the quadratic polynomial whose zeroes are $\left(5+2\sqrt{3}\right)$ and $\left(5-2\sqrt{3}\right)$

13. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)\,x - 9$ is negative of the other, then find the values of k.

14. What number should be subtracted to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained?

15. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

16. What will be the number of real zeroes of the polynomial $x^2 + 1$?

17. If α and β are zeroes of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeroes are 2α and 2β

18. If α and $\frac{1}{\alpha}$ are zeroes of $4x^2 - 17x + k - 4$, find the value of k.

19. What will be the number of zeroes of the polynomials whose graphs are parallel to (i) y-axis (ii) x-axis?

20. What will be the number of zeroes of the polynomials whose graphs are either touching or intersecting the axis only at the points:

(*i*) (–3, 0), (0, 2) & (3, 0) (*ii*) (0, 4), (0, 0) & (0, –4)

SHORT ANSWER TYPE (I) QUESTIONS

21. For what value of k, $x^2 - 4x + k$ touches x-axis?

22. If the product of zeroes of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeroes.

23. If zeroes of $x^2 - kx + 6$ are in the ratio 3 : 2, find k.

24. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k.

25. If α and β are the zeroes of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m.

26. If the sum of squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, find the value of k.

27. If α and β are zeroes of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. If α and β are zeroes of the polynomial $2x^2 + 7x + 5$, then find $(\alpha - \beta)$.

29. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

(CBSE, 2012)

30. Find a quadratic polynomial whose zeroes are $\frac{3+\sqrt{5}}{5}$ and $\frac{3-\sqrt{5}}{5}$.

(CBSE, 2013)

SHORT ANSWER TYPE (II) QUESTIONS

31. Find the zeroes of the polynomial $x^2 - 3x - m(m+3)$

32. Obtain zeroes of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

33. Form a quadratic polynomial, whose one zero is 8 and the product of zeroes is –56.

34. –5 is one of the zeroes of $2x^2 + px - 15$, and zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k.

35. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeroes as half of their product.

36. If zeroes of the polynomial $ax^2 + bx - c$, $a \neq 0$ are additive inverse of each other then what is the value of b?

37. If α and β are zeroes of $x^2 - x - 2$, find a polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$

38. If α, β are zeroes of the quadratic polynomial $2x^2 + 5x + k$, then find the value of 'k' such that $(\alpha + \beta)^2 - \alpha\beta = 24$.

39. If one zero of the polynomial $2x^2 - 3x + p$ is 3, find the other zero and the value of 'p'.

40. Find a quadratic polynomial, whose zeroes are in the ratio 2 : 3 and their sum is 15.

LONG ANSWER TYPE QUESTIONS

41. If $(x + a)$ is a factor of two quadratic polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = (n - q)/(m - p)$

42. If one zero of the quadratic polynomial $4x^2 - 8kx + 8x - 9$ is the negative of the other, then find the zeroes of $kx^2 + 3kx + 2$?

43. If α, β are zeroes of the quadratic polynomial $x^2 - 5x - 3$, then form a polynomial whose zeroes are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$.

44. If one zero of the polynomial $(k + 1)x^2 - 5x + 5$ is multiplicative inverse of the other, then find the zeroes of $kx^2 - 3kx + 9$.

45. If the product of the zeroes of the quadratic polynomial $kx^2 + 11x + 42$ is 7, then find the zeroes of the polynomial $(k - 4)x^2 + (k + 1)x + 5$.

46. If α and β are zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1+\dfrac{\beta}{\alpha}$ and $1+\dfrac{\alpha}{\beta}$.

47. Form a quadratic polynomial one of whose zero is $2+\sqrt{5}$ and sum of the zeroes is 4.

48. Form a polynomial whose zeroes are the reciprocal of the zeroes of $p(x) = ax^2 + bx + c$, $a \neq 0$.

49. If $(x + 2)$ is a factor of $x^2 + px + 2q$ and $p + q = 4$ then what are the values of p and q?

50. If sum of the zeroes of $5x^2 + (p + q + r)x + pqr$ is zero, then find $p^3 + q^3 + r^3$.

51. If the zeroes of $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$ find p and q.

ANSWERS AND HINTS

1. (b) 5

2. 1

3. (b) –10

4. (d) a = 0, b = –6

5. (b) 2

6. – 1

7. (b) Infinite

8. (d) $\alpha + \beta = 0$

9. (d) $5(9x^2 - 4)$

10. (c) Negative Integer

11. a = 1

12. $x^2 - 10x + 13$

13. $k = 0, 5$

14. (-2)

15. (i) 2 (ii) 0

16. 0

17. $k\,[3x^2 - 7x - 6]$

18. $k = 8$

19. (i) 1 (ii) 0

20. (i) 2 (ii) 1

21. 4

22. $a = -\frac{3}{2}$, sum of zeroes $= -4$

23. $-5, 5$

24. 5

25. 6

26. 12

27. $4t^2 + t - 1$

28. $\alpha - \beta = \pm\frac{3}{2}$

29. $\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn} = \frac{\left(-\frac{11}{3}\right)^2 - 2\left(-\frac{4}{3}\right)}{-\frac{4}{3}} = -\frac{145}{12}$

30. $\alpha + \beta = \frac{6}{5}, \quad \alpha\beta = \frac{4}{25},$
$k\,(25x^2 - 30x + 4)$

31. m+3, –m

32. $-\frac{2}{\sqrt{3}}, \quad \frac{\sqrt{3}}{4}$

33. $\alpha\beta = -56$ and $\beta = -7$
so, $\alpha = 8$, Now $\alpha + \beta = 1$
Required polynomial is $x^2 - x - 56$

34. $\frac{7}{4}$

35. 1

36. $b = 0$

37. $x^2 - 4x - 5$

38. $(\alpha + \beta) = -\frac{5}{2}$ and $\alpha\beta = \frac{k}{2}$

Substituting the above values in $(\alpha + \beta)^2 - \alpha\beta = 24$. Solve to get 'k' $= \frac{-71}{2}$.

39. 3 is a zero, so $2(3)^2 - 3 \times 3 + p = 0$

$p = 9$, Now $\alpha\beta = \frac{c}{a}$, solve to get the other zero $\frac{-3}{2}$.

40. $\alpha : \beta = 2:3$. So $\alpha = 2\beta/3$

Using $(\alpha + \beta) = 15$, solve to get α and β as 9 and 6 respectively.

Required polynomial is $x^2 - 15x + 54$

41. Since $(x + 2)$ is a factor of $x^2 + px + q$

$(-a)^2 - ap + q = 0$

$(-a)^2 = ap - q$ ....(1)

Similarly from $x^2 + mx + n$

$(a)^2 = am - n$ .... (2)

Comparing equatin (1) and (2)

$a = (n - q)/(m - p)$

42. $f(x) = 4x^2 + (8-8k)x - 9$

$(\alpha + \beta) = -(8-8k)/4$

$k = 1$

Substitute $k = 1$ and solve for $x = -2$ and -1

43. For given polynomial, $(\alpha + \beta) = 5$, $\alpha\beta = -3$

For Required polynomial

$$
\begin{aligned}
\text{Sum of zeroes} &= (2\alpha + 3\beta) + (3\alpha + 2\beta) \\
&= 5(a + p) \\
&= 25
\end{aligned}
$$

$$
\begin{aligned}
\text{Product of zeroes} &= (2\alpha + 3\beta)(3\alpha + 2\beta) \\
&= 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6(\alpha^2 + \beta^2) + 13\alpha\beta \\
&= 6\,[(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta \\
&= 147
\end{aligned}
$$

Required polynomial is $x^2 - 25x + 147$

44. $f(x) = (k + 1)x^2 - 5x + 5$

$(\alpha\beta) = 1$

$5/(k + 1) = 1$

$k = 4$

Substituting $k = 4$ in $kx^2 - 3kx + 9$ solve to get zeroes $x = 3/2$ and $3/2$

45. $f(x) = kx^2 + 11x + 42$

$(\alpha\beta) = 7$

$k = 6$

Substituting $k = 6$ in $(k - 4)x^2 + (k + 1)x + 5$, solve to get zeroes $x = -1$ and $x = -5/2$

46. $x^2 - \frac{16}{3}x + \frac{16}{3}$ or $\frac{1}{3}(3x^2 - 16x + 16)$

47. $\alpha + \beta = 4$

$(2 + \sqrt{5}) + \beta = 4$

$\beta = 2 - \sqrt{5}$

$\alpha\beta = -1 \quad \therefore$ Polynomial $= k[x^2 - 4x - 1]$

48. $k\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right]$

49. $p = 3$, $q = 1$

50. Product of the zeroes $= 3\,pqr$

51. $p = -5$ and $q = -6$

PRACTICE-TEST

Polynomials

Time : 45 Minutes | *M.M. : 20*

SECTION- A

1. If α and β are zeroes of a quadratic polynomial $p(x)$, then factorize $p(x)$. **1**

2. If α and β are zeroes of $x^2 - x - 1$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$. **1**

3. If one of the zeroes of quadratic polynomial $(k-1)\,x^2 + kx + 1$ is -3 then the value of k is, **1**

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

4. A quadratic polynomial, whose zeroes are – 3 and 4, is **1**

(*a*) $x^2 - x + 12$ (b) $x^2 + x + 12$

(*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

SECTION-B

5. If α and β are zeroes of $x^2 - (k + 6)x + 2(2k - 1)$. find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **2**

6. Find a quadratic polynomial one of whose zeroes is $(3+\sqrt{2})$ and the sum of its zeroes is 6. **2**

7. If zeroes of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$ then find the value of a. **2**

SECTION-C

8. If α and β are zeroes of the polynomial $p(s) = 3s^2 - 6s + 4$, then find the value of $\alpha/\beta + \beta/\alpha + 2(1/\alpha + 1/\beta) + 3\alpha\beta$ **3**

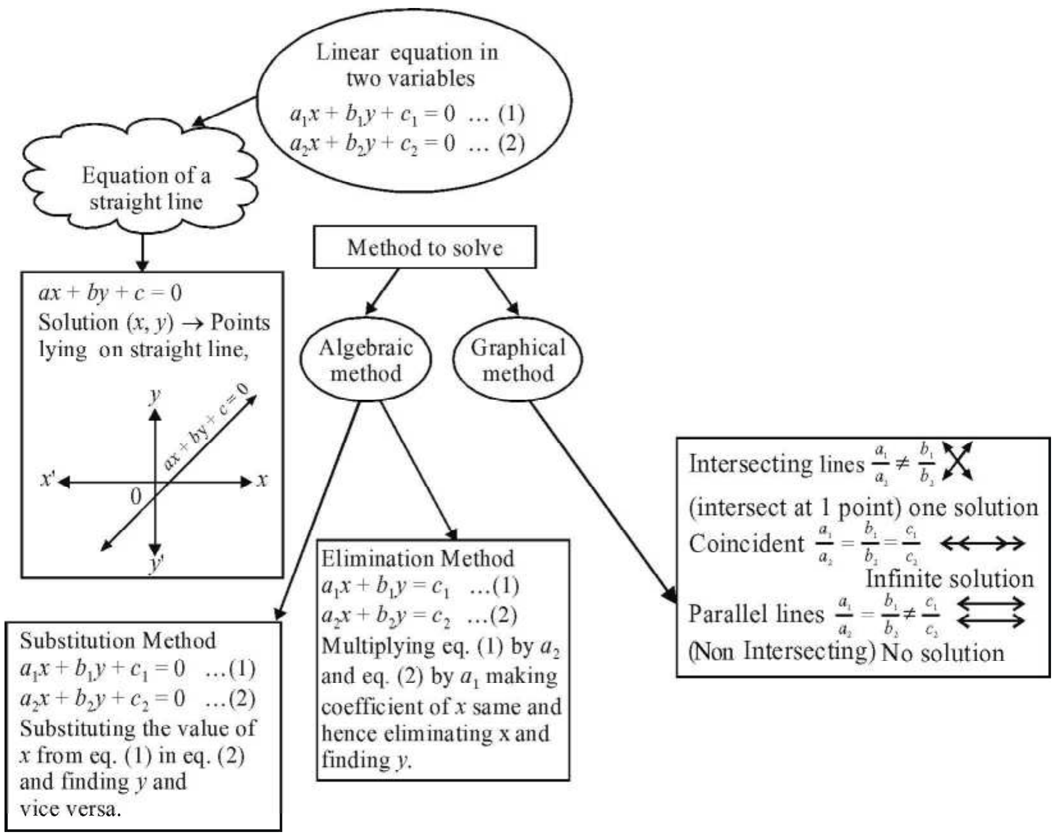
9. If truth and lie are zeroes of the polynomial $px^2 + qx + r$, $(p \neq 0)$ and zeroes are reciprocal to each other, Find the relation between p and r. **3**

SECTION-D

10. Find the zeroes of the polynomial $\sqrt{3}x^2 + 10x + 7\sqrt{3}$. Also verify the relationship between the zeroes and their coefficients. **4**

CHAPTER

3 Pair of Linear Equations in Two Variables



VERY SHORT ANSWER TYPE QUESTIONS

1. If the lines given by $3x + 2ky = 2$ and $2x + 5y = 1$ are parallel, then the value of k is _______ .

2. If $x = a$ and $y = b$ is the solution of the equation $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively _______ .

3. A pair of linear equations which has a unique solution $x = 2$ and $y = -3$ is

(a) $x + y = 1$ and $2x - 3y = -5$

(b) $2x + 5y = -11$ and $2x - 3y = -22$

(c) $2x + 5y = -11$ and $4x + 10y = -22$

(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

4. The area of the triangle formed by the lines $x = 3$, $y = 4$ and $x = y$ is _____ .

5. The value of k for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has a non-zero solutions is ____ .

6. If a pair of linear equations in two variables is consistent, then the lines represented by two equations are:

(a) Intersecting (b) Parallel

(c) always coincident (d) intersecting or coincident

7. For $2x + 3y = 4$, y can be written in terms of x as _______ .

8. One of the common solution of $ax + by = c$ and y axis is

(a) $\left(0, \frac{c}{b}\right)$ (b) $\left(0, \frac{b}{c}\right)$

(c) $\left(\frac{c}{b}, 0\right)$ (d) $\left(0, -\frac{c}{b}\right)$

9. If $ax + by = c$ and $lx + my = n$ has unique solution then the relation between the coefficient will be:

(a) $am \neq lb$ (b) $am = lb$ (c) $ab = lm$ (d) $ab \neq lm$

10. In ΔABC, $\angle C = 3\angle B$, $\angle C = 2(\angle A + \angle B)$ then, $\angle A$, $\angle B$, $\angle C$ are respectively.

(a) 30°, 60°, 90° (b) 20°, 40°, 120°

(c) 45°, 45°, 90° (d) 110°, 40°, 50°

11. If $x = 3m - 1$ and $y = 4$ is a solution of the equation $x + y = 6$, then find the value of m.

12. What is the point of intersection of the line represented by $3x - 2y = 6$ and the y-axis?

13. For what value of p, system of equations $2x + py = 8$ and $x + y = 6$ have no solution?

14. A motor cyclist is moving along the line $x - y = 2$ and another motor cyclist is moving along the line $x - y = 4$ find out their moving direction.

15. Find the value of k for which pair of linear equations $3x + 2y = -5$ and $x - ky = 2$ has a unique solution.

16. Write the solution of $y = x$ and $y = -x$.

17. If $2x + 5y = 4$, write another linear equation, so that lines represented by the pair are coincident.

18. Check whether the graph of the pair of linear equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ is intersecting lines or parallel lines.

19. What is the value of p, for which the pair of linear equations $x + y = 3$ and $3x + py = 9$ is inconsistent.

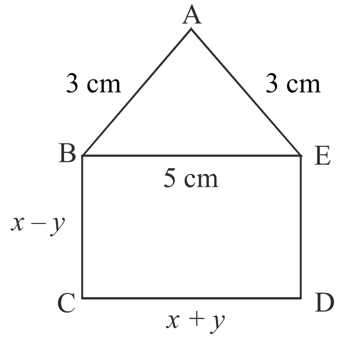
20. If we draw lines of $x = 2$ and $y = 3$, what kind of lines do we get?

SHORT ANSWER TYPE (I) QUESTIONS

21. Form a pair of linear equations for: The sum of the numerator and denominator of the fraction is 3 less than twice the denominator. If the numerator and denominator both are decreased by 1, the numerator becomes half the denominator.

22. For what value of p the pair of linear equations $(p + 2)x - (2p + 1)y = 3(2p - 1)$ and $2x - 3y = 7$ has a unique solution.

23. ABCDE is a pentagon with BE || CD and BC || DE, BC is perpendicular to CD If the perimeter of ABCDE is 21 cm, find x and y.



24. Solve for x and y

$$x - \frac{y}{2} = 3 \text{ and } \frac{x}{2} - \frac{2y}{3} = \frac{2}{3}$$

25. Solve for x and y

$3x + 2y = 11$ and $2x + 3y = 4$

Also find p if $p = 8x + 5y$

26. Solve the pair of linear equations by substitution method $x - 7y + 42 = 0$ and $x - 3y - 6 = 0$

27. Ram is walking along the line joining (1, 4) and (0, 6). Rahim is walking along the line joining (3, 4) and (1, 0). Represent on graph and find the point where both of them cross each other

28. Given the linear equation $2x + 3y - 12 = 0$, write another linear equation in these variables, such that. geometrical representation of the pair so formed is

(*i*) Parallel Lines (*ii*) Coincident Lines (*iii*) Intersecting lines.

29. The difference of two numbers is 66. If one number is four times the other, find the numbers.

30. For what value of k, the following system of equations will be inconsistent

$kx + 3y = k - 3$

$12x + ky = k$

SHORT ANSWERS TYPE (II) QUESTIONS

31. Solve graphically the pair of linear equations $5x - y = 5$ and $3x - 2y = -4$

Also find the co-ordinates of the points where these lines intersect y-axis.

32. Solve

$$\frac{x}{a} + \frac{y}{b} = a + b$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

33. For what values of a and b the following pair of linear equations have infinite number of solutions?

$$2x + 3y = 7$$

$$a(x + y) - b(x - y) = 3a + b - 2$$

34. Find the value of k for no solutions

$$(3k + 1)x + 3y - 2 = 0$$

$$(k^2 + 1)x + (k - 2)y - 5 = 0$$

35. Solve the pair of linear equations

$$152x - 378y = -74$$

$$-378x + 152y = -604$$

36. Pinky scored 40 marks in a test getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks were deducted for each wrong answer, then Pinky again would have scored 40 marks. How many questions were there in the test?

37. Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of the father.

38. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gain ₹ 2000. But if he sells the T.V. at 10% gain and fridge at 5% loss, he gains ₹ 1500 on the transaction. Find the actual price of the T.V. and the fridge

39. Sunita has some ₹ 50 and ₹ 100 notes amounting to a total of ₹ 15,500. If the total number of notes is 200, then find how many notes of ₹ 50 and ₹ 100 each, she has.

LONG ANSWER TYPE QUESTIONS

40. Solve graphically the pair of linear equations $3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$. Find the co-ordinates of vertices of triangular region formed by these lines and x-axis. Also calculate the area of this triangle.

41. A and B are two points 150 km apart on a highway. Two cars start with different speeds from A and B at same time. If they move in same direction, they meet in 15 hours. If they move in opposite direction, they meet in one hour. Find their speeds

42. The ratio of incomes of two persons A and B is 3 : 4 and the ratio of their expenditures is 5 : 7. If their savings are ₹ 15,000 annually find their annual incomes.

43. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

44. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket costs ₹ 2530. One reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket.

45. Determine graphically, the vertices of the triangle formed by the lines $y = x$, $3y = x$ and $x + y = 8$.

46. Draw the graphs of the equations x = 3, x = 5 and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x-axis.

47. Sarthak takes 3 hours more than Nishi to walk 30 km. But if Sarthak doubles his speed, he is ahead of Nishi by 1½ hours. Find their speed of walking.

48. In a two digit number, the ten's place digit is 3 times the unit's place digit. When the number is decreased by 54, digits get reversed. Find the original number.

49. A two-digit number is 3 more than 4 times the sum of the digits. If 18 is added to the number, digits reversed. Find the number.

50. Find the values of a and b for infinite solutions

(*i*) $2x - (a - 4)y = 2b + 1$

$4x - (a - 1)y = 5b - 1$

(*ii*) $2x + 3y = 7$

$2ax + ay = 28 - by$

ANSWERS AND HINTS

1. $k = \dfrac{15}{4}$

2. $a = 3$ and $b = 1$

3. (*c*) $2x + 5y = -11$ and $4x + 10y = -22$

4. $\dfrac{1}{2}$ sq. unit

5. $k \neq 6$

6. (*d*) intersecting or coincident

7. $y = \dfrac{4 - 2x}{3}$

8. (*a*) $\left(0, \dfrac{c}{b}\right)$

9. (*a*) am ≠ lb

10. (*b*) 20°, 40°, 120°

11. $m = 1$

12. (0, –3)

13. $p = 2$

14. move parallel

15. $k \neq \dfrac{-2}{3}$

16. (0, 0)

17. $4x + 10y = 8$

18. Parallel lines

19. $p = 3$

20. Intersecting lines

21. $x - y = -3,\ 2x - y = 1$

22. $p \neq 4$

23. $x = 5, y = 0$

24. 4, 2

25. $x = 5, y = -2, p = 30$

26. 42, 12

27. (2, 2)

28. (*i*) $4x + 6y + 10 = 0$

(*ii*) $4x + 6y - 24 = 0$

29. 88, 22

30. $k = -6$

31. (2, 5) (0, – 5) and (0, 2)

32. $x = a^2, y = b^2$

33. $a = 5, b = 1$

34. $k = -1$

35. 2, 1

36. 40 questions

37. 45 years

38. T.V. = ₹ 20,000 Fridge = ₹ 10,000

39. ₹ 50 notes = 90, ₹ 100 notes = 110

40. Solution (3, 3), Vertices (– 1, 0) (7, 0) and (3, 3), Area = 12 square units

41. 80 km/hr , 70 km/hr

42. ₹ 90,000, ₹ 1,20,000

43. Let the no. of bananas in lots A be x and in lots B be y

Case I : $\frac{2}{3}x + y = 400 \quad \Rightarrow \quad 2x + 3y = 1200$

Case 2 : $x + \frac{4}{5}y = 460 \quad \Rightarrow \quad 5x + 4y = 2300$

$x = 300, \quad y = 200$, Total bananas = 500.

44. Let the cost of full and half ticket be ₹ x & ₹ $\frac{x}{2}$ and reservation charge by ₹ y per ticket.

Case I : $x + y = 2530$

Case 2 : $x + y + \frac{x}{2} + y = 3810$

$x = 2500$, $y = 30$

Full first class fare is ₹ 2500 and reservation charge is ₹ 30.

45. Vertices of the triangle are (0, 0) (4, 4) (6, 2).

46. $A(3, 0)$, $B(5, 0)$

C(5, 6), $D(3, 2)$

Area of quad. ABCD $= \frac{1}{2} \times AB \times (AD + BC) = \frac{1}{2} \times 2 \times (6 + 2) = 8$ sq. units.

47. $\frac{10}{3}$ km/hr, 5 km/hr

48. 93

49. 35

50. (i) 7, 3

(ii) 4, 8

PRACTICE-TEST

Pair of Linear Equations In Two Variables

Time : 45 Minutes **M.M. : 20**

SECTION-A

1. For what value of k system of equations
$x + 2y = 3$ and $5x + ky + 7 = 0$ has a unique solution. **1**

2. Does the point (2, 3) lie on line represented by the graph of $3x - 2y = 5$. **1**

3. The pair of equations $x = a$ and $y = b$ graphically representes lines which are: **1**

(a) Parallel (b) Intersecting at (b, a)

(c) Coincident (d) Intersecting at (a, b)

4. For what value of K, the equations $3x - y + 8 = 0$ and $6x - Ky = -16$ represent coincident lines? **1**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) –2

SECTION-B

5. For what value of a and b the pair of linear equations have infinite number of solutions

$$2x - 3y = 7$$
$$ax + 3y = b$$ **2**

6. Solve for x and y

$$0.4x + 0.3y = 1.7$$
$$0.7x - 0.2y = 0.8$$ 2

7. If the system of equations $6x + 2y = 3$ and $kx + y = 2$ has a unique solution, find the value of k. **2**

SECTION-C

8. Solve for x and y

$$x + y = a + b$$
$$ax - by = a^2 - b^2$$ **3**

9. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their ages. **3**

SECTION-D

10. Solve the following pair of equations graphically.
$3x + 5y = 12$ and $3x - 5y = -18$. **4**

Also shade the region enclosed by these two lines and x-axis.

CHAPTER

4 Quadratic Equations

Quadratic Polynomial

A polynomial of the form $ax^2 + bx + c$, $a \neq 0$ is called a Quadratic polynomial, Degree of this polynomla is 2

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a Quadratic equation, in one variable x, where a, b, c are real numbers.

Roots of a Quadratic Equations

A real number α is called root of $ax^2 + bx + c = 0$, when it satisfies this equation i.e., $a\alpha^2 + b\alpha + c = 0$. A quadratic equation have at most 2 real roots.

Methods for solving Quadratic Equations

By factorisation

(a) By using identities
(b) By splitting the middle term

By Using Quadratic Formula (Given by Sridhar Acharya)

Quadratic equation $ax^2 + bx + c = 0$ has two roots α and β given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$D = b^2 - 4ac$ is called discriminant

Nature of roots

Case I : $D > 0$, $b^2 - 4ac > 0$
Roots are real and distinct

Case II : $D = 0$, $b^2 - 4ac = 0$
Roots are real and equal

Case III : $D < 0$, $b^2 - 4ac < 0$
Roots are not real

Real roots can be rational or irrational

NOTES:

1. Real and distinct roots are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. Real and equal roots are $\dfrac{-b}{2a}, \dfrac{-b}{2a}$

3. There are quadratic equation which donot have any real roots e.g. $x^2 + 1 = 0$

VERY SHORT ANSWER TYPE QUESTIONS

1. Which of the following is not a Quadratic Equation?

(*a*) $2(x-1)^2 = 4x^2 - 2x + 1$ (*b*) $3x - x^2 = x^2 + 6$

(*c*) $\left(\sqrt{3}x + \sqrt{2}\right)^2 = 2x^2 - 5x$ (*d*) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

2. Which of the following equation has 2 as a root

(*a*) $x^2 + 4 = 0$ (*b*) $x^2 - 4 = 0$

(*c*) $x^2 + 3x - 12 = 0$ (*d*) $3x^2 - 6x - 2 = 0$

3. If $\dfrac{1}{2}$ is a root of $x^2 + px - \dfrac{5}{4} = 0$ then value of p is

(*a*) 2 (*b*) –2

(*c*) $\dfrac{1}{4}$ (*d*) $\dfrac{1}{2}$

4. Every Quadratic Equation can have at most

(*a*) Three roots (*b*) One root

(*c*) Two roots (*d*) Any number of roots

5. Roots of Quadratic equation $x^2 - 7x = 0$ will be

(*a*) 7 (*b*) 0, –7

(*c*) 0, 5 (*d*) 0, 7

6. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

(*a*) 4 (*b*) ± 4

(*c*) -4 (*d*) 0 **(CBSE 2020)**

7. **Fill in the blanks:**

(*a*) If $px^2 + qx + r = 0$ has equal roots then value of r will be _______ .

(*b*) The quadratic equation $x^2 - 5x - 6 = 0$ if expressed as $(x + p)(x + q) = 0$ then value of p and q respectively are _______ and ________ .

(*c*) The value of k for which the roots of qaudratic equations $x^2 + 4x + k = 0$ are real is _______ .

(*d*) If roots of $4x^2 - 2x + c = 0$ are reciprocal of each other then the value of c is _________ .

(*e*) If in a quadratic equation $ax^2 + bx + c = 0$, value of a is zero then it become a _____ equation.

8. **Write the discriminant of the quadractic equation $(x+5)^2 = 2(5x-3)$**

9. Roots of $-x^2 + \frac{1}{2}x + \frac{1}{2} = 0$

(a) $-\frac{1}{2}, 1$ (b) $\frac{1}{2}, 1$

(c) $\frac{-1}{2}, -1$ (d) $\frac{1}{2}, \frac{-1}{2}$

SHORT ANSWER TYPE QUESTIONS-I

10. If the quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$ $(p \neq 0)$ has two equal roots then find the value of p.

11. **Solve for x by factorisation**

(*a*) $8x^2 - 22x - 21 = 0$

(*b*) $3\sqrt{5}x^2 + 25x + 10\sqrt{5} = 0$

(*c*) $2x^2 + ax - a^2 = 0$ **(CBSE 2014)**

(*d*) $3x^2 - 2\sqrt{6}x + 2 = 0$ **(CBSE 2010)**

(*e*) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

(*f*) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(*g*) $(x - 1)^2 - 5(x - 1) - 6 = 0$

12. For what vlaue of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots?

13. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots find the value of k.

(CBSE 2014, 2016)

14. If $x = \dfrac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$. Find the value of a and b. **(CBSE 2016)**

15. Find value of p for which the product of roots of the quadratic equation $px^2 + 6x + 4p = 0$ is equal to the sum of the roots.

16. The sides of two squares are x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 Find the sides of these two squares.

17. Find k if the difference of roots of the quadratic equation $x^2 - 5x + (3k - 3) = 0$ is 11.

SHORT ANSWER TYPE QUESTIONS-II

18. Find the positive value of k for which the quadratic equation $x^2 + kx + 64 = 0$ and the quadratic equation $x^2 - 8x + k = 0$ both will have real roots.

19. Solve for x

(*a*) $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$ $\quad a + b + x \neq 0,$ $a, b, x \neq 0$ **(CBSE 2005)**

(*b*) $\dfrac{1}{2a+b+2x} = \dfrac{1}{2a} + \dfrac{1}{b} + \dfrac{1}{2x}$ $\quad 2a + b + 2x \neq 0,$ $a, b, x \neq 0$

(*c*) $\dfrac{2x}{x-3} + \dfrac{1}{2x+3} + \dfrac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, \dfrac{-3}{2}$

(*d*) $4x^2 + 4bx - (a^2 - b^2) = 0$

(*e*) $\dfrac{1}{x-1} - \dfrac{1}{x+5} = \dfrac{6}{7}, x \neq 1, 5$ **(CBSE 2010)**

(*f*) $4x^2 - 2(a^2 + b^2)\,x + a^2b^2 = 0$

(*g*) $\dfrac{2}{x+1} + \dfrac{3}{2(x-2)} = \dfrac{23}{5x}, x \neq 0, -1, 2$

(*h*) $\left(\dfrac{2x}{x-5}\right)^2 + \dfrac{10x}{(x-5)} - 24 = 0, x \neq 5$

(*i*) $4x^2 - 4a^2x + a^4 - b^4 = 0$

(*j*) $2a^2x^2 + b(6a^2 + 1)x + 3b^2 = 0$

(*k*) $3\left(\frac{7x+1}{5x-3}\right) - 4\left(\frac{5x-3}{7x+1}\right) = 11, x \neq \frac{3}{5}, \frac{-1}{7}$

(*l*) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

(*m*) $\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}, x \neq 5, 7$ **(CBSE 2014)**

(*n*) $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4},$ $x \neq -1, -2, -4$

(*o*) $\frac{1}{2x-3} + \frac{1}{x-5} = 1,$ $x \neq \frac{3}{2}, 5$

(*p*) $x^2 + 5\sqrt{5}x - 70 = 0$

(*q*) $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$ **(CBSE 2014)**

20. Solve by using quadratic formula $abx^2 + (b^2 - ac)x - bc = 0$.

21. If the roots of the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$ are equal find p and then find the roots of this quadratic equation.

22. Find the nature of roots of the quadratic equation $3x^2 - 4\sqrt{3}x + 4 = 0$

If the roots are real, find them. **(CBSE 2020)**

23. Solve $9x^2 - 6a^2x + a^4 - b^4 = 0$ using quadratic formula.

LONG ANSWER TYPE QUESTIONS

24. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

25. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

26. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policemen will catch the thief?

27. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

28. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed, so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

29. A farmer wishes to grow a 100 sq.m recangular garden. Since he has only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of this house act as the fourth side fence. Find the dimensions of his garden.

30. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of a pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal at what distance from the hole is the snake caught?

31. If the price of a book is reduced by ₹ 5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

32. ₹ 6500 were divided equally among a certain number of persons. If there been 15 more persons, each would have got ₹ 30 less. Find the original number of persons.

33. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

34. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than the fast train, find the speed of the two trains.

35. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the orignal point in 4 hrs 30 minutes. Find the speed of the stream.

36. Sum of areas of two squares is 400 sq.cm. If the difference of their perimeter is 16 cm. Find the side of each square.

37. The area of an isosceles triangle is 60 sq.cm. The length of equal sides is 13 cm find length of its base.

38. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$. Find the fraction.

39. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

40. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

41. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers.

42. A piece of cloth costs ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, then the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

43. A motor boat whose speed is 24 km/hr in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream **(CBSE 2016)**

44. If the roots of the quadratic equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, Prove that $2b = a + c$.

45. If the equation $(1 + m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

46. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train. **(CBSE 2020)**

47. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park. **(CBSE 2020)**

ANSWERS AND HINTS

1. (***d***) [$x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^2 \Rightarrow 4x^3 = 3 \Rightarrow$ degree = 3]

2. (***b***) [Check by substituting $x = 2$ in the equation.]

3. (***a***) [Substitute $x = \frac{1}{2}$ in $x^2 + px - \frac{5}{4} = 0$.]

4. (***c***) [$\because$ A quadratic equation is of degree 2 and it has atmost two roots.]

5. (***d***) [$x(x - 7) = 0 \Rightarrow x = 0, x = 7$.]

6. (***b***) ± 4 ($D = 0$)

7. (*a*) [$r = \frac{q^2}{4p}$ ($D = 0 \Rightarrow q^2 - 4pr = 0$)]

(*b*) $p = -6, q = 1$

(*c*) $K \leq 4$ [$D \geq 0$]

(*d*) $c = 4$

(*e*) Linear equation

8. D = –124

9. (a) $-\frac{1}{2}, 1$

10. $D = 0 \Rightarrow 20p^2 - 60p = 0$, p $\neq 0$

$$20p(p - 3) = 0$$
$$p = 3$$

11. (*a*) $x = \frac{7}{2}, x = -\frac{3}{4}$

(*b*) $x = -\sqrt{5}, x = \frac{-2\sqrt{5}}{3}$

(*c*) $x = \frac{a}{2}, x = -a$

(*d*) $x = \sqrt{\frac{2}{3}}, x = \sqrt{\frac{2}{3}}$

(*e*) $x = -\sqrt{3}, x = \frac{-7\sqrt{3}}{3}$

(*f*) $x = -\sqrt{2}, x = \frac{-5\sqrt{2}}{2}$

(*g*) Take $(x - 1) = y$

$x = 0, x = 7$

12. $D < 0$, $(-6)^2 - 4(3a)(1) < 0$, $12a > 36 \Rightarrow a > 3$

13. $2(-5)^2 + p(-5) - 15 = 0 \Rightarrow p = 7$

$\therefore\ 7x^2 + 7x + k = 0, \quad D = 49 - 28k = 0$

$\Rightarrow k = \dfrac{49}{28} = \dfrac{7}{4}$

14. Substituting, $x = \dfrac{2}{3}$ we get $\quad 4a + 9b = -42 \quad$ ...(1)

Substituting, $x = -3$ we get $\quad 9a + b = 21 \quad$ ...(2)

Solve (1) and (2) to get $a = 3$, $b = -6$.

15. Product $= \dfrac{c}{a} = \dfrac{4p}{p} = 4$, $\quad$ sum $= \dfrac{-b}{a} = \dfrac{-6}{p}$

$= \dfrac{-6}{p} = 4 \quad \Rightarrow \quad p = \dfrac{-6}{4} = \dfrac{-3}{2}$

16. $x^2 + (x+4)^2 = 656$

$x^2 + 4x - 320 = 0$

$D = 1296 \quad x = \dfrac{-4 \pm \sqrt{1296}}{2} = \dfrac{-4+36}{2}, \dfrac{-4-36}{2}$

$x = \dfrac{32}{2} = 16$, (rejecting –ve value)

Sides are 16 cm, 20 cm

17. $\alpha - \beta = 11$

Sum of roots $\alpha + \beta = \dfrac{-b}{a} = 5$

Solve to get

$\alpha = 8, \beta = -3$

Product of roots $= \dfrac{c}{a}$

$-24 = 3k - 3$

$k = -7$

18. $x^2 + kx + 64 = 0 \rightarrow D_1 = k^2 - 256 \geq 0, \quad k^2 \geq 256$

$\Rightarrow \; k \geq 16 \quad ...(1)$

$k \leq -16$

$x^2 - 8x + k = 0 \rightarrow D_2 = 64 - 4k \geq 0, 64 \geq 4k$

$\Rightarrow \; k \leq 16 \quad ...(2)$

(1) and (2) gives $k = 16$

19. (*a*) $\dfrac{1}{a+b+x} - \dfrac{1}{x} = \dfrac{1}{a} + \dfrac{1}{b}$

$\dfrac{x-a-b-x}{(a+b+x)x} = \dfrac{a+b}{ab}$

$-(a+b)\,ab = (a+b)\,(a+b+x)\,x$

$x^2 + xa + bx + ab = 0$

$x = -a, x = -b$

(*b*) Similar to 19 (a)

(*c*) Take LCM to get $2x^2 + 5x + 3 = 0, x = -1, x \neq \dfrac{-3}{2}$. (given)

(*d*) $(4x^2 + 4bx + b^2) - a^2 = 0$

$(2x + b)^2 - a^2 = 0$ apply $A^2 - B^2 = (A + B)\,(A - B)$

Ans. $x = -\dfrac{(a+b)}{2}, x = \dfrac{(a-b)}{2}$

(*e*) Take LCM to get $x^2 + 4x - 12 = 0$

Ans. $x = 2, -6$

(*f*) $4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$2x(2x - a^2) - b^2(2x - a^2) = 0 \Rightarrow (2x - b^2)\,(2x - a^2) = 0$

$x = \dfrac{b^2}{2}, \dfrac{a^2}{2}$

(*g*) Take LCM to get $11x^2 - 21x - 92 = 0$

$x = 4, x = \dfrac{-23}{11}$

(*h*) $\left(\dfrac{2x}{x-5}\right)^2 + 5\left(\dfrac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y \quad \therefore\ y^2 + 5y - 24 = 0$. Solve to get $y = 3, y = -8$

Sub, $\frac{2x}{x-5} = 3,\ \frac{2x}{x-5} = -8$

Ans. $x = 15, x = 4$

(*i*) $4x^2 - 4a^2x + a^4 - b^4 = 0$

$(2x - a^2)^2 - (b^2)^2 = 0$

$(2x - a^2 - b^2)(2x - a^2 + b^2) = 0$

$x = \frac{a^2+b^2}{2}, \quad x = \frac{a^2-b^2}{2}$

(*j*) Find $D = b^2(6a^2 + 1)^2 - 24a^2b^2 = b^2\left[36a^4 + 1 + 12a^2 - 24b^2\right]$

$= b^2(6a^2 - 1)^2$

Use $x = \frac{-B \pm \sqrt{D}}{2A}$ to get answer

Ans. $x = \frac{-b}{2a^2}, -3b$

(*k*) Let $\frac{7x+1}{5x-3} = y$

$\therefore\ 3y - \frac{4}{y} = 11 \Rightarrow 3y^2 - 11y - 4 = 0$. Solve to get

$y = -\frac{1}{3}, y = 4$

Substitute y and get $x = 0, 1$

(*l*) Take LCM to get $x^2 - 3x + 2 = 0$

Solve to get $x = 1, x = 2$

(*m*) Take LCM to get $2x^2 - 27x + 88 = 0$

$x = 8, \frac{11}{2}$

(n) Take LCM to get $x^2 - 4x - 8 = 0$ (Use quadratic formula)

Ans. $x = 2 \pm 2\sqrt{3}$

(o) Take LCM to get $2x^2 - 16x + 23 = 0$

Solve using Quadratic formula

Ans. $x = \dfrac{-8 \pm 3\sqrt{2}}{2}$

(p) $x^2 + 7\sqrt{5}x - 2\sqrt{5}x - 70 = 0$

$(x + 7\sqrt{5})(x - 2\sqrt{5}) = 0$

$x = 2\sqrt{5}, -7\sqrt{5}$

(q) $\dfrac{16 - x}{x} = \dfrac{15}{x + 1}$

$x^2 - 16 = 0$

$x = \pm 4$

20. $abx^2 + b^2x - acx - bc = 0$

$(bx - c)(ax + b) = 0$

$x = -\dfrac{b}{a}, \dfrac{c}{b}$

21. $D = 0$

$\therefore p^2 - 2p - 3 = 0$; $p = -1, 3$

rejecting $p = -1$,

Ans. $p = 3$.

22. $D = \left(-4\sqrt{3}\right)^2 - 4(3)(4) = 0$

$\therefore$ Roots are equal and real

Roots are $\dfrac{-b}{2a}, \dfrac{-b}{2a} = \dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}$

23. $D = (-6a^2)^2 - 4(9)(a^4 - b^4)$

$= 36b^4$

$x = \dfrac{-(-6a^2) \pm \sqrt{36b^4}}{2 \times 9} = \dfrac{a^2 \pm b^2}{3}$

24. Equation $\frac{54}{x}+\frac{63}{x+6} = 3$, $x \longrightarrow$ speed of train at first, $x + 6 \longrightarrow$ Increased speed.

Ans. $x = 36, x \neq -3$.

25. Let the natural number be x.

$x + 12 = \frac{160}{x} \Rightarrow x^2 + 12x - 160 = 0$

$\Rightarrow x = 8, \quad x = -20$ (rejected)

26. Let time taken by thief be n minutes.

Policeman will catch the thief in $(n - 1)$ minutes.

Total distance covered by thief = $(100\ n)$ metres —(1)

(as distance covered in 1 min = 100 m)

Distance covered by policemen

$100 + 110 + 120 + ... +$ to $(n - 1)$ 10 —(2)

from (1) and (2) $\Rightarrow 100\ n = \frac{(n-1)}{2}\ [2 \times 100 + (n - 2)\ 10]$

Solve and get $\quad n^2 - 3n - 18 = 0$

$n = 6, \quad n \neq -3$

Policeman will catch the thief in 5 minutes.

27. Time taken by top of smaller diameter = x hrs

Time taken by larger tap = $(x - 9)$ hrs

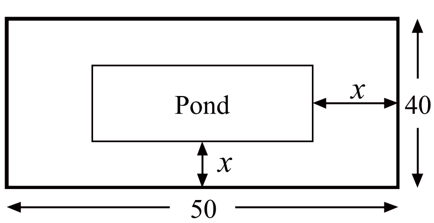
$\frac{1}{x}+\frac{1}{x-9} = \frac{1}{6}$ and get $x^2 - 21x + 54 = 0$

Ans. $x = 3, x = 18$

$x = 3$ rejected as $x - 9 = -6 < 0$

$\therefore\ x = 18$ hrs $x - 9 = 18 - 9 = 9$ hrs

28.



Length of rectangular lawn = 50 m

Breadth of rectangular lawn = 40 m

Length of pond = $(50 - 2x)$ m

Breadth of pond = $(40 - 2x)$ m

Area of lawn – Area of pond = Area of grass

$50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

get $x^2 - 45x + 296 = 0$

$x = 37, x = 8$

$x = 37$ rejected $\because$ $40 - 2x = 40 - 2(37) < 0$

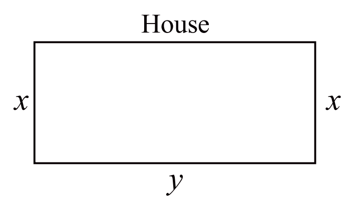
Length of pond = 34 m, Breadth of pond = 24 m

29. $x + y + x = 30$, $xy = 100$

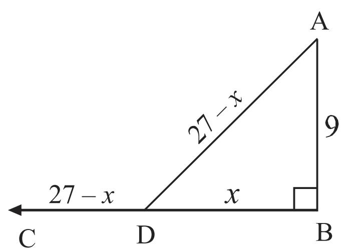
Solve $x = 5$m, 10 m,

$y = 20$ m, 10 m

$\therefore$ dim. are 5 m × 20 m or 10 m × 10 m



30.



In ΔABD, acc. to pythagoras theorem $9^2 + x^2 = (27 - x)^2$. Solve it to get $x = 12$ m.

31. Let original list price = ₹ x

$$\frac{300}{x-5} - \frac{300}{x} = 5$$

Solve and get $x = 20$, $x = -15$ rejected

Ans. ₹ 20

32. Let original number of persons be x

$$\frac{6500}{x} - \frac{6500}{x+15} = 30$$

Solve and get $x = 50$, $x = -65$ (rejected).

33. $\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$ [Speed of aircraft = x km/hr]

Solve to get $x = 600$, $x = -400$ (rejected).

Duration of flight $= \frac{600}{600} = 1$hr.

34. $\frac{600}{x} - \frac{600}{x+10} = 3$ (Speed of slow train x km/hr)

Solve to get $x = 40$, $x = -50$ (rejected).

Ans. 40 km/hr, 50 km/hr.

35. $\frac{30}{15-x} + \frac{30}{15+x} = \frac{9}{2}$. (Speed of stream x km/hr)

Solve to get x = 5, $x = -5$ (rejected)

Ans. 5 km/hr

36. $x^2 + y^2 = 400$ ...(1)

$4x - 4y = 16 \Rightarrow x - y = 4$ ...(2)

$y - x = 4$ ...(3)

Solve (1) and (2) to get $x = 16$, $x = -12$ (rejected)

Solve (1) and (3) to get $x = 12$, $x = -16$ (rejected)

Ans. $x = 16$ m, $y = 12$ m from (1) and (2)

$x = 12$ m, $y = 16$ m from (1) and (3)

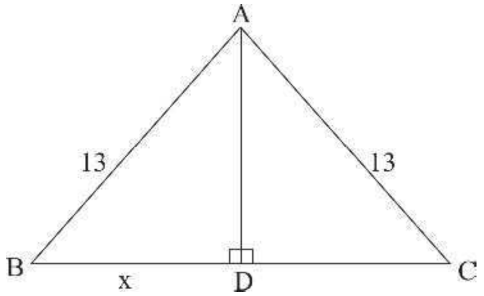
37. BC = 2x, BD = x (Draw a $\perp$ from A on BC)

Use pythagoras theorem

AD = $\sqrt{169 - x^2}$

$$A = \frac{1}{2} \times 2x \times \sqrt{169 - x^2} = 60$$

$x^2 = 144$, $x^2 = 25$



$x = 12$ or $x = 5$
$x = -12, -5$ (rejected)
$\therefore$ base $2x = 24$ cm or 10 cm

38. Fraction $\frac{x}{2x+1}$

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$$

$x = 3,\ x = \frac{-7}{11}$ (rejected)

Ans. Fraction $= \frac{3}{7}$.

39. Age of sister $= x$ years
Age of girl $= 2x$ years
$(x + 4)(2x + 4) = 160$
$x = 6$ years, $x = -12$ (rejected)
$2x = 12$ years
6 years, 12 years

40. Let tens place digit $= x$, then unit place digit $= \frac{18}{x}$.

Number $= 10x + \frac{18}{x}$

$$\left(10x + \frac{18}{x}\right) - \left(\frac{10 \times 18}{x} + x\right) = 63$$

$x = 9,\ x = -2$ (rejected).
Number 92

41. Let no. be $x, x + 1, x + 2$
$(x)^2 + (x + 1)(x + 2) = 46$
$2x^2 + 3x - 44 = 0$

$x = 4,\ x = -\frac{22}{4}$ (rejected)

$\therefore$ Numbers are 4, 5, 6.

42. Let length of piece be x metre.

$$\frac{200}{x} - \frac{200}{x+5} = 2$$

Solve to get $x = 20$, $x = -25$ (rejected)

Rate per meter $= \frac{200}{x} = \frac{200}{20} =$ ₹ 10

43. Let speed of stream $= x$ km/hr

$$\frac{32}{24-x} - \frac{32}{24+x} = 1$$

$x^2 + 64x - 576 = 0$

$x = 8$, $x = -72$ (rejected)

$x = 8$ km/hr

44. $D = 0$

$(c - a)^2 - 4(b - c)(a - b) = 0$

$\Rightarrow (a + c - 2b)^2 = 0$

$\therefore a + c = 2b$

45. $D = 0$

$(2\text{ mnc})^2 - 4(1 + m^2) n^2 (c^2 - a^2) = 0$

to get $4n^2c^2 = 4n^2a^2 (1 + m^2)$

$\therefore c^2 = a^2 (1 + m^2)$

46. Let the speed of the train $= x$ km/hr

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$x^2 - 8x - 1280 = 0$

$x = 40, -32$ (rejected)

$x = 40$ km/hr

47. Let Lm be the length of the rectangular park

Breadth = (L – 3) m

Altitude of the isosceles triangle = 12 m

$$L(L-3) = \frac{1}{2}(12)(L-3) + 4$$

$L^2 - 9L + 14 = 0$

$\Rightarrow \quad L = 7, 2$

So, L = 7m (L = 2 rejected $\because$ L – 3 = –1)

$\therefore$ Length = 7 m, Breadth = 4 m

Practice Test

Quadratic Equations

Time: 45 Minutes **M.M : 20**

SECTION-A

1. The value of k is .... if $x = 3$ is one root of $x^2 - 2kx - 6 = 0$. **1**
2. If the discriminant of $3x^2 + 2x + \alpha = 0$ is double the discriminant of $x^2 - 4x + 2 = 0$ then value of α is **1**
3. If discriminant of $6x^2 - bx + 2 = 0$ is 1 then value of b is **1**
4. $(x - 1)^3 = x^3 + 1$ is quadratic equation. (T/F) **1**

SECTION-B

5. If roots of $x^2 + kx + 12 = 0$ are in the ratio 1 : 3 find k. **2**
6. Solve for x : $21x^2 - 2x + \frac{1}{21} = 0$ **2**
7. Find k if the quadratic equation has equal roots : $kx(x - 2) + 6 = 0$. **2**

SECTION-C

8. Solve using quadratic formula **3**

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

9. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$ is a perfect square. **3**

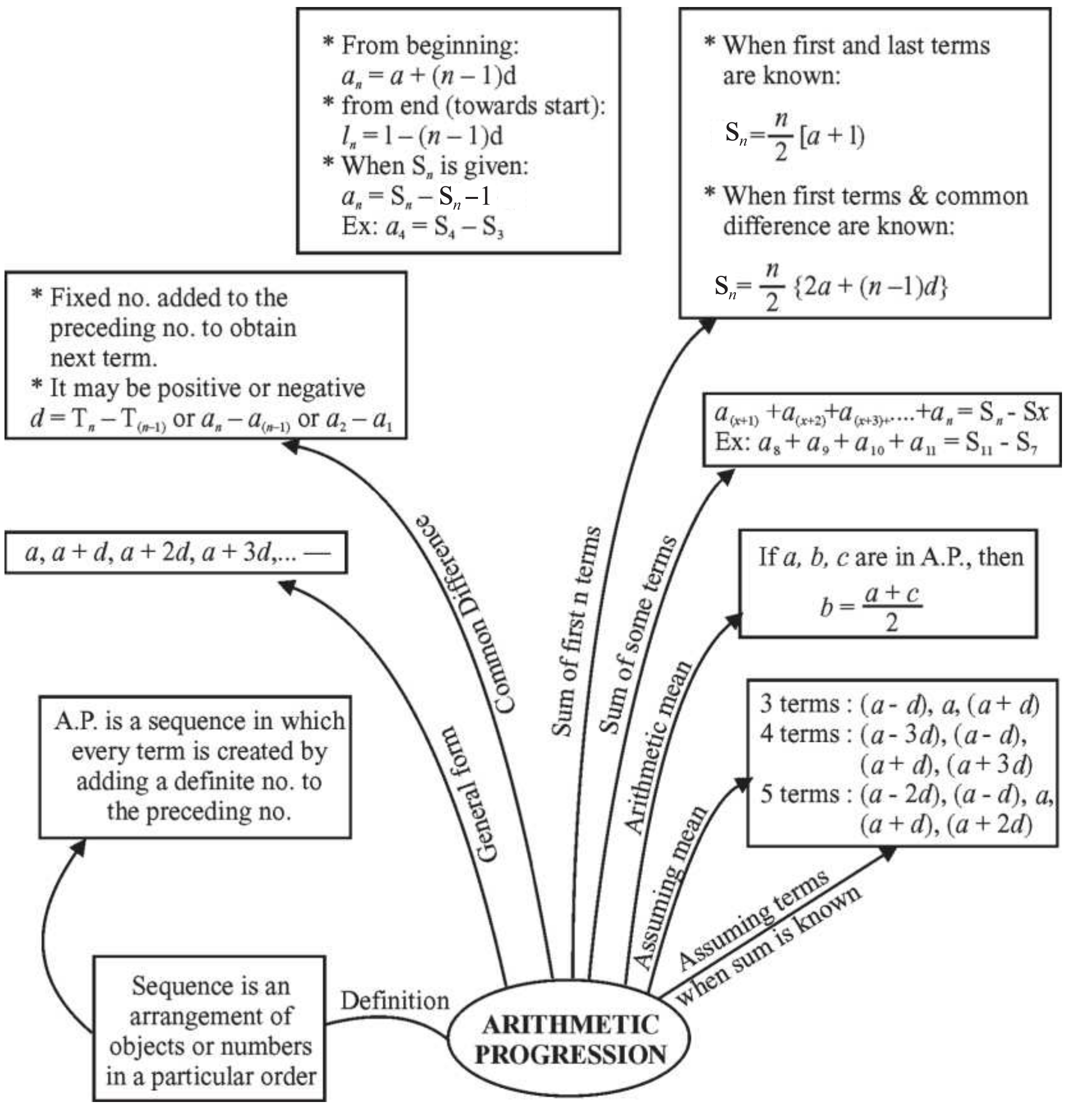
SECTION-D

10. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

4

CHAPTER

5 Arithmetic Progression



* $a \rightarrow$ first term, $d \rightarrow$ common difference; $a_n \rightarrow n^{th}$ term; S_n Sum of first n terms; $l \rightarrow$ last term

VERY SHORT ANSWER TYPE QUESTIONS

1. Find 5^{th} term of an A.P. whose n^{th} term is $3n - 5$

2. Find the sum of first 10 even numbers.

3. Write the n^{th} term of odd numbers.

4. Write the sum of first n natural numbers.

5. Write the sum of first n even numbers.

6. Find the n^{th} term of the A.P. – 10, – 15, – 20, – 25, ...

7. Find the common difference of A.P. $4\frac{1}{9}, 4\frac{2}{9}, 4\frac{1}{3}$, ...

8. Write the common difference of an A.P. whose n^{th} term is $a_n = 3n + 7$

9. What will be the value of $a_8 - a_4$ for the following A.P.

4, 9, 14, ..., 254

10. What is value of a_{16} for the A.P. – 10, – 12, – 14, – 16, ...

11. 3, $k - 2$, 5 are in A.P. find k.

12. For what value of p, the following terms are three consecutive terms of an A.P.

$\frac{4}{5}$, p, 2.

13. Determine the 36^{th} term of the A.P. whose first two terms are –3 and 4 respectively.

14. Multiple Choice Questions:

(i) 30^{th} term of the A.P. 10, 7, 4 .... is

(a) 97 (b) 77

(c) –77 (d) –87

(ii) 11^{th} term of an A.P. – 3, $-\frac{1}{2}$, , ... is

(a) 28 (b) 22

(c) –38 (d) $-48\frac{1}{2}$

(iii) In an A.P. if $d = -4$, $n = 7$, $a_n = 4$, then a is

(a) 6 (b) 7

(c) 120 (d) 28

(iv) The first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$ then y equals:

(a) -3 (b) 4

(c) 5 (d) 2

(v) The list of numbers $-10, -6, -2, 2, \ldots$ is

(a) An A.P. with $d = -16$ (b) An A.P. with $d = 4$

(c) An A.P. with $d = -4$ (d) Not an A.P.

(vi) The 11th term from the last term of an A.P. 10, 7, 4, ..., -62 is

(a) 25 (b) -32

(c) 16 (d) 0

(vii) The famous mathematician associated with finding the sum of the first 100 natural numbers is

(a) Pythagoras (b) Newton

(c) Gauss (d) Euclid

(viii) What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8

(c) -4 (d) 4

(ix) The nth term of the A.P. $\left(1+\sqrt{3}\right)$, $\left(1+2\sqrt{3}\right)$, $\left(1+3\sqrt{3}\right)$, ... is

(a) $1 + n\sqrt{3}$ (b) $n + \sqrt{3}$

(c) $n\left(1+\sqrt{3}\right)$ (d) $n\sqrt{3}$

(x) The first term of an A.P. is p and the common difference is q, then its 10th term is

(a) $a + 9p$ (b) $p - 9q$

(c) $p + 9q$ (d) $2p + 9q$

SHORT ANSWER TYPE QUESTIONS-I

15. Is 144 a term of the A.P. 3, 7, 11, ... ? Justify your answer.

16. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P.

17. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively, Find the sum of all terms of this A.P.

18. Find the sum of first 15 multiples of 8.

19. Find the sum of even positive integers between 1 and 200.

20. If $4m+8$, $2m^2+3m+6$, $3m^2+4m+4$ are three consecutive terms of an A.P. find m.

21. How many terms of the A.P. 22, 20, 18, ... should be taken so that their sum is zero.

22. If 10 times of 10^{th} term is equal to 20 times of 20^{th} term of an A.P. Find its 30^{th} term.

23. Solve for x: $1+4+7+10+\ldots+x=287$ **(CBSE 2020)**

24. Find how many two digit numbers are divisible by 6? **(CBSE 2011)**

25. If $\frac{1}{x+2}$, $\frac{1}{x+3}$ and $\frac{1}{x+5}$ are in A.P. find x. **(CBSE 2011)**

26. Find the middle term of an A.P. $-6, -2, 2, \ldots 58$. **(CBSE 2011)**

27. In an A.P. find S_n, where $a_n = 5n-1$. Hence find the sum of the first 20 terms. **(CBSE 2011)**

28. Which term of A.P. 3, 7, 11, 15 .... is 79? Also find the sum $3+7+11+\ldots+79$. **(CBSE 2011C)**

29. Find the 15^{th} term from the last term of the A.P. 3, 8, 13, ... 253. **(CBSE 2022)**

SHORT ANSWER TYPE QUESTIONS-II

30. Find the sum of integers between 10 and 500 which are divisible by 7.

31. The sum of 5^{th} and 9^{th} terms of an A.P. is 72 and the sum of 7^{th} and 12^{th} term is 97. Find the A.P.

32. If the m^{th} term of an A.P. be $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$, show that its $(mn)^{th}$ is 1.

33. If the m^{th} term of *an* A.P. is $\frac{1}{n}$ and the n^{th} terms is $\frac{1}{m}$, show that the sum of mn terms is $\frac{1}{2}(mn+1)$.

34. If the p^{th} term A.P. is q and the q^{th} term is p, prove that its n^{th} term is $(p+q-n)$.

35. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

36. The sum of 5^{th} and 9^{th} terms of an A.P. is 30. If its 25^{th} term is three times its 8^{th} term, find the A.P.

37. If m times the m^{th} terms of an A.P. is equal to n times of n^{th} term and $m \neq n$, show that its $(m+n)^{th}$ term is zero.

38. Which term of the A.P. 3, 15, 27, 39 .... will be 120 more than its 21^{st} term?

(CBSE 2018)

39. The sum of first n terms of an A.P. is given by $S_n = 3n^2 + 2n$. Find the A.P.

40. In an A.P., the first term is 12 and the common difference is 6. If the last term of the A.P. is 252, then find its middle term.

41. The 17^{th} term of an A.P. is 5 more than twice its 8^{th} term. If the 11^{th} term of the A.P. is 43, then find the n^{th} term of the A.P. **(CBSE 2020)**

42. If the sum of the first 14 terms of an A.P. is 1050 and its fourth term is 40, find its 20^{th} term. **(CBSE 2020)**

43. Find the number of terms in the series $20+19\frac{1}{3}+18\frac{2}{3}+...$ of which the sum is 300, explain the double answer.

44. Find the sum of n terms of the series: $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots$

LONG ANSWER TYPE QUESTIONS

45. The sum of third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first 16 terms of the A.P.

46. Determine the A.P. whose 4^{th} term is 18 and the difference of 9^{th} term from the 15^{th} term is 30.

47. The sum of first 9 terms of an A.P. is 162. The ratio of its 6^{th} term to its 13^{th} term is 1:2. Find the first and fifteenth terms of the A.P.

48. The sum of the first 9 terms of an A.P. is 171 and the sum of its first 24 terms is 996. Find the first term and common difference of the A.P. **(CBSE 2020)**

49. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 term is 161. Find the 28^{th} term of this A.P.

50. If the sum of the first four terms of an AP is 40 and the sum of the first fourteen terms of an AP is 280. Find the sum of first n terms of the A.P. **(CBSE 2018)**

51. A man saved ₹ 16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year?

52. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of last 15 terms is 2565. Find the A.P. **(CBSE 2014)**

53. The sum of first n terms of an A.P. is $5n^2 + 3n$. If the m^{th} term is 168, find the value of m. Also find the 20^{th} term of the A.P. **(CBSE 2013)**

54. If the 4^{th} term of an A.P. is zero, prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

55. In an A.P. if $S_5 + S_7 = 167$ and $S_{10} = 235$. Find the A.P., where S_n denotes the sum of its first n terms. **(CBSE 2015)**

56. In an A.P. prove $S_{12} = 3\,(S_8 - S_4)$ where S_n represent the sum of first n terms of an A.P.

57. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

58. Find the sum of first 16 terms of an Arithmetic Progression whose 4^{th} and 9^{th} terms are –15 and –30 respectively.

59. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P.

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. $a_n = 3n - 5 \qquad a_5 = 10$

2. $S_n = \dfrac{10}{2}\,[2 \times 2 + 9 \times 2] = 110$

3. 1, 3, 5, ...

$a_n = 1 + (n - 1)2 = 2n - 1.$

4. $1 + 2 + \text{........} + n = \dfrac{n}{2}\,[1 + n]$

5. $2 + 4 + 6 + \dots + 2n = \dfrac{n}{2}[2 + 2n] = n(n + 1)$

6. $a_n = a + (n - 1)d = -\,5(n + 1)$

7. $d = a_2 - a_1 = \dfrac{1}{9}$

8. $a_1 = 3 + 7 = 10,\ a_2 = 6 + 7 = 13,\ d = 3$

9. $(a + 7d) - (a + 3d) = 4d = 20$

10. $a_{16} = a + 15d = -\,40$

11. 3, $k - 2$, 5 are in A.P.

$\therefore \quad k - 2 = \dfrac{3+5}{2} = 4 \qquad k = 6$

12. $p = \dfrac{7}{5}$ (same as Q.11)

13. $a = -3; a_2 = 4; d = 7$

$a_n = a + (n-1)d$

$a_{36} = -3 + 35 \times 7$

$a_{36} = 242$

14. (i) c (ii) b

(iii) d (iv) c

(v) b (vi) b

(vii) c (viii) a

(ix) a (x) c

15. $144 = 3 + (n - 1)\ 4$

$\dfrac{141}{4} + 1 = n$ which is not possible

16. $a_1 = (a - b)^2 \quad a_2 = a^2 + b^2 \quad a_3 = (a + b)^2$

$a_2 - a_1 = a^2 + b^2 - (a - b)^2$

$= 2ab$

$a_3 - a_2 = (a + b)^2 - (a^2 + b^2)$

$= 2ab$

$a_2 - a_1 = a_3 - a_2$

$\therefore$ in A.P.

17. $a = 12,\ d = 6,\ a_n = 252 \quad \Rightarrow \quad n = 41$

Find $S_{41} = 5412$, use $S_n = \dfrac{n}{2}\ [2a + (n - 1)\ d]$

18. $S_{15} = \dfrac{15}{2}[2a + 14d]$

where $a = 8,\ d = 8$

Ans. 960

19. $2 + 4 + 6 + ... + 198$

$a = 2, d = 2, a_n = 198 \Rightarrow n = 99$

$S_n = \frac{n}{2}[a+l] = 9900$

20. $b = \frac{a+c}{2}$

$\therefore\ 2m^2 + 3m + 6 = \frac{4m+8+3m^2+4m+4}{2}$

Solve to get $m^2 - 2m = 0$

$m = 0, 2$

21. $S_n = 0 \Rightarrow \frac{n}{2}\ [44 + (n - 1)(- 2)] = 0.$

Solve $n = 23$

22. ATQ $10\ a_{10} = 20\ a_{20}$

$\Rightarrow a_{10} = 2a_{20}$

$a + 9d = 2a + 38d$

$a = - 29d$ ...(1)

$a_{30} = a + 29d$

Substitute a from (1)

Ans. $a_{30} = 0$

23. a = 1, $d = 3, a_n = x$

$S_n = 287$

$287 = \frac{n}{2}\left[2\times1+(n-1)3\right]$

$\Rightarrow 3n^2 - n - 574 = 0$

$n = 14, \frac{-41}{3}$ (rejected)

$\therefore\ n = 14$

$\therefore\ x = a_{14} = 40$

24. Two digit numbers divisible by 6 are 12, 18, 24, ... 96.

$a_2 - a_1 = a_3 - a_2 = 6$

$\therefore$ A.P., $a_n = 96 \Rightarrow n = 15$

25. $\dfrac{2}{x+3} = \dfrac{1}{x+2} + \dfrac{1}{x+5}$ $(2b = a + c)$

Solve to get $x = 1$.

26. $a_n = a + (n-1)\,d$

$58 = -6 + (n-1)\,4$

find $n = 17$

Find Middle term using concept of median

$= \left(\dfrac{n+1}{2}\right)^{\text{th}}$ term = 9th term

$a_9 = -6 + 8(4) = 26$

27. $a_n = 5n - 1$

Find A.P. $a_1 = 4$, $a_2 = 9$, $a_3 = 14$

4, 9, 14, ...

$a_2 - a_1 = 5 = a_3 - a_2$

$S_n = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}[8 + (n-1)\,5]$

$= \dfrac{n}{2}[5n + 3]$

$S_{20} = \dfrac{20}{2}\,[100 + 3] = 10 \times 103 = 1030$

28. $79 = 3 + (n-1)\,4$

$n = 20$

$S_{20} = \dfrac{20}{2}\,[3 + 79] = 10[82]$

$S_{20} = 820$

29. 15th term from end using $[l - (n-1)\,d]$

$= 253 - 14 \times 5$

$= 253 - 70 = 183$

30. Numbers between 10 and 500 which are divisible by 7, 14, 21, 28 ..., 497

Find n, using $a_n = a + (n - 1)\, d$, then use $S_n = \frac{n}{2}\,[2a + (n - 1)\, d]$

Ans. $S_n = 17885.$ $(n = 70)$

31. $a_5 + a_9 = 72$

$a_7 + a_{12} = 97$

Solve these equations to get a and d, $a = 6$, $d = 5$

$\therefore$ A.P., 6, 11, 16, 21, 26, ...

32. $a_m = \frac{1}{n} \quad \Rightarrow a + (m - 1)d = \frac{1}{n}$

$a_n = \frac{1}{m} \quad \Rightarrow a + (n - 1)d = \frac{1}{m}$

$\quad\quad\quad\; - \;\; - \quad\quad\;\; -$

$$(m - n)\, d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$\therefore\ d = \frac{1}{mn}$, find $a = \frac{1}{mn}$

$a_{mn} = a + (mn - 1)\, d$

$= \frac{1}{mn} + (mn-1)\frac{1}{mn}$

$a_{mn} = 1.$

33. $a_m = a + (m-1)d = \frac{1}{n}$ ...(1)

$a_m = a + (n-1)d = \frac{1}{m}$ ...(2)

Subtracting equation 2 from equation 1, we get

$d = \frac{1}{mn}$

$$a = \frac{1}{mn}$$

$$S_{mn} = \frac{mn}{2}\{2a + (mn - 1)d\}$$

$$S_{mn} = \frac{1}{2}(mn + 1)$$

34. $a_p = q, \quad a_q = p$

Soved to get a and d, $a = q + p - 1$, $d = -1$

$a_n = p + q - n$

35. Numbers divisible by both 2 and 5

$\Rightarrow$ Numbers divisible by 10.

Numbers between 101 and 999 divisible by 2 and 5 both 110, 120, 130, 140, ..., 990.

Use $a_n = 990$ to get $n = 89$.

36. ATQ $a_5 + a_9 = 30$

$a_{25} = 3\ a_8$

Solve to get $a = 3$, $d = 2$

A.P. 3, 5, 7, 9, ...

37. $m \times a_m = n \times a_n$

$$a(m - n) = d[(m - n) - (m^2 - n^2)]$$

$$(m - n)\{a + (m + n - 1)d\} = 0$$

$$(m - n)a_{(m+n)} = 0$$

$$a_{(m+n)} = 0$$

38. Let $a_n = 120 + a_{21}$

$3 + (n - 1)d = 120 + [3 + 20d]$

$3 + (n - 1)12 = 120 + [3 + 20 \times 12]$

$= 120 + 243$

$(n - 1)12 = 363 - 3 = 360$

$n = 31$

39. $S_n = 3n^2 + 2n$

$S_1 = 5;\ S_2 = 16; S_3 = 33$

$a_n = S_n - S_{(n-1)}$

$a = S_1 = 5$

$a_2 = S_2 - S_1 = 16 - 5 = 11$

$a_3 = S_3 - S_2 = 33 - 16 = 17$

A.P. : 5, 11, 17, ...

40. $a = 12; d = 6; a_n = 252$

$a_n = a + (n - 1)d$

Substitute the values and find n

$n = 41$

Middle terms = $\frac{41+1}{2} = 21^{st}$ term

$a_{21} = 132$

Middle term of A.P. is 132

41. ATQ,

$a_{17} = 5 + 2 \times a_8$

$a + 16d = 5 + 2a + 14\,d$

$a - 2d = -5$ ...(1)

$a_{11} = a + 10d = 43$ ....(2)

Solving (1) & (2), we get

$a = 3, d = 4$

$\therefore a_n = 4n - 1$

42. $S_{14} = 1050, a_4 = 40$

$S_{14} = \frac{14}{2}\,[2 \times a + 13d]$

$$\frac{1050}{7} = 2a + 13d$$

Solve $2a + 13d = 150$ and $a + 3d = 40$ to get $a = 10, d = 10$

$a_{20} = a + 19d = 10 + 190 = 200$

43. $a = 20; d = \frac{-2}{3}$

$S_n = 300$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

Substitute the values and find n

$n = 25$ or 36

Sum of 26^{th} to 36^{th} term is 0.

44. $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) ...$

$= (4 + 4 + 4 + ...) - \frac{1}{n}(1 + 2 + 3 + ...)$

$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$

$= \frac{7n - 1}{2}$

LONG ANSWER TYPE QUESTIONS

45. $a_3 + a_7 = 6, \quad a_3 \times a_7 = 8$

On Solving

$a = 1, \quad d = \frac{1}{2} \quad S_n = 16$

$a = 5, \quad d = \frac{-1}{2} \quad S_n = 20$

Ans. 76, 20

46. **ATQ** $a_4 = 18 \quad ...(1), \quad a_{15} - a_9 = 30 \quad ...(2)$

equation (2) will give $d = 5$

Substitute $d = 5$ in (1) to get $a = 3$

A.P. 3, 8, 13, ...

47. ATQ $S_9 = 162 \Rightarrow \frac{9}{2}$ $[2a + 8d] = 162$ ...(1)

ATQ $\frac{a_6}{a_{13}} = \frac{1}{2}$ solve and get $a = 2d$

Sub $a = 2d$ in (1) to get $d = 3$, $a = 6$

$a_{15} = a + 14d$

Ans. $a_{15} = 48$, $a = 6$

48. $S_9 = 171$, $S_{24} = 996$

$a + 4d = 19$, $2a + 23d = 83$

Solve to get,

$d = 3$, $a = 7$

49. ATQ $S_7 = 63$, ...(1)

Sum of next 7 terms $= S_{14} - S_7 = 161$ ...(2)

Use $S_n = \frac{n}{2}$ $[2a + (n - 1)\, d]$

Solve (1) and (2) to get a and d then find a_{28} using $a_n = a + (n - 1)\, d$.

$a = 3$, $d = 2$

Ans. $a_{28} = 57$

50. $S_4 = 40 \Rightarrow \frac{4}{2}[2a + 3d] = 40$

$S_{14} = 280 \Rightarrow \frac{14}{2}[2a + 13d] = 280$

Solve to get $a = 7$, $d = 2$

Ans. $S_n = n^2 + 6n$ (using $S_n = \frac{n}{2}$ $[2a + (n - 1)\, d]$)

51. ₹1200

52. $S_{10} = 210 \Rightarrow 5\, [2a + 9d] = 210$

$2a + 9d = 42$ ...(1)

$S_{50} - S_{35} = 2565 \Rightarrow \frac{50}{2}[2a + 49d] - \frac{35}{2}[2a + 34d] = 2565$

or $3a + 126d = 513$ ...(2)

Solve (1) and (2) $d = 4$, $a = 3$.

53. $S_n = 5n^2 + 3n$

$S_1 = a_1 = 8$

$S_2 = a_1 + a_2$

$26 = 8 + a_2 \Rightarrow a_2 = 18$

$d = 18 - 8 = 10$

$a_m = 168 \quad \Rightarrow \quad a + (m-1)d = 168$

$8 + (m-1)10 = 168 \Rightarrow m = 17$

$a_{20} = a + 19d = 8 + 190 = 198$

54. $a_4 = 0 \quad \Rightarrow \quad a + 3d = 0 \quad \Rightarrow \quad a = -3d$

$a_{25} = a + 24d = -3d + 24d = 21d$

$\therefore a_{25} = 3a_{11}$

$a_{11} = a + 10d = -3d + 10d = 7d$

55. Use $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_5 + S_7 = 167 \quad S_{10} = 235$

Solve to get $a = 1$, $d = 5$

A.P. = 1, 6, 11, 16, 21, ...

56. L.H.S. = $S_{12} = \frac{12}{2}[2a + 11d] = 6[2a + 11d]$

R.H.S. = $3\left[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)\right] = 3[4a + 22d] = 6[2a + 11d]$

$\therefore$ L.H.S. = R.H.S.

57. Four consecutive terms are :

$a - 3d, a - d, a + d, a + 3d$

$a = 8$

$$\frac{\text{Product of Extremes}}{\text{Product of means}} = \frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

Put $a = 8$ and solve to get

$\Rightarrow \quad d^2 = 4$

$d = \pm 2$

$\therefore$ for $a = 8$, $d = 2$ terms are 2, 6, 10, 14

for $a = 8$, $d = -2$ terms are 14, 10, 6, 2

58. $a_4 = -15$, $a_9 = -30$

$a + 3d = -15$, $a + 8d = -30$

Solve to get $a = -6$, $d = -3$

$S_{16} = -456$ $[S_n = \frac{n}{2}\{2a + (n-1)d\}]$

59. $a, a_2, a_3, ..., a_{36}, a_{37}$

3 middle most terms – a_{18}, a_{19}, a_{20}

$a_{18} + a_{19} + a_{20} = 225 \Rightarrow a + 18d = 75$ ...(1)

$a_{35} + a_{36} + a_{37} = 429 \Rightarrow a + 35d = 143$ ...(2)

Solving (1) and (2)

$a = 3$; $d = 4$

A.P. $\rightarrow$ 3, 7, 11, ..., 147

Practice Test

Arithmetic Progression

Time: 45 Minutes **M.M. : 20**

Section-A

1. Find the sum of first 10 natural numbers. **1**
2. What is the common difference of an A.P. $8\frac{1}{8}, 8\frac{2}{8}, 8\frac{3}{8}, ...$ **1**
3. If k, $2k - 1$ and $2k + 1$ are in A.P. then value of k is **1**
4. The 10th term from the end of the A.P. 8, 10, 12, ..., 126 is **1**

Section-B

5. How many 2 digit number are there in between 6 and 102 which are divisible by 6. **2**
6. The sum of n terms of an A.P. is $n^2 + 3n$. Find its 20th term. **2**
7. Find the sum $(-5) + (-8) + (-11) + ... + (-230)$ **2**

Section-C

8. Find the five terms of an A.P. whose sum is $12\frac{1}{2}$ and first and last term ratio is 2 : 3. **3**
9. Find the middle term of an A.P. 20,16,12,.......,– 176. **3**

Section-D

10. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers. **4**

CHAPTER

6 Triangles

TRIANGLES

Similarity

(a) Corresponding angles are equal

(b) Corresponding sides are proportional

SIMILARITY CRITERIA

(a) AAA criterion: If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (proportion) and hence the two triangles are similar.

(b) SSS criterion: If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

(c) SAS criterion: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Thales/Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Converse of Thales/B.P.T.

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Key Points:

1. Two polygons of the same number of sides are similar, if (i) all the corresponding angles are equal and (ii) all the corresponding sides are in the same ratio (proportion).

2. **Criteria for Similarity of triangles**

   In ΔABC and ΔDEF

   (*i*) If $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, then $\Delta ABC \sim \Delta DEF$ by AAA Similarity.

   (*ii*) If $\frac{AB}{DE} = \frac{BC}{EF}$ and $\angle B = \angle E$, then $\Delta ABC \sim \Delta DEF$ by SAS Similarity.

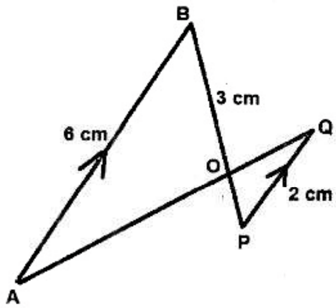
   (*iii*) If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ then $\Delta ABC \sim \Delta DEF$ by SSS Similarity.

3. **(a) (Prove) Basic Proportionality Theorem :** If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

   (b) (Motivate)Converse of BPT Theorem : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (without proof).

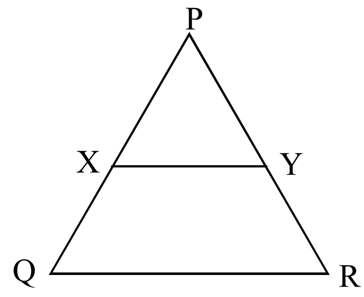
MULTIPLE CHOICE QUESTIONS

1. In the given figure, $AB \parallel PQ$. If AB = 6cm, PQ = 2cm and OB = 3cm, then the lenght of OP is:



(a) 9cm (b) 3cm (c) 4cm (d) 1cm

2. In the following figure, $XY \parallel QR$ and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



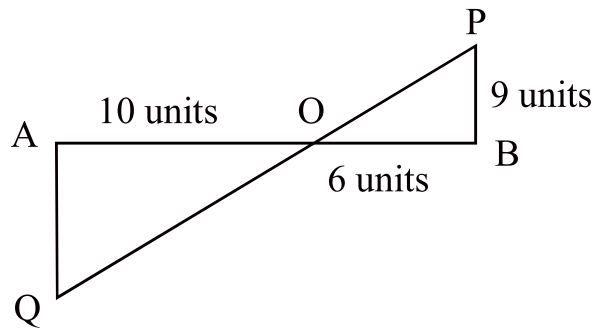
(*a*) $XY = QR$

(*b*) $XY = \frac{1}{3}QR$

(*c*) $XY^2 = QR^2$

(*d*) $XY = \frac{1}{2}QR$

3. In the following figure, $QA \perp AB$ and $PB \perp AB$, then AQ is



(*a*) 15 units

(*b*) 8 units

(*c*) 5 units

(*d*) 9 units

4. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

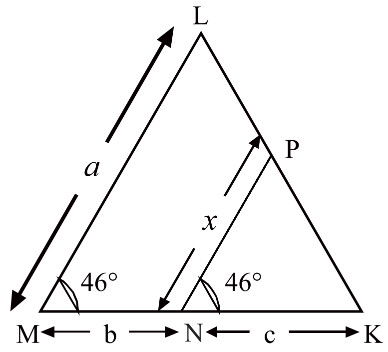
(*a*) $BC.EF = AC.FD$

(*b*) $AB.EF = AC.DE$

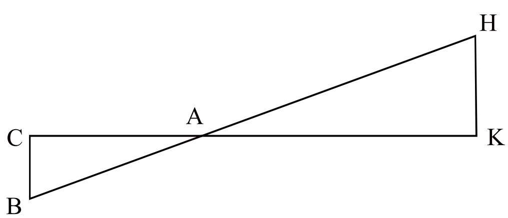
(*c*) $BC.DE = AB.EF$

(*d*) $BC.DE = AB.FD$

5. In the given figure, $\angle M = \angle N = 46°$, Express x in terms of a, b and c.



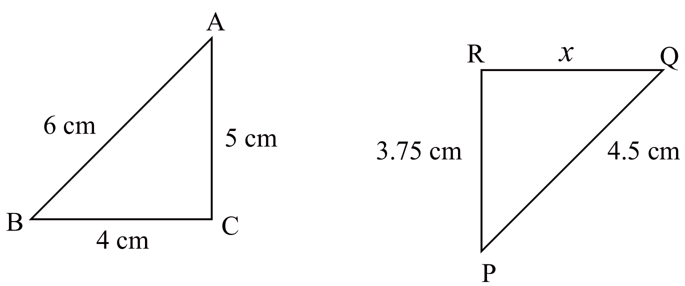
6. In the given figure, $\Delta AHK \sim \Delta ABC$. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. **(CBSE 2010)**



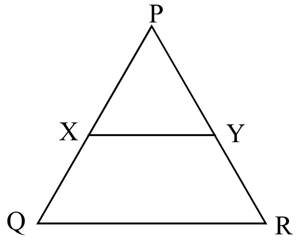
7. If $\Delta DEF \sim \Delta RPQ$, then is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$?

8. If the corresponding medians of two similar triangles are in the ratio 5 : 7, then find the ratio of their sides.

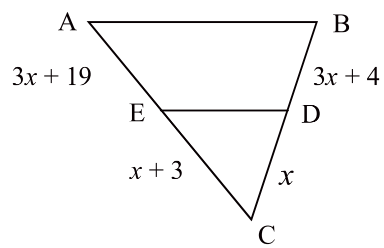
9. In the given figure, if $\Delta ABC \sim \Delta PQR$, find the value of x?



10. In the given figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, find XY : QR.



11. In the given figure, find the value of x which will make DE || AB ?

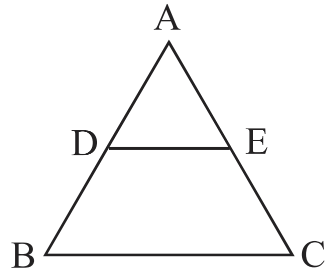


12. If ΔABC and ΔDEF are similar triangles such that $\angle A = 45°$ and $\angle F = 56°$, then find the value of $\angle C$.

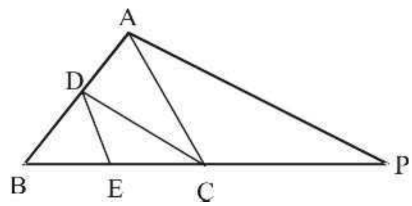
13. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then find the ratio of their corresponding attitudes.

SHORT ANSWER TYPE QUESTIONS-I

14. In the given figure $\frac{BD}{AB} = \frac{CE}{AC}$, then prove that DE || BC.

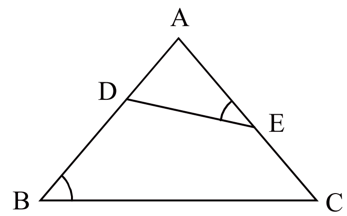


15. In the given figure, DE || AC and DC || AP Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.

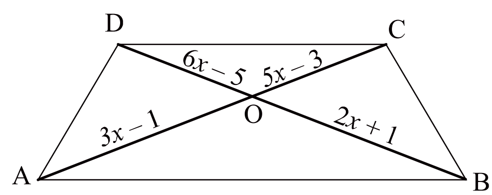


16. In ΔPQR, MN || QR, using B.P.T. prove that $\frac{PM}{PQ} = \frac{PN}{PR}$.

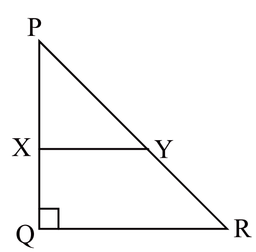
17. In the given figure, D and E are points on sides AB and CA of ΔABC such that $\angle B = \angle AED$. Show that $\Delta ABC \sim \Delta AED$.



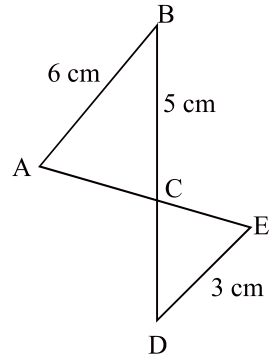
18. In the given figure, AB || DC and diagonals AC and BD intersects at O. If OA = $3x - 1$ and OB = $2x + 1$, OC = $5x - 3$ and OD = $6x - 5$, find the value of x.



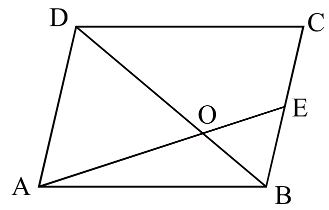
19. In the given figure, PQR is a right angled triangle in which $\angle Q = 90°$. If XY || QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2, then find the lengths of PR and QR.



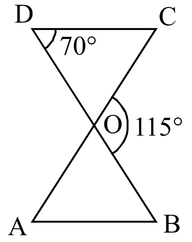
20. In the given figure, AB || DE. Find the length of CD.



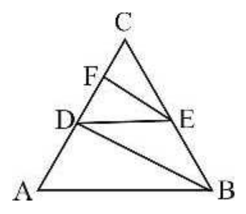
21. In the given figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm, find BC.



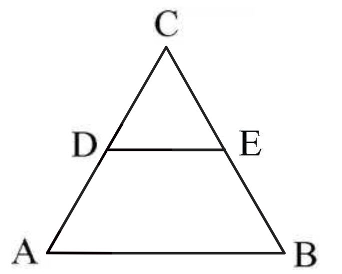
22. In the given figure, ΔODC ~ ΔOBA, $\angle$BOC = 115° and $\angle$CDO = 70°. Find, (*i*) $\angle$DOC, (*ii*) $\angle$DCO, (*iii*) $\angle$OAB, (*iv*) $\angle$OBA.



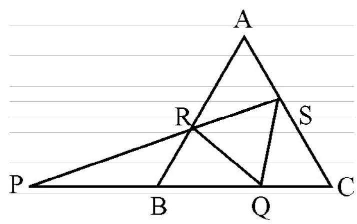
23. In the given figure, AB || DE and BD || EF prove that $DC^2 = CF \times AC$



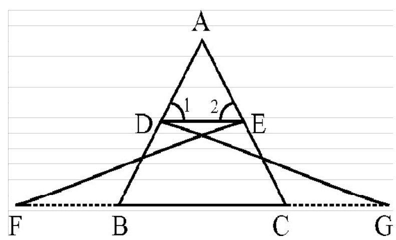
24. In the given figure, $\frac{AD}{DC} = \frac{BE}{EC}$ and $\angle CDE = \angle CED$. Prove that ΔCAB is isosceles.



25. In the given figure, QS || BA, QR || CA and PQ = 10 cm. Find PB × PC.



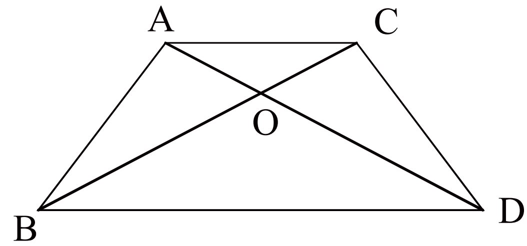
26. In the given figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that ΔADE ~ ΔABC.



SHORT ANSWER TYPE QUESTIONS-II

27. In ΔABC, $\angle ACB = 90°$ and $CD \perp AB$. Prove that: $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$.

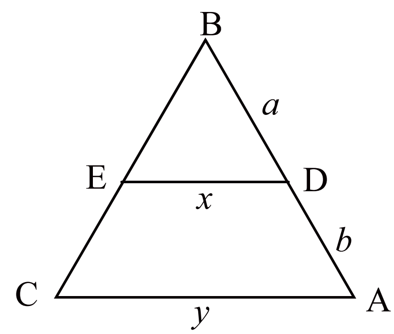
28. In the adjoining figure ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\dfrac{\text{area }(\Delta\, ABC)}{\text{area }(\Delta\, DBC)} = \dfrac{AO}{DO}$. **(CBSE 2020)**



39. If AD and PS are medians of ΔABC and ΔPQR respectively where $\Delta ABC \sim \Delta PQR$, Prove that $\dfrac{AB}{PQ} = \dfrac{AD}{PS}$.

30. In the given figure, $DE \parallel AC$. Which of the following is correct?

$x = \dfrac{a+b}{ay}$ or $x = \dfrac{ay}{a+b}$

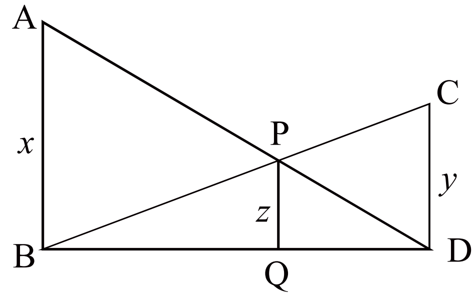


31. If three parallel lines are intersected by two transversals, then prove that the intercepts made by them on the transversals are proportional.

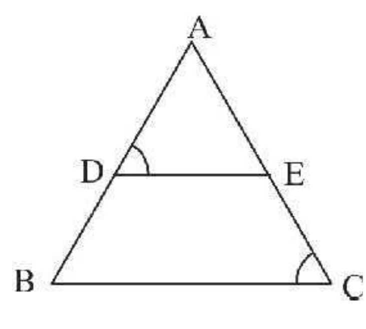
32. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how far she is away from the base of the pole.

33. Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

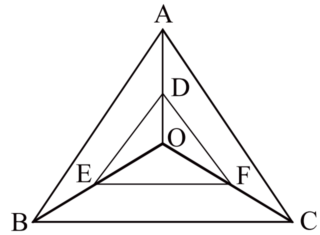
34. In the given figure, AB || PQ || CD, AB = x, CD = y and PQ = z. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



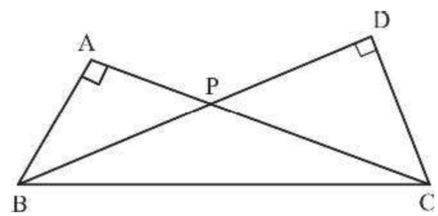
35. In the given figure, $\angle D = \angle E$ and $\frac{AD}{DB}=\frac{AE}{EC}$. Prove that ΔBAC is an isosceles triangle. **(CBSE 2020)**



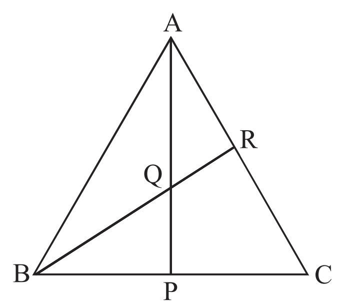
36. In the given figure, a point O inside ΔABC is joined to its vertices. From a point D on AO, DE is drawn parallel to AB and from a point E on BO, EF is drawn parallel to BC. Prove that DF || AC.



37. Two triangles ΔBAC and ΔBDC, right angled at A and D respectively are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P, then prove that $AP \times PC = DP \times PB$. **(CBSE 2019)**

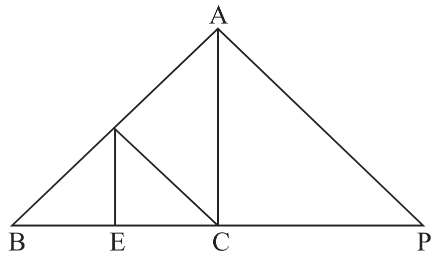


38. In the given figure, P is the mid point of BC and Q is the mid point of AP. If BQ when produced meets AC at R, prove that $RA = \frac{1}{3} CA$.

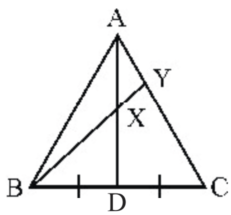


LONG ANSWER TYPE QUESTION

39. In the given figure, DE || AC and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that DC || AP.



40. In ΔABC, AD is a median, X is a point on AD such that AX : XD = 2 : 3. Ray BX intersects side AC in Y. Prove that BX = 4XY.



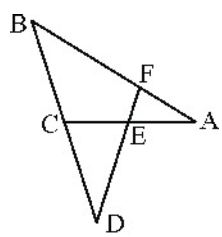
41. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that $\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$.

42. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

(CBSE 2019, 2020)

43. Through the mid point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL.

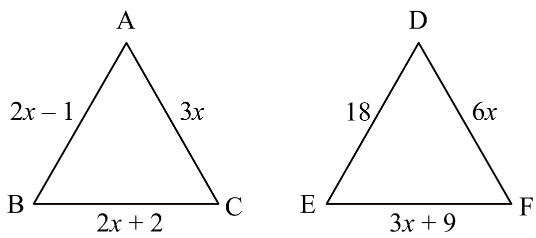
44. In the given figure, $\angle AEF = \angle AFE$ and E is the mid-point of CA. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$.



45. Sides AB and AC and median AD of ΔABC are respectively proportional to sides PQ and PR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$.

(CBSE 2020)

46. In the given figure $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle. **(CBSE 2020)**



47. The perimeters of two similar triangles are 30 cm and 20 cm repsectively. If one side of the first triangle is 9 cm long. Find the length of the corresponding side of the second triangle. **(CBSE 2020)**

48. If in ΔABC, D be a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$, then show that AD is bisector of $\angle A$.

ANSWERS AND HINTS

1. (d) 1cm

2. (b) $XY = \frac{1}{3}QR$

3. (a) 15 units

4. (c) BC.DE = AB.EF

5. $\Delta KPN \sim \Delta KLM$

$$\frac{x}{a} = \frac{c}{b+c}$$

$$x = \frac{ac}{b+c}$$

6. $\frac{AK}{AC} = \frac{HK}{BC} \quad \Rightarrow \quad \frac{10}{AC} = \frac{7}{3.5} \quad \Rightarrow \quad AC = 5 \text{ cm}$

7. $\angle D = \angle R$ (True)

$\angle F = \angle P$ (False)

8. 5 : 7

9. $\frac{AB}{PQ} = \frac{BC}{QR} \quad \Rightarrow \quad \frac{6}{4.5} = \frac{4}{x} \quad \Rightarrow \quad x = 3\text{cm}$

10. $\Delta PXY \sim \Delta PQR$

$$\frac{PX}{PQ} = \frac{XY}{QR} = \frac{1}{3}$$

$\therefore$ XY : QR = 1 : 3

11. $\frac{x+3}{3x+19} = \frac{x}{3x+4}$ (By B.P.T.)

$x = 2$

12. $\angle F = \angle C = 56°$

13. 2 : 3

14. $\frac{BD}{AB}=\frac{CE}{AC}$

Subtracting 1 from reciprocal

$\frac{AB}{BD}-1=\frac{AC}{CE}-1$

$\frac{AD}{BD}=\frac{AE}{CE}$

$\Rightarrow DE \parallel BC$

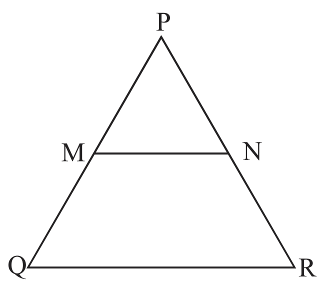
15. $DE \parallel AC$, $\frac{AD}{DB}=\frac{EC}{BE}$ ...(1) [$\because$ BPT]

$DC \parallel AP$, $\frac{AD}{DB}=\frac{CP}{BC}$ ...(2) [$\because$ BPT]

From (1) and (2), we get

$\frac{BE}{EC}=\frac{BC}{CP}$

16. In ΔPQR, $MN \parallel QR$



$\frac{MQ}{PM}=\frac{NR}{PN}$

Adding 1 to both sides and we get

$\frac{PQ}{PM}=\frac{PR}{PN}$

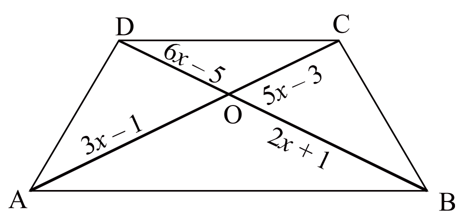
$\Rightarrow \frac{PM}{PQ}=\frac{PN}{PR}$

17. $\angle B=\angle AED$ (Given)

$\angle A=\angle A$ (Common)

$\therefore \Delta ABC \sim \Delta AED$ [AA similarity criterion]

18. $\Delta AOB \sim \Delta COD$



$$\frac{3x-1}{5x-3} = \frac{2x+1}{6x-5} \quad \Rightarrow \quad x = \frac{1}{2} \text{ or } 2$$

But $x = \frac{1}{2}$ is neglected because $(5x - 3)$ and $(6x–5)$ get negative value.

So, $x = 2$ is the required value.

19. $\frac{PX}{XQ} = \frac{PY}{YR} \quad \Rightarrow \quad \frac{1}{2} = \frac{4}{YR} \quad \Rightarrow \quad YR = 8 \text{ cm}$

$\therefore$ PR = 8 + 4 = 12 cm

$QR = \sqrt{(12)^2 - (6)^2} = 6\sqrt{3}$ cm

20. $\Delta ABC \sim \Delta EDC$ (AA Similarity criterion)

$$\frac{6}{3} = \frac{5}{CD}$$

CD = 2.5 cm

21. $\Delta BOE \sim \Delta DOA$ (AA Similarity criterion)

$$\frac{BO}{DO} = \frac{BE}{DA}$$

$$\frac{1}{2} = \frac{1.5}{DA}$$

DA = 3 cm

BC = DA = 3 cm (Opposite sides of a parallelogram)

22. (*i*) 65°

(*ii*) 45°

(*iii*) 45°

(*iv*) 70°

23. In ΔCAB, $DE \parallel AB$

$$\Rightarrow \frac{DC}{AC} = \frac{CE}{BC} \quad ...(1)$$

In ΔCDB, $BD \parallel EF$

$$\frac{CF}{DC} = \frac{CE}{BC} \qquad ...(2)$$

$$\Rightarrow \frac{DC}{AC} = \frac{CF}{DC}$$

$\Rightarrow DC^2 = CF \times AC$

24. In ΔCAB

$$\Rightarrow \frac{AD}{DC} = \frac{BE}{EC}$$

$\Rightarrow DE \parallel AB$ (Converse of B.P.T.)

$\Rightarrow \angle A = \angle D, \angle B = \angle E$

$\Rightarrow \angle A = \angle B$

$\Rightarrow \Delta ABC$ is isosceles.

25. In ΔPSQ

$$\frac{PB}{PQ} = \frac{PR}{PS} \quad ...(1)$$

In ΔPSC

$$\frac{PQ}{PC} = \frac{PR}{PS}$$

$$\frac{PB}{PQ} = \frac{PQ}{PC}$$

$\Rightarrow PB \times PC = (PQ)^2$

$\Rightarrow PB \times PC = 100\ cm^2$

26. $EC = BD \ (\because \Delta FEC \cong \Delta GBD)$

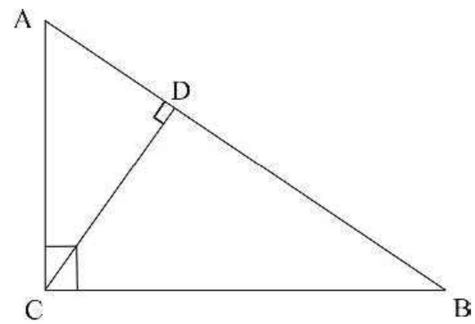
$AD = AE \ (\because \angle 1 = \angle 2)$

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$\Rightarrow DE \parallel BC$

$\Rightarrow \Delta ADE \sim \Delta ABC$

27. $\Delta ABC \sim \Delta CBD$



$\therefore BC^2 = AB.BD$ ...(1)

$\Delta ABC \sim \Delta ACD$

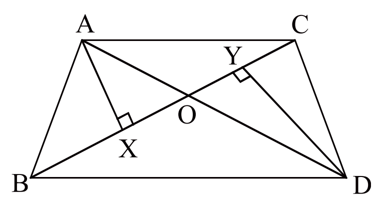
$\therefore AC^2 = AB.AD$ ...(2)

Divide (1) by (2), we get

$$\frac{BC^2}{AC^2} = \frac{BD}{AD}$$

28. Draw $AX \perp BC$ and $DY \perp BC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AX}{\frac{1}{2} \times BC \times DY} = \frac{AX}{DY} \quad ...(1)$$



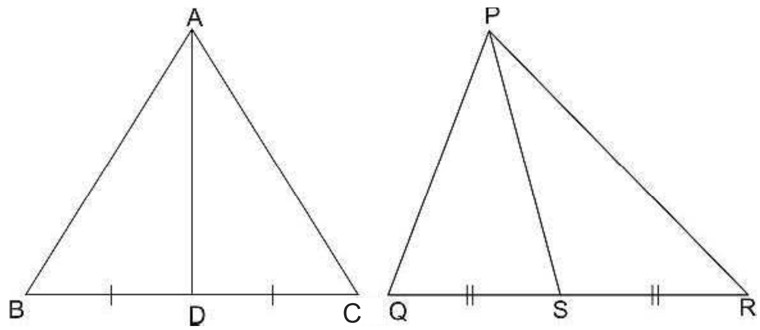
$\Delta AXO \sim \Delta DYO$ [AA similarity criterion]

$$\frac{AX}{DY} = \frac{AO}{DO} \quad ...(2) \quad \text{(C.P.S.T.)}$$

From (1) and (2), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

29.



As $\Delta ABC \sim \Delta PQR$, Hence $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QS}$

In ΔABD and ΔPQS

$\frac{AB}{PQ} = \frac{BD}{QS}$ and $\angle B = \angle Q$.

$\therefore$ $\Delta ABD \sim \Delta PQS$ (SAS Similarity criterion).

Hence, $\frac{AB}{PQ} = \frac{AD}{PS}$ (C.P.S.T.)

30. $\Delta BED \sim \Delta BCA$

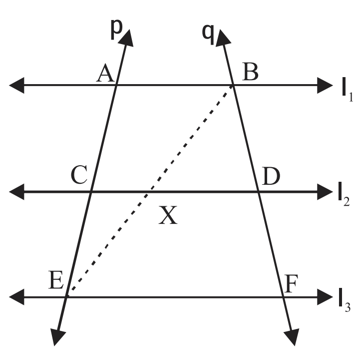
$$\frac{x}{y} = \frac{a}{a+b}$$

$$\Rightarrow x = \frac{ay}{a+b}$$

31. $l_1 \parallel l_2 \parallel l_3$

Constr: Join BE

Proof: In ΔABE



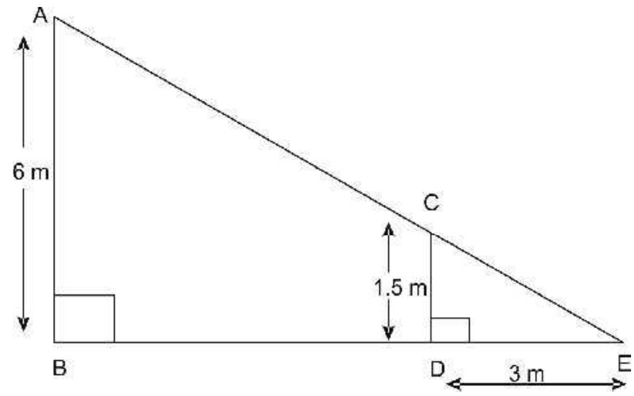
$\frac{AC}{CE} = \frac{BX}{XE}$ ...(1)

In ΔBEF

$\frac{BX}{XE} = \frac{BD}{DF}$ ...(2)

$\Rightarrow \frac{AC}{CE} = \frac{BD}{DF}$

32. $\Delta ABE \sim \Delta CDE$



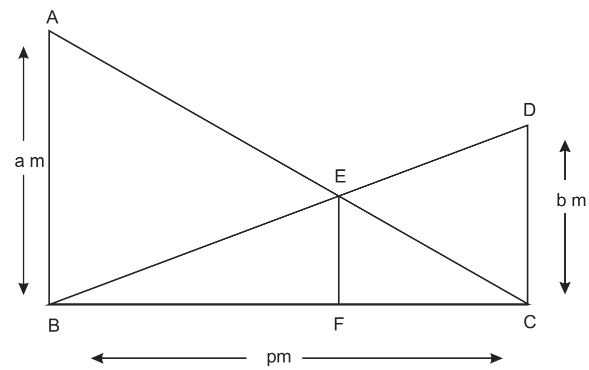
$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\frac{6}{1.5} = \frac{3+BD}{3}$$

$BD = 9m$

33. **To prove :** $EF = \dfrac{ab}{a+b}$

Proof : AB || EF || DC



$\Delta EFC \sim \Delta ABC$

$$\frac{EF}{AB} = \frac{FC}{BC} \quad ...(1)$$

$\Delta BFE \sim \Delta BCD$

$$\frac{EF}{CD} = \frac{BF}{BC} \quad ...(2)$$

Adding (1) and (2), we get

$$\frac{EF}{AB} + \frac{EF}{CD} = \frac{FC+BF}{BC}$$

$$EF\left[\frac{1}{AB} + \frac{1}{CD}\right] = \frac{BC}{BC}$$

$$EF\left[\frac{1}{a} + \frac{1}{b}\right] = 1$$

$$EF = \frac{ab}{a+b}$$

34. Same as Q. 33.

35. $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, DE || BC

$\therefore$ $\angle D = \angle B$ and $\angle E = \angle C$ (Corresponding Angles)

But $\angle D = \angle E$

So, $\angle B = \angle C$

$\therefore$ $AB = AC$

So, ΔABC is an isosceles triangle.

36. In ΔOAB, $\frac{OD}{DA} = \frac{OE}{EB}$ .... (1) ($\because$ BPT)

In ΔOBC, $\frac{OE}{EB} = \frac{OF}{FC}$ .... (2) ($\because$ BPT)

From (1) and (2), we get

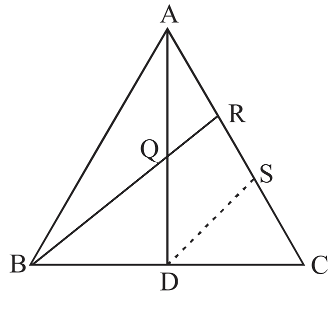
$\frac{OD}{DA} = \frac{OF}{FC}$

By converse of BPT, DF || AC.

37. $\Delta APB \sim \Delta DPC$ (AA Similarity criterion)

$\frac{AP}{DP} = \frac{PB}{PC}$ ($\because$ C.P.S.T.)

AP.PC = DP.PB



38. Draw PS || BR

In ΔCBR

PS || BR

$\Rightarrow$ CS = SR ....(1)

In ΔAPS

AR = RS ....(2)

From (1) and (2)

$AR = RS = SC$

$AR = \frac{1}{3}AC$

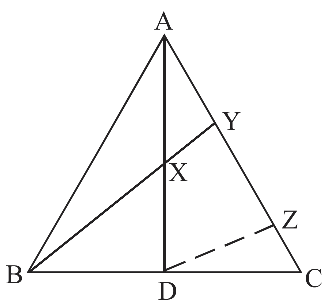
39. In ΔBCA

$\frac{BE}{EC} = \frac{BD}{DA}$ (B.P.T.) and $\frac{BE}{EC} = \frac{BC}{CP}$ (given)

$\Rightarrow \frac{BD}{DA} = \frac{BC}{CP}$

$\Rightarrow DC \parallel AP$ (Converse of B.P.T.)

40. Draw $DZ \parallel BY$



$\Delta AXY \sim \Delta ADZ$

$\Rightarrow \frac{AX}{AD} = \frac{XY}{DZ}$

$\Rightarrow 2DZ = 5XY$ (1)

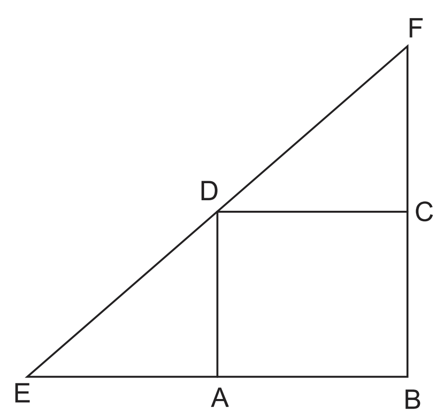
Now, $\Delta CDZ \sim \Delta CBY$

$\frac{CD}{CB} = \frac{DZ}{BY} \Rightarrow BY = 2DZ$ (2)

from (1) and (2)

$\Rightarrow BX = 4XY$

41. $\Delta EAD \sim \Delta EBF$



$\frac{EA}{EB} = \frac{AD}{BF}$

$\Rightarrow \frac{BF}{BE} = \frac{AD}{AE}$ (1)

$\Delta DCF \sim \Delta EBF$

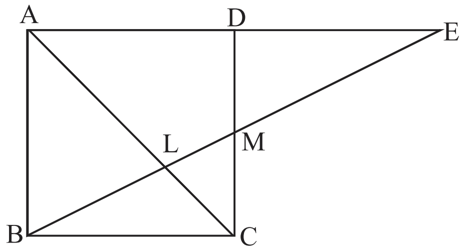
$\frac{DC}{EB} = \frac{CF}{BF}$

or $\dfrac{BF}{EB} = \dfrac{CF}{CD}$ (2)

from (1) and (2) $\dfrac{AD}{AE} = \dfrac{FB}{BE} = \dfrac{FC}{CD}$

42. Theorem 6.1 of NCERT.

43. $\Delta BMC \cong \Delta EMD$



$BC = DE$

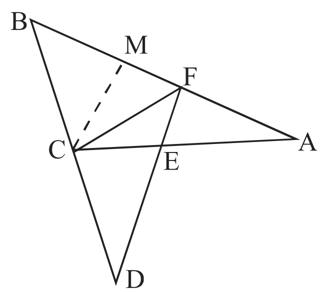
& $AD = BC$

$\Rightarrow AE = 2BC$

Now, $\Delta AEL \sim \Delta CBL$

$\Rightarrow EL = 2BL$

44. Draw $CM \parallel DF$,



In ΔACM

$EF \parallel CM$

$\Rightarrow \dfrac{AE}{CE} = \dfrac{AF}{FM}$

$\Rightarrow CE = MF$

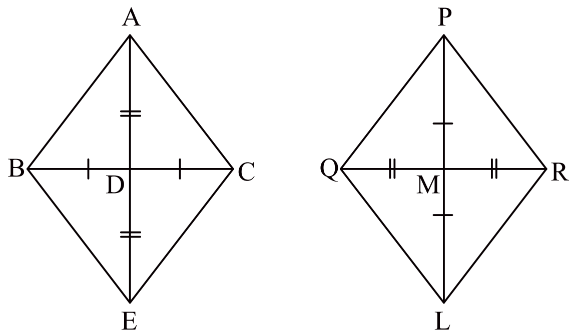
In ΔBDF

$\dfrac{BD}{CD} = \dfrac{BF}{MF} \Rightarrow \dfrac{BD}{CD} = \dfrac{BF}{CE}$

45. In ΔABC and ΔPQR

$\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{AD}{PM}$ ...(1)

Extend AD to a point E such that AD = DE and PM to point L such that PM = ML



$\therefore$ quadrilateral of ABEC and PQLR are parallelogram

($\because$ diagonals bisect each other)

$\therefore \quad AC = BE, AB = EC$

$PR = QL, PQ = LR$ ....(2)

From (1) and (2)

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2\,AD}{2\,PM} = \frac{AE}{PL}$$

$\therefore \quad \Delta ABE \sim \Delta PQL$

$\therefore \quad \angle BAE = \angle QPL$

Similarly, $\Delta AEC \sim \Delta PLR$

$\Rightarrow \quad \angle CAE = \angle RPL$ ...(4)

$\Rightarrow \quad \angle CAB = \angle RPQ$ [from (3) and (4)]

$\therefore \quad$ In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ and } \angle CAB = \angle RPQ$$

$\therefore \quad \Delta ABC \sim \Delta PQR$

46. $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$ ($\because \Delta ABC \sim \Delta DEF$)

$$\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

Solving, we get $x = 5$

$\therefore$ AB = 9 cm BC = 12 cm AC = 15 cm

DE = 18 cm EF = 24 cm FD = 30 cm

47. $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$$

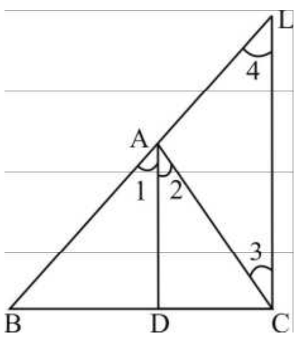
$\Rightarrow$ AB = kDE, BC = kEF, AC = kDF

$\therefore$ AB + BC + AC = k (DE + EF + DF)

$$\therefore \quad \frac{30}{20} = \frac{9}{x} \quad \Rightarrow \quad x = 6 \text{ cm}$$

48. Constr: Produce BA upto L such that AL = AC, Join CL

Proof: In ΔACL $\angle 3 = \angle 4$



In ΔBCL

$$\frac{BD}{DC} = \frac{AB}{AL} (\because AC = AL)$$

DA || CL

$\Rightarrow \angle 1 = \angle 4$

$\angle 2 = \angle 3$

$\Rightarrow \angle 1 = \angle 2$

Hence, AD is bisector of $\angle A$.

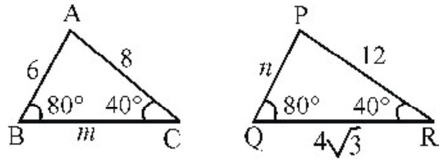
PRACTICE-TEST

Triangles

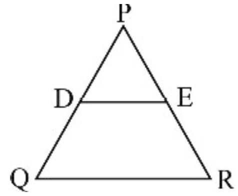
Time : 45 minutes *M.M. : 20*

SECTION - A

1. In the given figure, $\Delta ABC \sim \Delta PQR$, then find $(m + n)$ **1**



2. In the given figure, DE || QR, PQ = 5.6 cm and PD = 1.6 cm. Find PE : ER. **1**



3. ΔABC is such that AB = 3cm, BC = 2cm and CA = 2.5 cm. If $\Delta PQR \sim \Delta ABC$ and QR = 6 cm, then find the perimeter of ΔPQR is _________. **1**

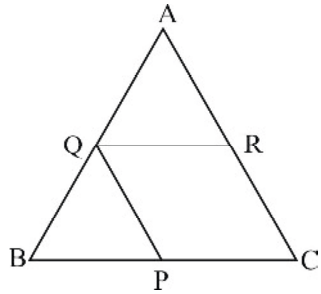
4. If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{FD}$, then **1**

(a) $\Delta BCA \sim \Delta FDE$ (b) $\Delta FDE \sim \Delta ABC$

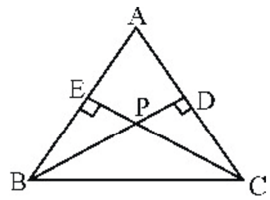
(c) $\Delta CBA \sim \Delta FDE$ (d) $\Delta FDE \sim \Delta CAB$

SECTION B

5. In the given figure, QR || BC and QP || AC. If PB = 12 cm, PC = 20 cm and AR = BQ = 15 cm, calculate AQ and CR. **2**



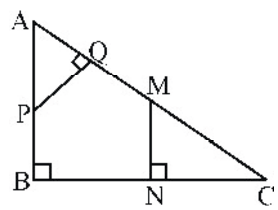
6. In the given figure, BD $\perp$ AC and CE $\perp$ AB. Prove that BP × PD = EP × PC.



7. If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 3, prove that one of the parallel sideds is three times the other. **2**

SECTION C

8. In the given figure, if AB $\perp$ BC, PQ $\perp$ AC and MN $\perp$ BC, prove that $\Delta APQ \sim \Delta MCN$.



9. E is a point on the side AD produced of a prallelogram ABCD and BE interects CD at F. Show that AB × BC = AE × CF. **3**

SECTION D

10. State and prove Basic Proportionality Theorem. **4**

CHAPTER 7 Co-ordinate Geometry

1.

Coordinate Geometry

↓

A system of geometry where the position of points on the plane is described using an ordered pair P(x, y)

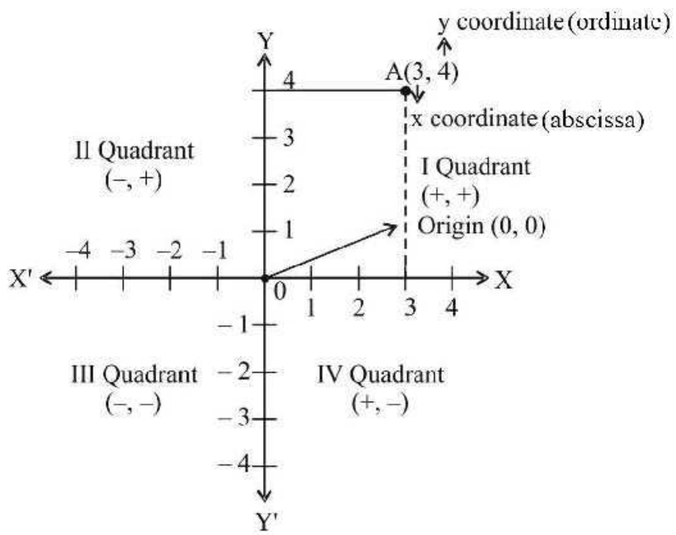
↓

The distance of a point from y-axis is called x-coordinate or abscissa	The distance of a point from x-axis is called y-coordinate or ordinate

↓

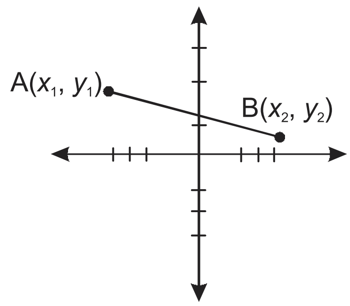
The coordinate of point on x-axis is (x, 0). The coordinate of point on y-axis is (0, y). The coordinate of point on 2-Dimensional plane i.e., cartesian plane is (x, y)

Cartesian Plane



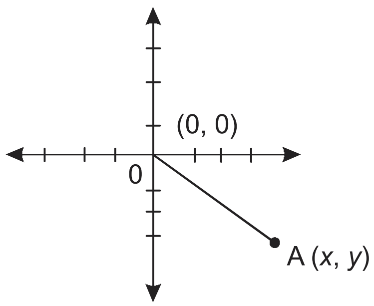
2. Distance Formula

Finding distance between two given points :



AB (Distance between A and B) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. ***Distance of a point from origin :***



Using distance formula

OA = $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

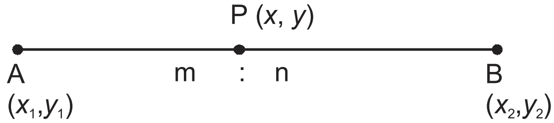
4. ***Midpoint formula :***

Coordinates of mid points of AB where $A(x_1, y_1)$ and $B(x_2, y_2)$ are : $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

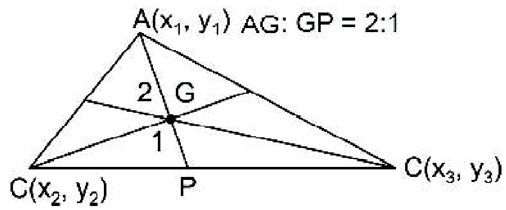
5. ***Section formula:***

The coordinates of a point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



6. ***Centroid of given triangle is given by :***



$$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

VERY SHORT ANSWER TYPE QUESTIONS

1. P is a point on x-axis at a distance of 3 unit from y-axis to its left. The co-ordinates of P are :

(*a*) (3, 0) (*b*) (0, 3)

(*c*) (– 3, 0) (*d*) (0, – 3)

2. The distance of $P(3, -2)$ from y-axis is :

(*a*) 3 units (*b*) 2 units

(*c*) – 2 units (*d*) $\sqrt{13}$ units

3. The co-ordinates of two points are (6, 0) and (0, – 8). The co-ordinates of the mid points are:

(*a*) (3, 4) (*b*) (3, – 4)

(*c*) (0, 0) (*d*) (– 4, 3)

4. If the distance between $P(4, 0)$ and $Q(0, x)$ is 5 units, the value of x will be:

(*a*) 2 (*b*) ± 3

(*c*) 4 (*d*) 5

5. The co-ordinates of the point where line $\frac{x}{a}+\frac{y}{b}=7$ intersects y-axis are:

(*a*) (a, 0) (*b*) (0, b)

(*c*) (0, 7b) (*d*) (2a, 0)

6. The area of triangle OAB, the co-ordinates of whose vertices are A(4, 0), B(0, – 7) and *O* origin, is :

(*a*) 11 sq. units (*b*) 18 sq. units

(*c*) 28 sq. units (*d*) 14 sq. units

7. The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is:

(*a*) 6 units (*b*) 4 units

(*c*) 3 units (*d*) 2 units

8. The co-ordinate of the point which is reflection of point (–3, 5) in *x* axis are:

(*a*) (3, 5) (*b*) (3, –5)

(*c*) (–3, –5) (*d*) (–3, 5)

9. The co-ordinates of vertex A of ΔABC are (– 4, 2) and a point D which is mid point of BC are (2, 5). The coordinates of centroid of ΔABC are:

(*a*) (0, 4) (*b*) $\left(-1, \frac{7}{2}\right)$

(*c*) $\left(-2, \frac{7}{3}\right)$ (*d*) (0, 2)

10. The distance between the line $2x + 4 = 0$ and $x - 5 = 0$ is:

(*a*) 9 units (*b*) 1 unit

(*c*) 5 units (*d*) 7 units

11. The perimeter of triangle formed by the points (0, 0), (2, 0) and (0, 2) is:

(*a*) 4 units (*b*) 6 units

(*c*) $6\sqrt{2}$ units (*d*) $4+2\sqrt{2}$ units

12. If the centroid of the triangle formed by (9, *a*), (*b*, – 4) and (7, 8) is (6, 8), then the value *a* and *b* are :

(*a*) $a = 4, b = 5$ (*b*) $a = 5, b = 4$

(*c*) $a = 5, b = 2$ (*d*) $a = 20, b = 2$

13. The centre of circle having end points of its diameter as (–4, 2) and (4, –3) is:

(*a*) (2, –1) (*b*) (0, –1)

(*c*) $(0, -\frac{1}{2})$ (*d*) $(4, -\frac{5}{2})$

14. The distance between the points (0, 0) and $(a - b, a + b)$ is:

(*a*) $2\sqrt{ab}$ (*b*) $\sqrt{2a^2 + ab}$

(*c*) $2\sqrt{a^2 + b^2}$ (*d*) $\sqrt{2a^2 + 2b^2}$

SHORT ANSWER TYPE QUESTIONS-I

15. For what value of p, the points (2, 1), $(p, -1)$ and (– 1, 3) are collinear.

16. Three consecutive vertices of a parallelgram are (–2, –1), (1, 0) and (4, 3). Find the co-ordinates of the fourth vertex.

17. Find the points of trisection of the line segment joining the points (1, – 2) and (– 3, 4).

18. A circle has its centre at (4, 4). If one end of a diameter is (4, 0) then find the coordinates of the other end.

19. Find the ratio in which P(4, m) divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.

20. Show that the points (– 2, 3), (8, 3) and (6, 7) are the vertices of a right angled triangle.

21. Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

(CBSE 2019)

22. Find the ratio in which y-axis divides the line segment joining the points A(5, – 6) and B(– 1, – 4).

23. Find the co-ordinates of a centroid of a triangle whose vertices are (3, – 5), (– 7, 4) and (10, – 2).

24. Find the relation between x and y such that the points (x, y) is equidistant from the points (7, 1) and (3, 5).

25. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis. Also find the coordinates of the point on x-axis.

26. What is the value of a if the points (3, 5) and (7, 1) are equidistant from the point (a, 0) ?

27. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3). then find the value of x.

28. A(5, 1), B(1, 5) and C(–3, –1) are the vertices of ΔABC. Find the length of median passing through A.

29. Name the type of triangle formed by the points A(– 5, 6), B(– 4, – 2) and C(7, 5).

30. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, – 4). How many such points are there?

31. A line intersects the y-axis and x-axis at the point P and Q. If (2, –5) is the midpoint of PQ then find the co-ordinates of P and Q. **(CBSE 2017)**

32. If A(–2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

(CBSE 2018)

33. Let P and Q be the points of trisection of the line segment joining the points A(2, – 2) and B(– 7, 4) such that P is nearer to A. Find the co-ordinates of P and Q.

SHORT ANSWER TYPE QUESTIONS-II

34. The line segment joining the points A(2, 1) and B(5, – 8) is trisected at the point P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k. **(CBSE 2019)**

35. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (– 2, – 5) and (6, 3). Find the co-ordinates of the point of intersection.

36. Find the ratio in which line x + 3y – 14 = 0 divides the line segment joining A (–2, 4) and B(1, 7).

37. Find the centre of circle passing through (5, –8), (2, –9) and (2, 1).

38. Point P divides the line segment joining the points $A(2, 1)$ and $B(5, -8)$ such that $\frac{\text{AP}}{\text{PB}} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$. Find the value of k.

39. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal then prove that $3x = 2y$. **(CBSE 2017)**

40. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divides the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? **(CBSE 2017)**

41. If $A(-3, 2)$, $B(x, y)$ and $C(1, 4)$ are the vertices of an isosceles triangle with AB = BC. Find the value of $(2x + y)$.

42. If the point $P(3, 4)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$ then prove that $3b - 4a = 0$.

LONG ANSWER TYPE QUESTIONS

43. If the co-ordinates of the mid-points of the sides of a triangle are (3, 1), (5, 6) and (–3, 2). Find the co-ordinates of its vertices and centroid.

44. If $P(x, y)$ is any point on the line joining $A(a, 0)$ and $B(0, b)$ then show that $\frac{x}{a} + \frac{y}{b} = 1$.

45. Find the co-ordinates of the point which divides the line segment joining the points A(2, 6) and B(10, –10) in to 4 equal parts.

46. Find the relation between x and y if $A(x, y)$, $B(-2, 3)$ and $C(2, 1)$ form an isosceles triangle with $AB = AC$.

47. Prove that the point $\left(x, \sqrt{1 - x^2}\right)$ is at a distance of 1 unit from the origin.

48. Prove that the points (1, 2), (9, 3) and (17, 4) are collinear by using section formula. **(CBSE 2017)**

49. Determine the ratio in which the line 3x + y – 9 = 0 divides the line segment joining the points (1, 3) and (2, 7).

50. In a triangle PQR, the co-ordinates of points P, Q and R are (3, 2), (6, 4) and (9, 3) respectively. Find the co-ordinates of centroid G.

51. If co-ordinates of two adjacent vertices of a parallelogram are (3, 2) and (1, 0) and diagonats bisect each other at (–2, 5). Find the co-ordinates of the other vertices.

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. (c) (– 3, 0)

2. (a) 3 units

3. (b) (3, – 4)

4. (b) ± 3

5. (c) (0, 7b)

6. (d) 14 sq. units

7. (c) 3 units

8. (c) (–3, –5)

9. (a) (0, 4)

10. (d) 7 units

11. (d) $\left(4+2\sqrt{2}\right)$ units

12. (d) a = 20, b = 2

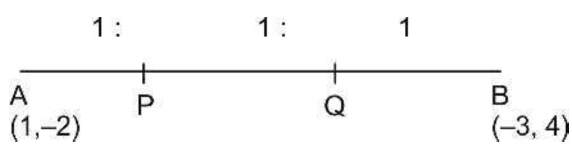
13. (c) $0,-\frac{1}{2}$

14. (d) $\sqrt{2a^2+2b^2}$

15. (1, 2)

16. 18 sq. units

17.



AP : PB = 1 : 2

AQ : QB = 2 : 1

$$P=\left(-\frac{1}{3},0\right)$$

$$Q=\left(-\frac{5}{3},2\right)$$

18. (4, 8)

19. Ratio 1 : 1, m = 0

20. Show using pythagoras theorem and distance formula.

21. (0, –2)

22. 5 : 1

23. (2, – 1)

24. $x - y = 2$

25. 3 : 5 ; $\left(\frac{17}{8}, 0\right)$

26. $a = 2$

27. $x = 2$

28. $\sqrt{37}$ units

29. Using distance formula, scalene triangle.

30. $x = 1, x = -15$

Two such points are there.

31. (4, –10)

32. $a = 1, b = 1, AB = CD = \sqrt{10}, AD = BC = \sqrt{10}$

33. $P(-1, 0), Q(-4, 2)$

34. $P(3, -2)$

Put value of $x = 3, y = -2$ in equation, then $k = -8$.

35. Let $P(x, y)$ be the point and $m : n$ is the ratio

then $x = \dfrac{6n - 2m}{m + n}, \quad y = \dfrac{3n - 5m}{m + n}$ ...(1)

From equation of line $x = 3y \Rightarrow \dfrac{x}{y} = 3$

By putting $x = 3y$ or $\dfrac{x}{y} = 3$ in (1)

$m : n = 3 : 13$

Then $P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$

36. 1 : 2

37. Centre (2, –4)

38. $K = \dfrac{-17}{4}$

39. $PA = PB$, Use distance formula

40. 2 : 9

41. $2x + y = 1$

42. $3b - 4a = 0$ proved by using distance formula.

43. A(–1, 7), B(–5, –3), C(11, 5), co-ordinate of centroid $\left(\dfrac{5}{3}, 3\right)$

44. Use section formula.

45. (4, 2), (6, –2) and (8, –6)

46. $y = 2x+2$ is required relation

49. Required ratio is 3:4

50. $G(x, y) = (6, 3)$

51. Other vertices (–5, 10) and (–7, 8)

PRACTICE-TEST

Coordinate Geometry

Time : 45 Minutes ***M.M. : 20***

SECTION - A

1. x axis divides the line segment joining A(2, –3) and B(5, 6) in the ratio **1**

(i) 2: 3 (ii) 3:5

(iii) 1 : 3 (iv) 2:1

2. What is the distance between the points A(c, 0) and B(0, $-c$) **1**

3. The distance of point P(– 6, 8) from the origin is _______ . **1**

4. Find the value of 'a' so that the point (3, a) lies on the line segment $2x - 3y = 5$. **1**

SECTION B

5. Find the point on y-axis which is equidistant from (–5, –2) and (3, 2). **2**

6. If the points A(8, 6) and B(x, 10) lie on the circle whose centre is (4, 6), then find the value of x. **2**

7. Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0). **2**

SECTION C

8. Show that the points A(–3, 2), B(– 5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus. **3**

9. Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y. **3**

SECTION D

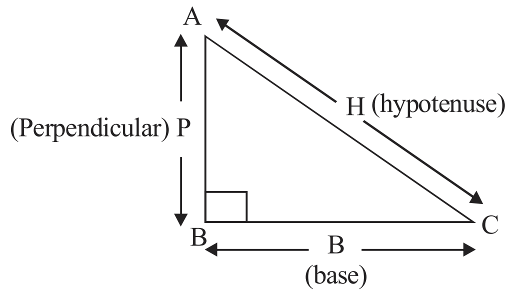
10. If the point P divides the line segment joining the points A(–2, –2) and B(2, – 4) such that $\frac{AP}{AB} = \frac{3}{7}$, then find the coordinate of P. 4

CHAPTER

8 Introduction to Trigonometry

KEY POINTS

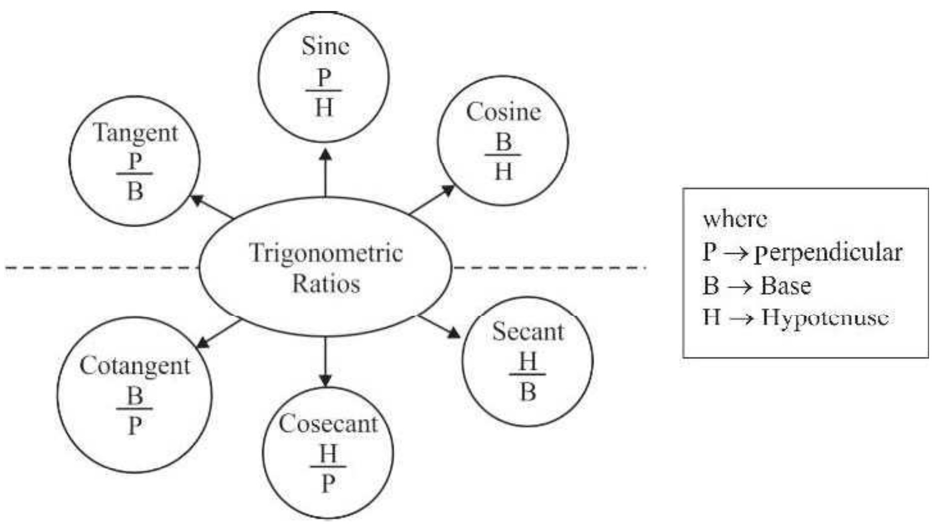
- A branch of mathematics which deals with the problems related to right angled triangles. It is the study of relationship between the sides and angles of a right angled triangle.



  Note : For $\angle A$, Perpendicular is BC and Base is AB.

  For $\angle C$, Perpendicualr is AB and Base is BC.

  Trigonometric Ratios of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.

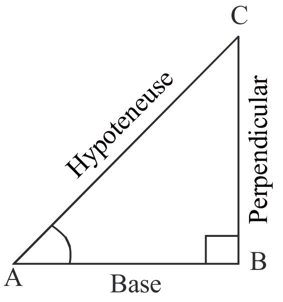


Mind Trick: To learn the relationship of sine, cosine and tangent follow this sentence.

Some **P**eople **H**ave **C**urly **B**rown **H**air **T**hrough **P**roper **B**rushing

$\sin A = \frac{P}{H}$ $\quad\cos A = \frac{B}{H}$ $\quad\tan A = \frac{P}{B}$

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Reciprocal ratios:

$$\sin\theta = \frac{1}{\text{cosec }\theta}, \qquad \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \qquad \cot\theta = \frac{1}{\tan\theta}$$

3. $$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

4. Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. Trigonometric ratios of some specific angles

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\sin\theta = \cos\theta$, find the value of θ.
2. Find the value of $\tan^4\theta + \cot^4\theta$, if $\sin\theta - \cos\theta = 0$.
3. Find the value of $\tan\theta + \cot\theta$, if $\tan^2\theta - 3\tan\theta + 1 = 0$.
4. If $\tan\theta = \frac{4}{3}$, then find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.
5. If $3x = \operatorname{cosec}\theta$ and $\frac{3}{x} = \cot\theta$, then find $3\left(x^2 - \frac{1}{x^2}\right)$.
6. If $x = a\sin\theta$ and $y = a\cos\theta$, then find the value of $x^2 + y^2$.
7. If $\cos A = \frac{3}{5}$, find the value of $4 + 4\tan^2 A$.
8. Find the value of $9\sec^2 A - 9\tan^2 A$.
9. Express $\sec\theta$ in terms of $\cot\theta$.
10. If $x = a\sec\theta$, $y = b\tan\theta$, then find the value of $b^2x^2 - a^2y^2$.

11. Find the value of $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$, if $\tan\theta = \dfrac{4}{3}$.

12. Find the value of $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}$

13. Given $\tan\theta = \dfrac{1}{\sqrt{3}}$, find the value of $\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$. (CBSE, 2010)

14. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\tan^2\theta + \cot^2\theta$.

15. If $5\tan\theta - 4 = 0$, then value of $\dfrac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is :

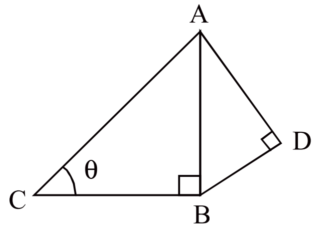
(a) $\dfrac{5}{3}$ (b) $\dfrac{5}{6}$ (c) 0 (d) $\dfrac{1}{6}$

16. $3\tan^2\theta - 3\sec^2\theta + 4$ is equal to :

(a) 3 (b) 2

(c) 1 (d) 0

17. In Fig. if $AD = 4$ cm, BD = 3 cm and CB = 12 cm. then $\cot\theta$ is :



(a) $\dfrac{12}{5}$ (b) $\dfrac{5}{12}$

(c) $\dfrac{13}{12}$ (d) $\dfrac{12}{13}$

18. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then $x^2 + y^2$ is :

(a) 25 (b) 45 (c) 7 (d) 49

19. If $\sin\theta = \frac{a}{b}$, then the value of $\sec\theta + \tan\theta$ is :

(a) $\sqrt{\frac{a+b}{a-b}}$ (b) $\frac{a+b}{a-b}$ (c) $\sqrt{\frac{b+a}{b-a}}$ (d) $\frac{b+a}{b-a}$

SHORT ANSWER TYPE QUESTIONS (I)

Prove that :

20. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

21. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \operatorname{Sec}\theta$

22. If $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$.

23. If $7\sin^2\theta + 3\cos^2\theta = 4$, then show that $\tan\theta = \frac{1}{\sqrt{3}}$.

24. Find the value of $\cos\theta$, if $\sec\theta + \tan\theta = 5$.

25. If $3\cot A = 4$, find the value of $\frac{\operatorname{cosec}^2 A + 1}{\operatorname{cosec}^2 A - 1}$.

26. Find the value of $\tan^3\theta + \cot^3\theta$, if $\tan\theta + \cot\theta = 2$.

27. Find the value of $\tan\theta$, if $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$.

(CBSE 2011)

28. In ΔABC, right angled at B, AB = 5 cm and $\angle ACB = 30°$. Find BC and AC.

29. Show that : $\frac{1-\sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

30. Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, $\theta \le 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS-II

Prove that :

31. $\dfrac{1}{\sec x - \tan x} - \dfrac{1}{\cos x} = \dfrac{1}{\cos x} - \dfrac{1}{\sec x + \tan x}$

32. $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta = \sec\theta\,\text{cosec}\,\theta + 1$

33. $\sec A\,(1 - \sin A)(\sec A + \tan A) = 1$ **(CBSE 2023)**

34. If $\sec\theta = x + \dfrac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$.

35. If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

36. Prove that $\cos\theta = \dfrac{p^2 - 1}{p^2 + 1}$, if $p = \text{cosec}\,\theta + \cot\theta$.

37. Show that: $x^2 + y^2 + z^2 = r^2$, if $x = r\cos\alpha\sin\beta$, $y = r\cos\alpha\cos\beta$ and $z = r\sin\alpha$

38. Find the value of $\sin^{10}\theta + \text{cosec}^{19}\theta$, if $\sin\theta + \text{cosec}\,\theta = 2$.

39. Prove that: $2\sec^2 x - \sec^4 x - 2\text{cosec}^2 x + \text{cosec}^4 x = \cot^4 x - \tan^4 x$

40. Find the value of $\text{cosec}\,\theta$, *if* $\text{cosec}\,\theta - \cot\theta = \dfrac{1}{3}$.

41. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

42. Evaulate : $\dfrac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\text{cosec}\,30° + \sec 60° - \cot^2 30°}$

43. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$ (CBSE, 2023)
Prove that : $a^2 + b^2 = m^2 + n^2$

LONG ANSWER TYPE QUESTIONS

Prove that:

44. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$

45. $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$

46. $(1+\cot A+\tan A)(\sin A-\cos A)=\sin A\tan A-\cot A\cos A$

47. If $\sin\theta+\cos\theta=m$ and $\sec\theta+\operatorname{cosec}\theta=n$ then show that $n(m^2-1)=2m$

48. Prove that: $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\operatorname{cosec}\theta$ **(CBSE 2023)**

49. Prove that :

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

50. If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\sin\beta}=n$, then prove that $(m^2+n^2)\cos^2\beta=n^2$

51. Prove that :

$$\sec^2\theta-\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}=1$$

52. Prove that : $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$

53. Prove that: $\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\frac{\sin\theta}{1-\cos\theta}$

54. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$ **(CBSE 2020)**

55. Prove $\frac{\cot A-\cos A}{\cot A+\cos A}=\sec^2 A+\tan^2 A-2\sec A\tan A$

56. Prove $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

57. If $\cos(A+B)=\sin(A-B)=\frac{1}{2}$, $0<A+B<90°$ and $A>B$ then find the value of A and B.

58. If $\tan\theta+\sin\theta=m$, $\tan\theta-\sin\theta=n$, then prove that $m^2-n^2=4\sqrt{mn}$.

59. Prove that : $l^2m^2(l^2+m^2+3)=1$, if

$l=\operatorname{cosec}x-\sin x, m=\sec x-\cos x$

60. Prove that : $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}$

61. Prove that : $\frac{(1+\sin x-\cos x)^2}{(1+\sin x+\cos x)^2}=\frac{1-\cos x}{1+\cos x}$ **(CBSE 2019)**

62. Prove that : $\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}=2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$ **(CBSE 2019)**

63. If $4\tan\theta=3$, then find the value of $\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}$ **(CBSE 2018)**

64. Prove that : $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\sec\theta+\tan\theta$ **(CBSE 2018)**

65. Prove that : $\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\operatorname{cosec}^2\theta}=2$

66. Prove that : $\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$

67. If $\operatorname{cosec}\theta=4x+\frac{1}{16x}$, then prove that $\operatorname{cosec}\theta\pm\cot\theta=8x$ or $\frac{1}{8x}$

ANSWERS AND HINTS

1. $45°$
2. 2
3. 3
4. 7
5. $\frac{1}{3}$
6. a^2
7. $\frac{100}{9}$
8. 9
9. $\frac{\sqrt{1+\text{Cot}^2\theta}}{\text{Cot}\,\theta}$
10. a^2b^2
11. $\frac{16}{9}$
12. $\tan^2\theta$
13. $\frac{1}{2}$
14. $\frac{10}{3}$
15. (c) 0
16. (c) 1
17. (a) $\frac{12}{5}$
18. (a) 25
19. (c) $\sqrt{\frac{b+a}{b-a}}$
20. LHS = $\sec^2\theta\,(\sec^2\theta - 1)$
    RHS = $\tan^2\theta\,(\tan^2\theta + 1)$
    Use $1 + \tan^2\theta = \sec^2\theta$
21. Rationalize and proceed in LHS
22. Squaring both sides of x and y and then subtracting.
23. Divide both sides by $\cos^2\theta$
24. $\cos\theta = \frac{5}{13}$
25. $\frac{17}{8}$
26. 2
27. $\sqrt{2}-1$
28. AC = 10, BC = $5\sqrt{3}$, use Pythagoras theorem

30. 60°

38. 2

40. $\text{cosec}\ \theta = \frac{5}{3}$

41. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Square both sides and get $1 + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow 2\cos\theta\sin\theta = 2\cos^2\theta - 1$ ...(1)

Now square $(\cos\theta - \sin\theta)^2$ and get

$(\cos\theta - \sin\theta)^2 = 1 - 2\cos\theta\sin\theta$ ...(2)

Substitute (1) in (2)

42. 9.

43. Find m^2 and n^2 and then add

49. Rationalise $\frac{1}{\text{cosec}\,\theta + \cot\theta}$ in LHS and proceed, use $\frac{1}{\sin\theta} = \text{cosec}\ \theta$.

Rationalise $\frac{1}{\text{cosec}\,\theta - \cot\theta}$ on RHS and proceed, use $\frac{1}{\sin\theta} = \text{cosec}\ \theta$.

50. Find m^2 and n^2 and substitute in LHS.

51. Take common $\sin^2\theta$ in Numerator and $\cos^2\theta$ in Denominator of 2nd term on LHS and replace 1 by $\sin^2\theta + \cos^2\theta$.

54. $(\sin\theta + \cos\theta) = \sqrt{3}$

square both sides and get value of $\frac{1}{\sin\theta \times \cos\theta}$

Change $\tan\theta + \cot\theta$ into $\sin\theta$ and $\cos\theta$ proceed.

55. Change $\cot A = \frac{\cos A}{\sin A}$, take cos A common from Numerator and Denominator, Rationalise remaining term and change into sec A and tan A.

56. LHS $= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$, write $1 = \sin^2\theta + \cos^2\theta$ and proceed.

57. cos (A + B) = $\frac{1}{2}$ = cos 60°

$\Rightarrow A + B = 60°$

sin (A – B) = $\frac{1}{2}$ = sin 30°

$\Rightarrow$ A – B = 30°

on solving, A = 45°, B = 15°

58. Find m^2 and n^2 substitute in $m^2 - n^2$ and substitute m and n in $4\sqrt{mn}$

62. Convert cot θ and cosec θ into sin θ and cos θ

and use $\sin^2\theta = 1 - \cos^2\theta$

63. Divide Numerator and Denominator by cos θ, and use sec θ = $\sqrt{1+\tan^2\theta}$

or use pythagoras theorem and trigonometeric ratios,

Ans. $\frac{13}{11}$

PRACTICE-TEST

Introduction to Trigonometry

Time : 45 Minutes ***M.M.: 20***

SECTION-A

1. If $\sin\theta = \frac{4}{5}$, then find the value of $\cos\theta$. **1**

2. Find the value of $\tan^4\theta + \cot^4\theta$, if $\tan\theta + \cot\theta = 2$ **1**

3. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then find the value of $5\left(x^2 - \frac{1}{x^2}\right)$.

4. If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is : **1**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

SECTION-B

5. If $5\tan\theta = 4$, then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$. **2**

6. Find the value of $5\sin\theta - 3\cos\theta$, if $3\sin\theta + 5\cos\theta = 5$. **2**

7. Prove that : $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \operatorname{cosec}\alpha$ **2**

SECTION-C

8. Prove that : $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$ **3**

9. Prove that : $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ **3**

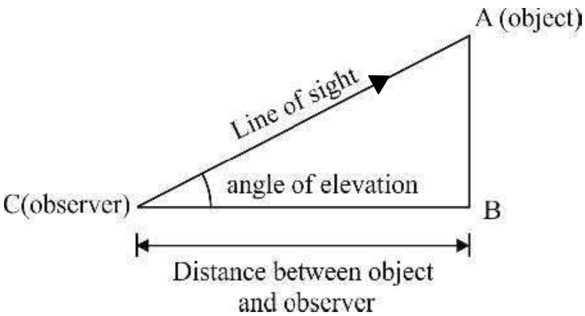
SECTION-D

10. Prove that : $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1-\sin\theta}$. **4**

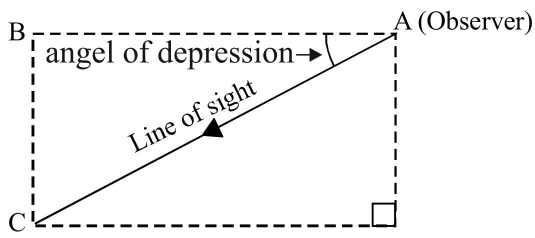
CHAPTER

9 Some Applications of Trigonometry

KEY POINTS



- **Angle of Elevation:** Let AB be height of object. C is the observer looking upto to A (the top of AB). AC is called the line of sight and $\angle ACB$ is angle of elevation.
- **Angle of Depression :** Let A is the observer looking at C (the object) from a height BC. AC is line of sight and $\angle BAC$ is angle of depression.



- If the observer moves towards the object the angle of elevation increases and if the observer moves away from the object, the angle of elevation decreases.
- Numerically, angle of elevation is equal to angle of depression (both are measured with the same horizontal parallel planes).

VERY SHORT ANSWER TYPE QUESTIONS

1. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is :

   (a) 45° (b) 30° (c) 60° (d) 90°

2. The tops of the poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, then l =

(*a*) 26 m (b) 16 m (c) 12 m (d) 10 m

3. A pole of height 6 m casts a shadow $2\sqrt{3}$ m long on the ground. the angle of elevation of the sun is (CBSE 2017)

(a) 30° (b) 60° (c) 45° (d) 90°

4. A ladder leaning aginast a wall makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, then the length of the ladder is —

(CBSE 2016)

(a) 3 m (b) 4 m (c) 5 m (d) 6 m

5. If a tower is 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then the angle of elevation of the sun is: (CBSE, 2017)

(a) 30° (b) 45° (c) 60° (d) 90°

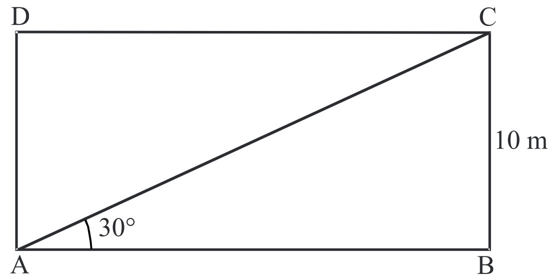
6. A tower is 50 m high. When the sun's altitude is 45° then what will be the length of its shadow?

7. The length of shadow of a pole 50 m high is $\frac{50}{\sqrt{3}}$ m. Find the sun's altitude.

8. Find the angle of elevation of a point which is at a distance of 30 m from the base of a tower $10\sqrt{3}$ m high.

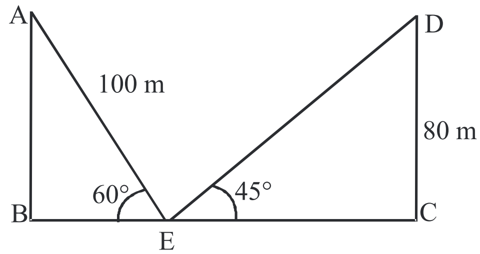
9. A kite is flying at a height of $50\sqrt{3}$ m from the horizontal. It is attached with a string and makes an angle 60° with the horizontal. Find the length of the string.

10. In the given figure find the perimeter of rectangle ABCD.

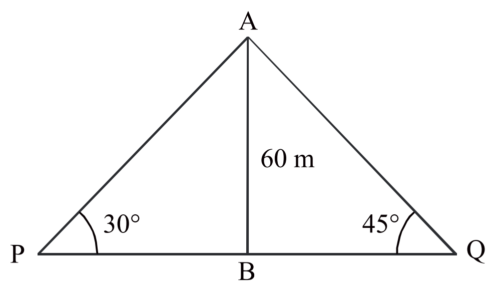


SHORT ANSWER TYPE QUESTIONS – I

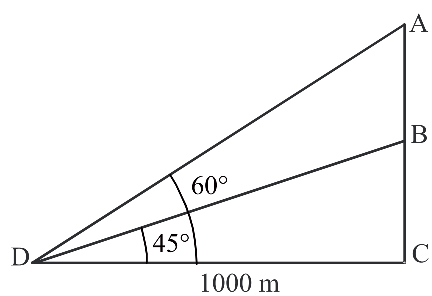
11. In the figure, find the value of BC.



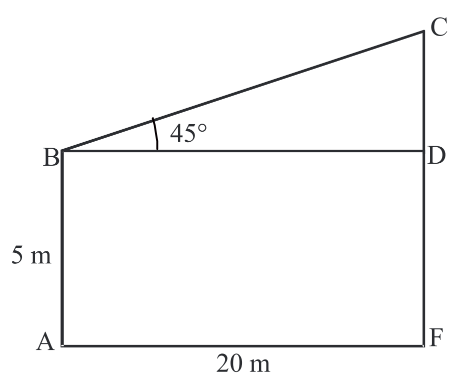
12. In the figure, two persons are standing at the opposite direction P & Q of the tower. If the height of the tower is 60 m then find the distance between the two persons.



13. In the figure, find the value of AB.



14. In the figure, find the value of CF.



15. If the horizontal distance of the boat from the bridge is 25 m and the height of the bridge is 25 m, then find the angle of depression of the boat from the bridge.

16. The string of a kite is 150 m long and it makes an angle 60° with the horizontal. Find the height of the kite above the ground. (Assume string to be tight)

17. The shadow of a vertical tower on level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower.

(Use $\sqrt{3} = 1.73$)

18. An aeroplane at an altitude of 200 m observes angles of depression of opposite points on the two banks of the river to be 45° and 60°, find the width of the river. (Use $\sqrt{3} = 1.732$)

19. The angle of elevation of a tower at a point is 45°. After going 40 m towards the foot of the tower, the angle of elevation of the tower becomes 60°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

20. The upper part of a tree broken over by the wind makes an angle of 30° with the ground and the distance of the foot of the tree from the point where the top touches the ground is 25 m. What was the total height of the tree?

21. A vertical flagstaff stands on a horizontal plane. From a point 100 m from its foot, the angle of elevation of its top is found to be 45°. Find the height of the flagstaff.

22. The length of a string between kite and a point on the ground is 90 m. If the string makes an angle α with the level ground and $\sin\alpha = \frac{3}{5}$. Find the height of the kite. There is no slack in the string.

23. An aeroplane, flying 3000 m high, passes vertically above another plane at an instant when the angle of elevation of two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes. (Use $\sqrt{3} = 1.732$)

24. A 7 m long flagstaff is fixed on the top of a tower on the horizontal plane. From a point on the ground, the angle of elevation of the top and the bottom of the flagstaff are 45° and 30° respectively. Find the height of the tower.

(Use $\sqrt{3} = 1.732$)

25. Anand is watching a circus artist climbing a 20m long rope which is tightly stretched and tied from the top of vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.

LONG ANSWER TYPE QUESTIONS

26. A man standing on the deck of a ship, 10 m above the water level observes the angle of elevation of the top of a hill as 60° and angle of depression of the bottom of the hill as 30°. Find the distance of the hill from the ship and height of the hill.

27. From a window 60 m high above the ground of a house in a street, the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are 60° and 45° respectively. Show that the height of opposite house is $60(1+\sqrt{3})$ metres.

28. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed in km/hour of the plane.

29. A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird. (Use $\sqrt{3}$ = 1.732)

30. The shadow of a tower standing on a level ground is found to be 30 m longer when the sun altitude is 30° longer when the sun altitude is 30° than when it is 60°. Find the height of the tower.

31. The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building. **(CBSE 2020)**

32. An observer from the top of a light house, 100 m high above sea level, observes the angle of depression of a ship, sailing directly towards him, changes from 30° to 60°. Determine the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ = 1.732)

33. The angles of elevation and depression of the top and bottom of a light house from the top of a 60 m high building are 30° and 60° respectively. Find

$= 1.73$)

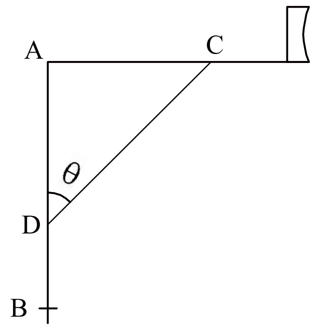
39. From the top of a 120 m high tower a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as 60° and 45°. Find the distance between the cars.

(Use $\sqrt{3} = 1.732$)

40. A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

(CBSE 2020)

41. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in the figure. If AC = 1.5 m long and CD = 3 m, find (*i*) tan θ (*ii*) sec θ + cosec θ. **(CBSE 2020)**



42. At a point on level ground, the angle of elevation of a vertical tower is found to be α such that $\tan\alpha = \frac{1}{3}$. After walking 200 m towards the tower, then angle of elevation β becomes such that $\tan\beta = \frac{3}{4}$, find the height of the tower.

43. A vertically straight tree, 20m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

44. If the angle of elevations of a cloud from a point *h* meters above a lake be 30° and the angle of depression of its reflection in the lake be 60°. Prove that the height of cloud is 2h, also find the distance of observer from cloud.

45. The angles of elevation of the top of a tower of height h meter from two points P and Q at a distance of x m and y m from the base of the tower respectively and in the same straight line with it, are 60° and 30°, respectively prove that height of tower be $\sqrt{xy}\ m$.

46. Two poles of heights 18 m and 30 m stand vertically on the ground. The tops of two poles are connected by a wire, which is inclined to the horizontal at an angle of 60°. Find the length of wire and the distance between the poles.

47. The angles of depression of the top and bottom of a 10 m tall pole from the top of a transimission tower are 45° and 60° respectively. Find the height of the transmission tower and the distance between the pole and tower.

(Use $\sqrt{3}$ = 1.732)

48. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making and angle of 30° with it. The height of the breaking point from the ground is 10 m. Find the total height of the tree.

ANSWERS AND HINTS

1. (b) 16 m
2. (c) 45°
3. (b) 60°
4. (c) 5 m
5. (c) 60°
6. 50 m
7. 60°
8. 30°
9. 100 m
10. $20\left(\sqrt{3}+1\right)$ m
11. 130 m
12. $60\left(\sqrt{3}+1\right)$ m
13. $1000\left(\sqrt{3}-1\right)$ m
14. 25 m
15. 45°
16. $75\sqrt{3}m$
17. 13.65 m
18. 315.46 m
19. 94.64 m
20. $25\sqrt{3}$ m
21. 100 m
22. 54 m
23. 1268 m
24. 9.562 m
25. 10 m
26. $10\sqrt{3}$ m, 40 m
28. 864 km/hr
29. 29.28 m/s
30. $15\sqrt{3}$ m
31. 20 m
32. 115.46 m
33. 20 m, $20\sqrt{3}$ m
34. Station P, 7.4 km (approx)
35. 20 m
36. height = 64.95 m, distance (Position) = 37.5m from the pillar having angle of elevation 60°

37. $10(\sqrt{3}+1)\ m$

38. 1902 m/h (approx.)

39. 189.28 m

40. h = $20(\sqrt{3}-1)$ m

41. (*i*) $\tan\theta = \frac{1}{\sqrt{3}}$

(*ii*) $\sec\theta + \operatorname{cosec}\theta = \frac{2}{\sqrt{3}}+2$

42. $h = 120$ m

43. $20\sqrt{3}(2-\sqrt{3})$m

44. 2 *h*

46. Length of wire = $8\sqrt{3}$ m, distance = $4\sqrt{3}$ m

47. Height = 23.66m, distance = 13.66m

48. Height of tree = 30m

PRACTICE-TEST

Some Applications of Trigonometry

Time : 45 Minutes | | ***M.M.: 20***

SECTION-A

1. A pole which is 6 m high cast a shadow $2\sqrt{3}$ on the ground. Find the sun's angle of elevation. **1**

2. The height of a tower is 100 m. When the angle of elevation of sun is 30°, then find the length of the shadow of the tower. **1**

3. The angle of elevation of the sun, when the shadow of a pole h meters high is $\sqrt{3}\,h$ is. :

   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90° **1**

4. An observer 1.5 metre tall is 20.5 metre away from a tower 22 metres high. The angle of elevation of the top of the tower from the eye of the observer is:

   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 0° **1**

SECTION-B

5. From a point on the ground 20 m away from the foot of a tower the angle of elevation is 60°. Find the height of the tower. **2**

6. The ratio of height and shadow of a tower is $1:\frac{1}{\sqrt{3}}$. What is the angle of elevation of the sun? **2**

7. The angle of elevation of the top of a tower is 30°. If the height of the tower is tripled, then prove that the angle of elevation would be doubled. **2**

SECTION-C

8. The tops of the two towers of height x and y standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

3

9. The angle of elevation of the top of a rock from the top and foot of a 100 m high tower are 30° and 45° respectively. Find the height of the rock. **3**

SECTION-D

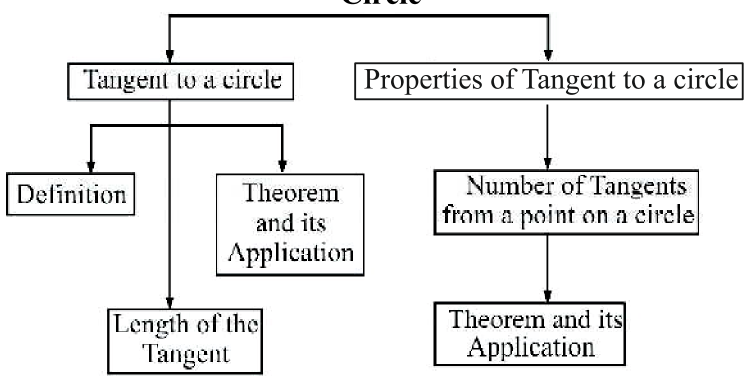
10 A man standing on the deck of a ship, 10 m above the water level observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and height of the hill. **4**

CHAPTER

10 Circles

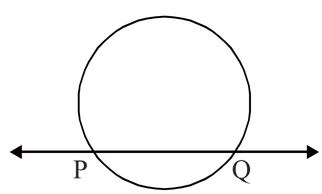
Mind–Maping

Circle



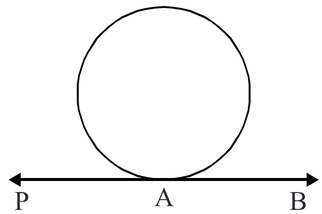
KEY POINTS

1. A **circle** is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the **centre** and constant distance is called the **radius**.
2. **Secant:** A line which intersects a circle in two distinct points is called a secant of the circle.



3. **Tangent**: It is a line that intersects the circle at only one point. The point where tangent touches the circle is called the point of contact.

   Here A is the point of contact.



4. **Number of Tangent**: Infinitely many tangents can be drawn on a circle.
5. **Number of Secant**: There are infinitely many secants which can be drawn to a circle.
6. (i) (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

   (ii) (Prove) The lengths of tangents drawn from an external point to a circle are equal.
7. The tangent to a circle is a special case of the secant, when the two end points of the corresponding chord coincide.
8. There is no tangent to a circle passing through a point lying inside the circle.
9. There is one and only one tangent to a circle passing through a point lying on the circle.
10. There are exactly two tangents to a circle through a point lying outside the circle.

VERY SHORT ANSWER TYPE QUESTIONS

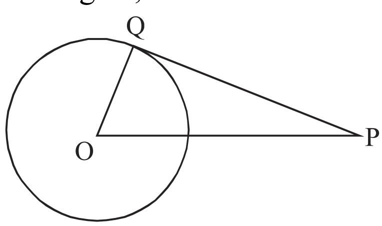
1. How many tangents can a circle have?

   (a) Only one (b) Two

   (c) None (d) Infinitely many
2. A tangent to a circle intersects it in:

   (a) Only one point (b) Two points

   (c) No point (d) Infinitely many points
3. In the given figure, if PQ is a tangent, then the value of $2(\angle POQ + \angle QPO)$ is:



(a) $60°$ (b) $90°$

(c) $120°$ (d) $180°$

4. A tangent PQ at point P of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ = 12cm. The length of PQ is:

(a) 12 cm (b) 13 cm

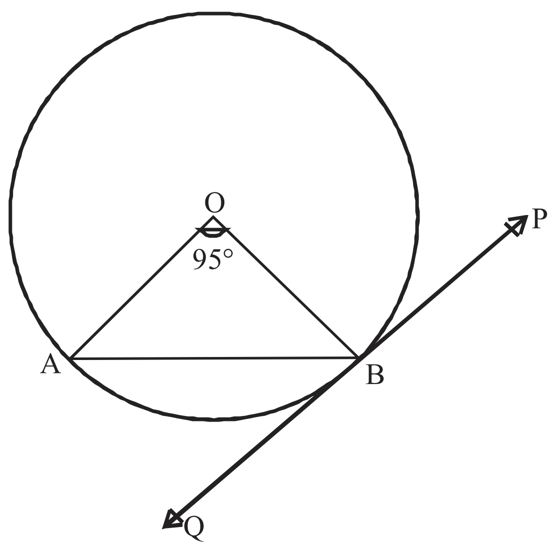
(c) 15 cm (d) $\sqrt{119}$ cm

5. A circle can have _____ parallel tangents at the most.

(a) Two (b) Four

(c) Six (d) Infinitely many

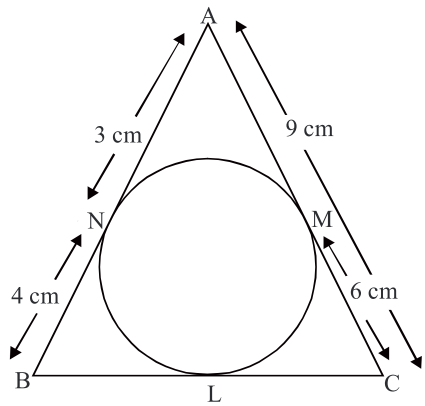
6. In the given figure, PQ is Tangent to the circle centered at O. If $\angle AOB = 95°$, then the measure of $\angle ABQ$ is:



(a) $42.5°$ (b) $47.5°$

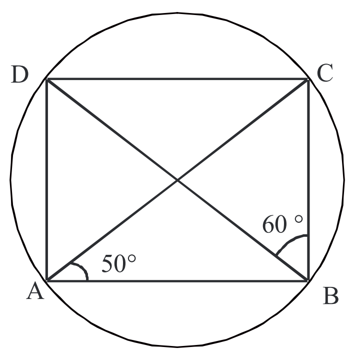
(c) $85°$ (d) $95°$

7. In the given figure, ΔABC is circumscribing a circle. Find the length of BC.

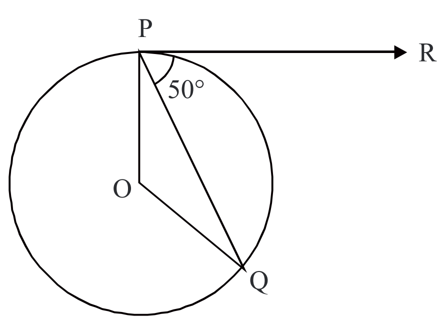


8. If the length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm, then find the radius of the circle.

9. In the given figure, ABCD is a cyclic quadrilatreral. If $\angle BAC = 50°$ and $\angle DBC = 60°$, then find $\angle BCD$.



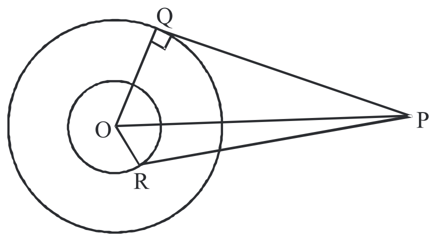
10. In figure, O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angles of 50° with PQ. Find $\angle POQ$.



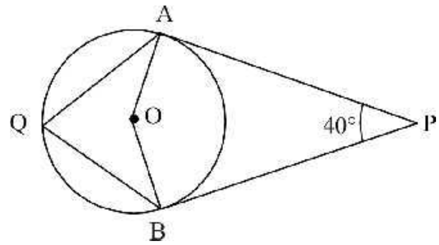
11. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

12. If radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of that circle which is tangent to the other circle.

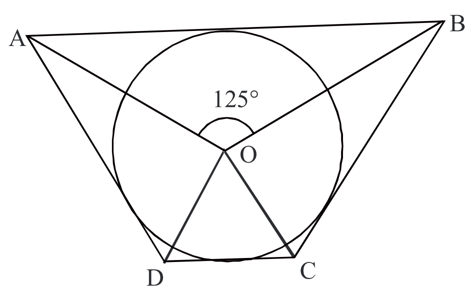
13. In the given figure, PQ is tangent to outer circle and PR is tangent to inner circle. If PQ = 4cm, OQ = 3 cm and OR = 2 cm then find the length of PR.



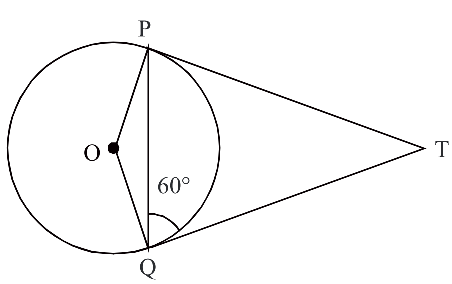
14. In the given figure, O is the centre of the circle, PA and PB are tangents to the circle. Find $\angle AQB$. **(CBSE 2016)**



15. In the given figure, If $\angle AOB = 125°$ then find $\angle COD$.



16. If two tangent TP and TQ are drawn from an external point T such that $\angle TQP = 60°$, then find $\angle OPQ$.



17. Find the distance between two points of contact of two parallel tangents to a given circle of radius 9 cm.

18. Find the radius of a circle, if distance between two parallel tangents be 10 cm.

19. How many common tangents can be drawn to two circles touching internally?

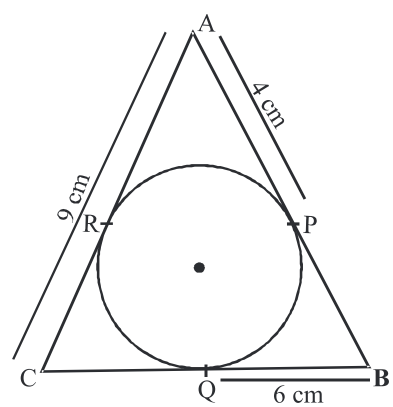
SHORT ANSWER TYPE QUESTIONS

20. If diameters of two concentric circles are d_1 and d_2 ($d_2 > d_1$) and c is the length of chord of bigger circle which is tangent to the smaller circle. Show that $d_2^2 = c^2 + d_1^2$.

21. The length of tangent to a circle of radius 2.5 cm from an external point P is 6 cm. Find the distance of P from the nearest point of the circle.

22. TP and TQ are the tangents from the external point T of a circle with centre O. If $\angle OPQ = 30°$ then find the measure of $\angle TQP$.

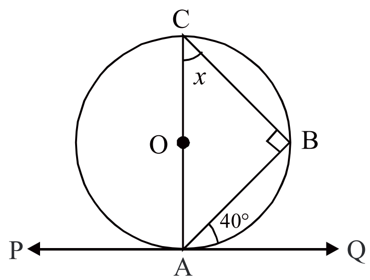
23. In the given figure, AP = 4 cm, BQ = 6 cm and AC = 9 cm. Find the semi perimeter of ΔABC.



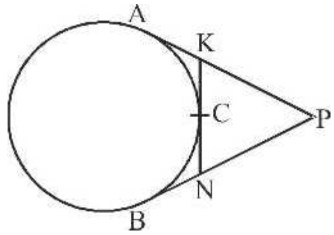
24. A circle is drawn inside a right angled triangle whose sides are a, b and c where c is the hypotenuse, which touches all the sides of the triangle. Prove that $r = \dfrac{a + b - c}{2}$ where r is the radius of the circle.

25. Prove that in two concentric circles the chord of the larger circle which is tangent to the smaller circle is bisected at the point of contact.

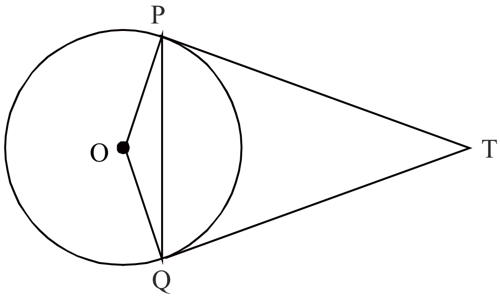
26. In the given figure, AC is diameter of the circle with centre O and A is the point of contact. Find x.



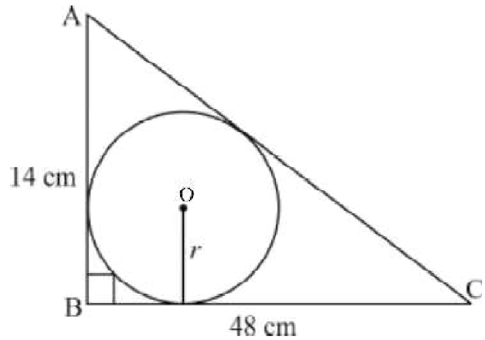
27. In the given figure, KN, PA and PB are tangents to the circle. Prove that KN = AK + BN.



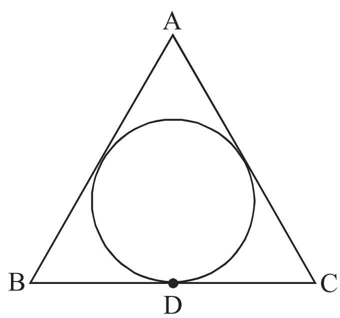
28. In the given figure, PQ is a chord of length 6 cm and the radius of the circle is 6 cm. TP and TQ are two tangents drawn from an external point T. Find $\angle PTQ$.



29. In the given figure, ABC is a triangle in which $\angle B = 90°$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find the radius (r) of the incircle.

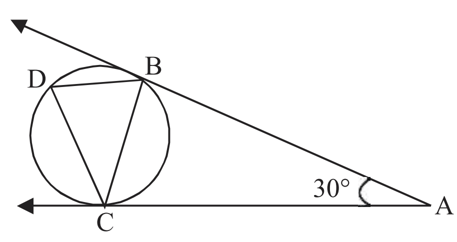


30. If the inscribed circle of the ΔABC touches BC at D. Prove that AB – BD = AC – CD.

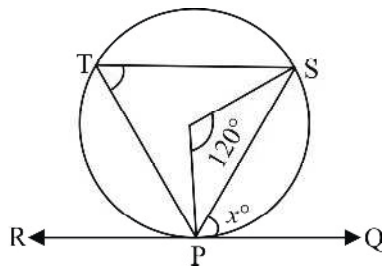


31. From a point P which is at distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn, then find the area of the quadrilateral PQOR.

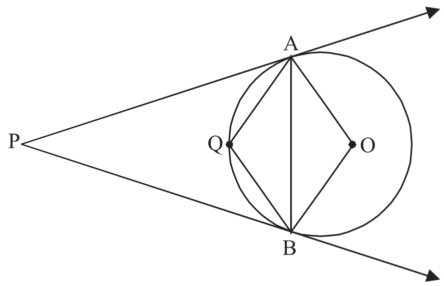
32. In the given figure, tangents AC and AB are drawn to a circle from a point A such that $\angle BAC = 30°$ and a chord BD is drawn parallel to the tangent AC. Find $\angle DBC$.



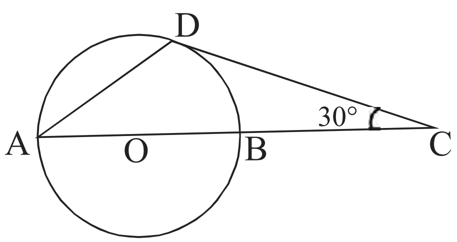
33. Find the value of x.



34. PA and PB are tangents to the circle with centre at O. If $\angle APB = 70°$, then find $\angle AQB$.

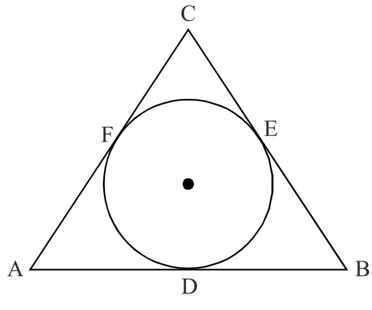


35. In the given figure, CD is a tangent and AB is a diameter of the circle. If $\angle DCB = 30°$, then find $\angle ADC$.

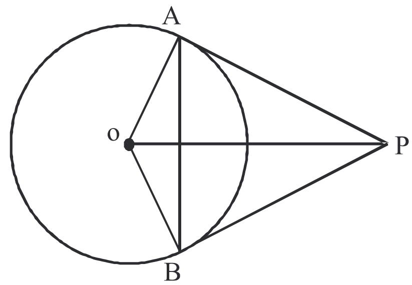


LONG ANSWER TYPE QUESTIONS

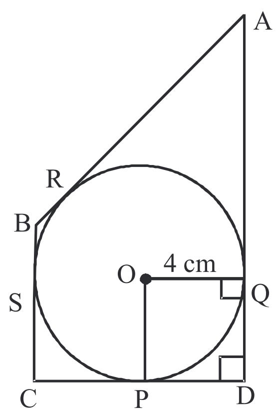
36. In the given figure, find AD, BE, CF where AB = 12 cm, BC = 8 cm and AC = 10 cm.



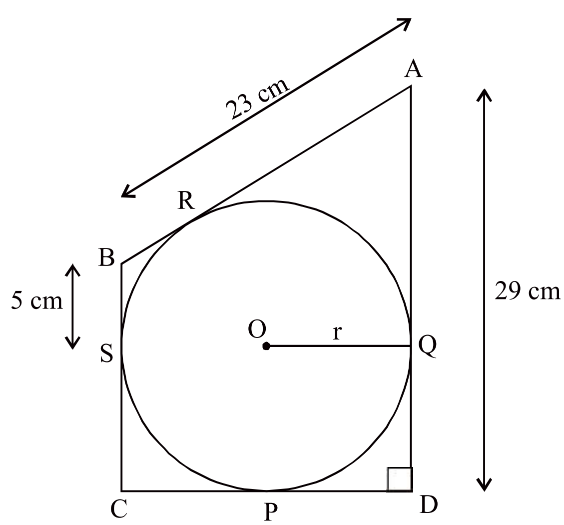
37. In the given figure, OP is equal to the diameter of the circle with centre O. Prove that ΔABP is an equilateral triangle.



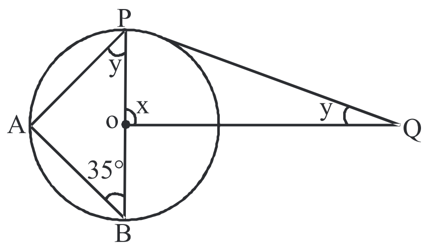
38. In the given figure, find PC. If AB = 13 cm, BC = 7 cm and AD = 15 cm.



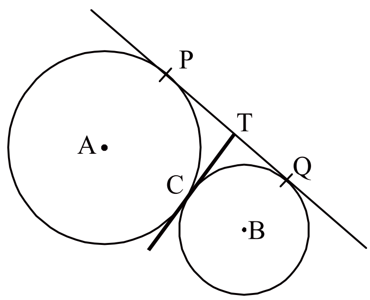
39. In the given figure, find the radius of the circle.



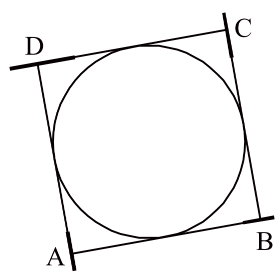
40. In the given figure, PQ is tangent and PB is diameter. Find the values of angle x and y.



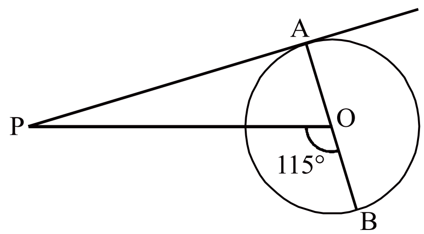
41. In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent PQ at T.



42. In the given figure, a circle touches all the four sides of a quadrilateral ABCD. If AB = 6 cm, BC = 9 cm and CD = 8 cm, then find the length of AD.

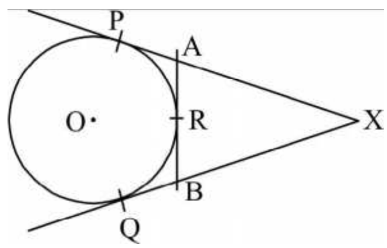


43. In the figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115°$, then find $\angle APO$.

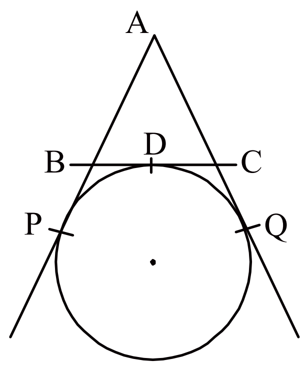


44. In the given figure, XP and XQ are tangents from X to the circle with centre O, R is a point on the circle and AB is tangent at R. Prove that :

XA + AR = XB + BR



45. In the given figure, find the perimeter of ΔABC, if AP = 12 cm.



ANSWERS AND HINTS

1. (d) Infinitely many

2. (a) Only one point

3. (d) 180°

4. (d) $\sqrt{119}$ cm

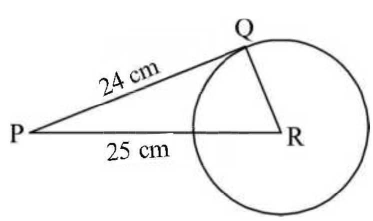
5. (a) Two

6. (b) 47.5°

7. Since length of both the tangents from a point outside the circle is equal, So

$$BN = BL, \ CM = CL$$

$$BL + CL = BC = 10 \text{ cm}$$

8. 

By Pythagoras Theorem, QR = 7 cm.

9. Angle in the same segment are euqal.

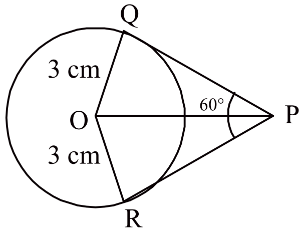
$\angle DAC = \angle DBC = 60°$.

The sum of the opposite angles of a cyclic quadrilateral is 180°.

So $\angle BCD = 70°$

10. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

So, $\angle RPO = 90°$
$\angle OPQ = \angle OQP = 40°$
$\angle POQ = 100°$

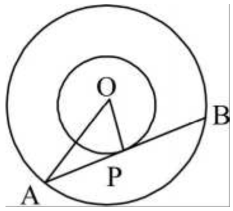
11. 

$$\Delta QPO \cong \Delta RPO$$

$\Rightarrow$ $\angle QPO = \angle RPO = \frac{60°}{2} = 30°$

In ΔQPO, $\angle OQP = 90°$(Tangent is perpendicular at the point of contact).

$$\tan 30° = \frac{OQ}{QP} \quad \Rightarrow \quad QP = 3\sqrt{3} \text{ cm}$$

12. 

In ΔAOP, right angled at P.

$$\Rightarrow \quad (5)^2 = AP^2 + 4^2 \quad \Rightarrow \quad AP^2 = 9$$

$\Rightarrow$ $AP = 3$

$\therefore$ $AB = 6$ cm $(\because OP \perp AB$ so OP bisects AB)

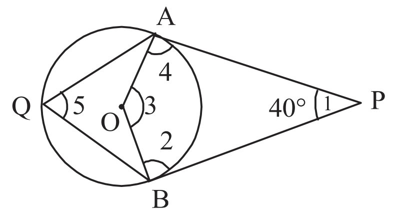
13. In ΔPQO, $(4)^2 + (3)^2 = (OP)^2$

$5 = OP$

In ΔPRO, $(5)^2 = (2)^2 + (PR)^2$

$PR = \sqrt{21}$ cm

14.



In Quadrilateral OAPB

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$$

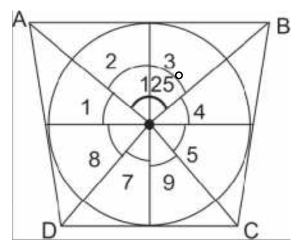
$$\angle 1 + \angle 3 = 180°$$

$$\angle 3 = 140°$$

Now, $\angle 3 = 2 \angle 5$

$$\angle 5 = 70° \text{ or } \angle AQB = 70°$$

15. $\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 3 = \angle 4 \\ \angle 5 = \angle 6 \\ \angle 7 = \angle 8\end{array}\right]$ (CPCT)



$$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$$

$$\angle AOB + \angle COD = 180°$$

$$\angle COD = 55°$$

16. $\angle OQT = 90°$ (Angle between tangent & radius)

$$\angle PQO = 30°$$

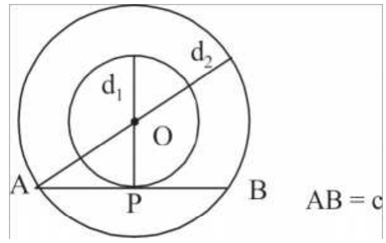
$$\angle PQO = \angle OPQ = 30°$$

17. 18 cm

18. 5 cm

19. 1

20.



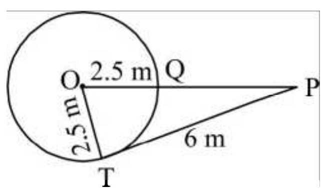
$$AO^2 = OP^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$\frac{d_2^{\,2}}{4} = \frac{d_1^{\,2}}{4} + \frac{c^2}{4}$$

$$d_2^{\,2} = c^2 + d_1^{\,2}$$

21.



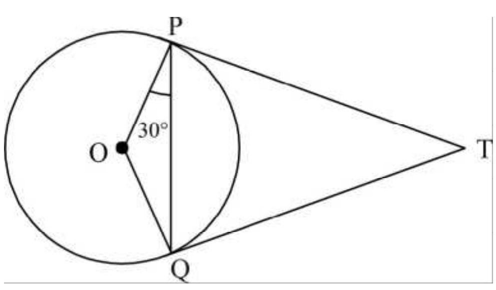
$$(OP)^2 = (OT)^2 + (PT)^2$$

$$(OP)^2 = (2.5)^2 + (6)^2$$

$$= 42.25$$

$$\Rightarrow \quad OP = 6.5 \text{ cm}, QP = 4 \text{ cm}$$

22.



$\angle OQP = \angle OPQ = 30°$

$\angle OQT = 90°$ (Angle between radius and tangent)

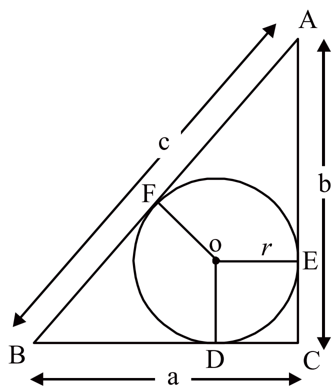
$$\angle TQP = \angle OQT - \angle OQP$$
$$= 90° - 30° = 60°$$

23. $AP = AR = 4$ cm

$CR = CQ = (9 - 4)$ cm $= 5$ cm

$$\text{Semi perimeter} = \frac{1}{2}[AC + AB + BC]$$
$$= \frac{1}{2}[9 + 10 + 11] = 15 \text{ cm}$$

24.



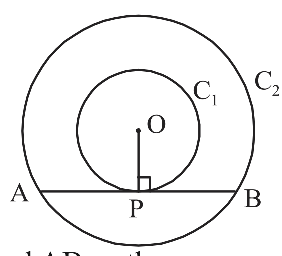
$AE = AF = b - r$; $BD = BF = a - r$

$AB = AF + BF$

$c = b - r + a - r$

This gives, $r = \dfrac{a + b - c}{2}$

25. Join OP



AB is tangent to circle C_1 at P and OP is radius

$OP \perp AB$

AB is chord of circle C_2 and $OP \perp AB$.

Therefore OP is the Perpendicular besector of the chord AB as the perpendicular from the centre bisects the chord i.e,

$AP = BP$

26. $\angle OAB = 50°$

$x + \angle B + \angle OAB = 180°$

$x + 90° + 50° = 180°$

$x = 40°$

27. $AK = KC$

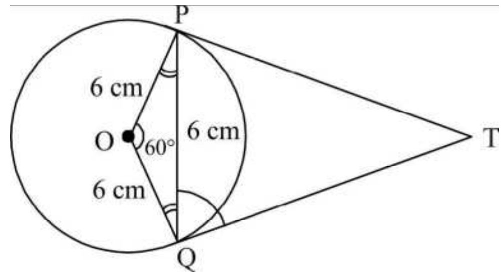
$BN = NC$

$\therefore \quad KN = KC + NC = AK + BN$

28. $\angle POQ + \angle PTQ = 180°$

$60° + \angle PTQ = 180°$

$\angle PTQ = 120°$



29. $r = 6$ cm

30. $AP = AQ \quad (1)$

$BP = BD \quad (2)$

$CD = CQ \quad (3)$

Adding (1) and (2)

$AP + BP = AQ + BD$

$AB - BD = AQ \quad (4)$

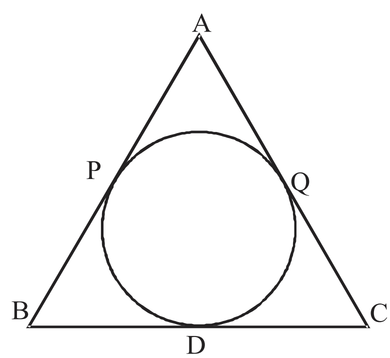
Adding (1) and (3)

$AP + CD = AQ + CQ$

$AP = AC - CD \quad (5)$

From (1), (4) and (5)

$AB - BD = AC - CD$



31. 60 cm^2

32. $\angle DBC = 75°$

33. $x = 60$

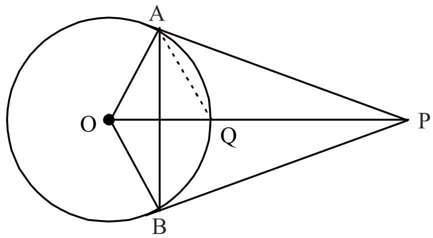
34. $\angle AQB = 125°$

35. $\angle ADC = 120°$ (Join OD)

36. AD = 7 cm, BE = 5 cm, CF = 3 cm

37. $$OP = 2r$$

$$\Rightarrow \quad OQ = QP = r$$



Consider ΔAOP in which $OA \perp AP$ and OP is the hypotenuse.

$$OQ = AQ = OA$$

(Mid point of hypotenuse is equidistance from the vertices).

$\Rightarrow$ OAQ is an equilitateral triangle.

$$\Rightarrow \quad \angle AOQ = 60°$$

$$\angle OAP = 90° \quad \Rightarrow \quad \angle APO = 30°$$

$$\angle APB = 2\angle APO = 60°$$

$$PA = PB \text{ (tangents)}$$

$$\Rightarrow \quad \angle PAB = \angle PBA$$

$$\angle APB = 60°$$

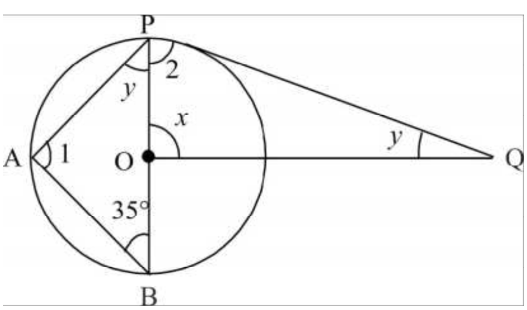
$$\angle PAB = \angle PBA = 60°$$

$\therefore$ Δ ABP is an equilateral triangle.

38. PC = 5 cm

39. 11 cm

40.



In ΔABP, $\angle 1 = 90°$ (Angle in semi-circle)

$$\angle 1 + 35° + \angle y = 180°$$

$$90° + 35° + \angle y = 180°$$

$$\angle y = 55°$$

In ΔOPQ, $\angle 2 = 90°$ (Angle between tangent and radius)

$$\angle 2 + \angle x + \angle y = 180°$$

$$90° + \angle x + 55° = 180°$$

$$\angle x = 35°$$

42. AD = 5 cm

43. 25°

45. 24 cm

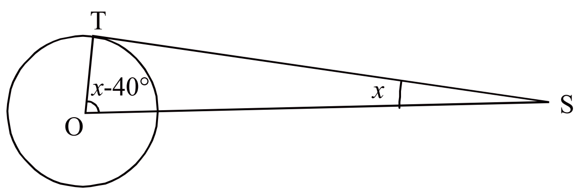
PRACTICE-TEST

CIRCLES

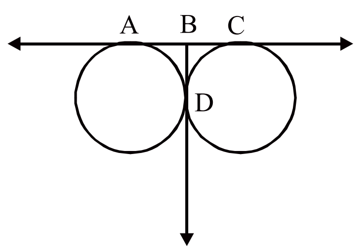
Time : 45 Minutes | | *M.M.: 20*

SECTION-A

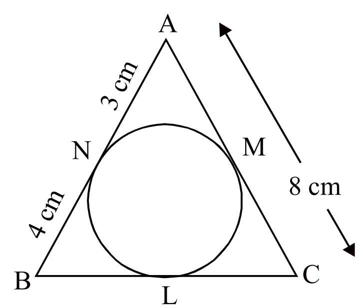
1. In the given figure find x, where ST is the tangent. **1**



2. In the given figure if AC = 9 cm, find BD. **1**



3. In the given figure, ΔABC is circumscribing a circle, then find the length of BC. **1**

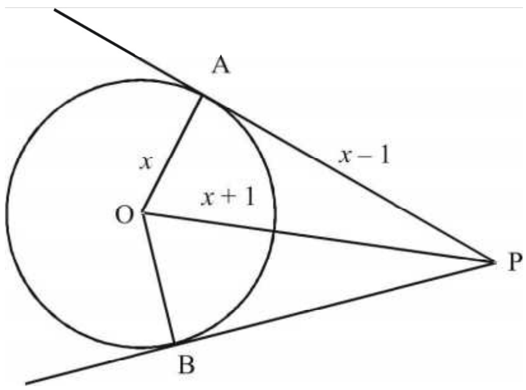


4. From the external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle$PAB = 50°, then find $\angle$AOB. **1**

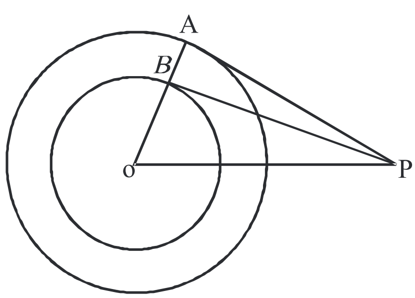
SECTION-B

5. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O is 60° then find the length of OP. **2**

6. In the following figure, find x. **2**

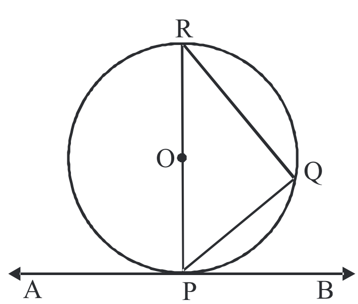


7. Two concentric circle with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle as shown in the figure. If AP = 10 cm, then find BP. **2**

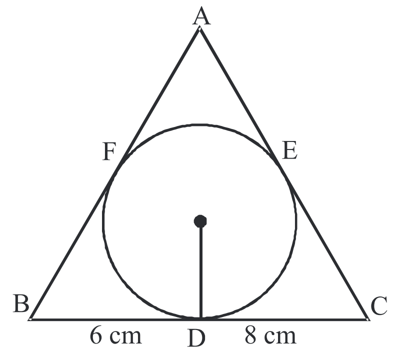


SECTION-C

8. In the given figure, AB is a tangent to a circle with centre O. Prove that $\angle BPQ = \angle PRQ$. **3**

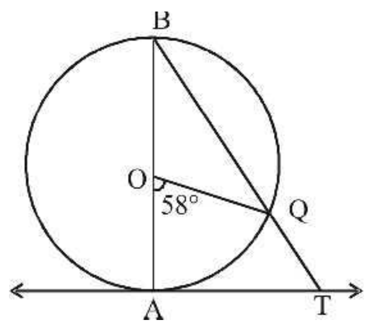


9. In the given figure, ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segment BD and DC into which BC is divided by the point of contact D are of length 6 cm and 8 cm respectively, find side AB if the *ar*(ΔABC) = 63 cm² **3**



SECTION-D

10. AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, then find $\angle ATQ$. **4**



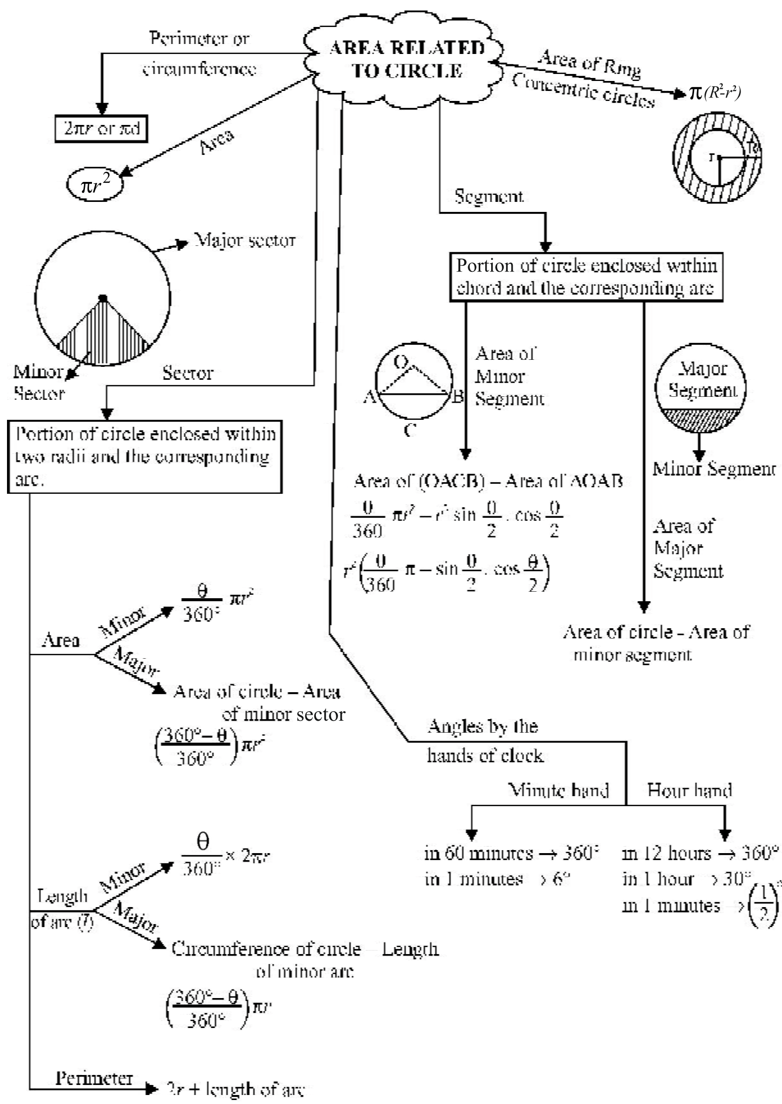
CHAPTER

11 Areas Related to Circles

TOPICS

Perimeter and Area of a circle.

Area of sector and segment of a circle.



KEY POINTS

If r is radius of a circle, then

(i) Area of semi circle = $\dfrac{\pi r^2}{2}$

(ii) Area of quadrant of a circle = $\dfrac{\pi r^2}{4}$

(iii) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(iv) If two circles touch externally, then distance between their centres is equal to the sum of their radii.

(v) Distance covered by rotating wheel in one revolution is equal to the circumference of the wheel.

(vi) The number of revolutions completed by a rotating wheel in

$$\text{one minute} = \frac{\text{Distance moved in one minute}}{\text{Circumference of the wheel}}$$

(vii) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(viii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

VERY SHORT ANSWER QUESTIONS

1. If the diameter of a semi circular protactor is 14 cm, then find its perimeter.

2. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

3. Find the area of the circle 'inscribed' in a square of side *a* cm.

4. Find the area of a sector of a circle whose radius is *r* and length of the arc is *l*.

5. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.

6. If the area of a circle is 616 sq.cm, then what is its circumference?

7. What is the area of the circle that can be inscribed in a square of side 6 cm?

8. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 24 cm and 7 cm?

9. A wire can be bent in the form of a circle of radius 35 cm. If it is bent in the form of a square, then what will be its area?

10. What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 3π cm?

11. If the circumference of two circles are in the ratio 2:3, what is the ratio of their areas?

12. If the difference between the circumference and radius of a circle is 37 cm, then find the circumference of the circle. (Use $\pi = \frac{22}{7}$)

13. If diameter of a circle is increased by 40%, find by how much percentage its area increases?

14. The minute hand of a clock is 6 cm long. Find the area swept by it between 11:20 am and 11:55 am.

15. The perimeter of a sector of a circle of radius 14 cm is 68 cm. Find the area of the sector. **(CBSE 2020)**

16. The circumference of a circle is 39.6 cm. Find its area. (Use $\pi = \frac{22}{7}$) **(CBSE 2020)**

17. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use $\pi = \frac{22}{7}$)

18. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is :

(a) 22:7 (b) 14:11

(c) 7:22 (d) 11:14

19. The area of circle that can be inscribed in a square of side 6 cm is:

(a) 36π sq.cm (b) 18π sq.cm

(c) 12π sq.cm (d) 9π sq.cm

20. If the circumference of a circle increases from 4π to 8π, then area is:

(a) Halved (b) Doubled

(c) Tripled (d) Quadrupled

21. If the perimeter of a semi- circular protractor is 36 cm , then its diameter is:

(a) 10 cm (b) 14 cm

(c) 12 cm (d) 16 cm

22. The length of a minute hand of clock is 14 cm. What is the area swept by the mimute hand in 15 minutes?

(a) 154 sq.cm (b) 87 sq.cm

(c) 154π sq.cm (d) 87π sq.cm

23. The wheel of a cycle is of radius 35 cm. How many revolutions are required to travel a distance of 11 m ?

(a) 2 (b) 5

(c) 10 (d) 15

24. Four horses are tied each with 7 m long rope at four corners of a square field of sides 20 m. What is the area of field which can be grazed by the horses?

(a) 49π sq.m (b) 98π sq.m

(c) 74π sq.m (d) 154π sq.m

SHORT ANSWER TYPE QUESTIONS (I)

25. Find the area of a quadrant of a circle whose circumference is 22 cm.

(Use $\pi = \frac{22}{7}$)

26. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?

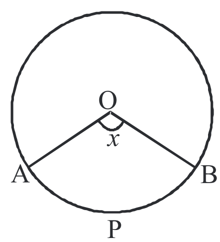
27. If a square is inscribed in a circle, what is the ratio of the area of the circle and the square?

28. Find the area of a circle whose circumference is 44 cm. **(CBSE 2020)**

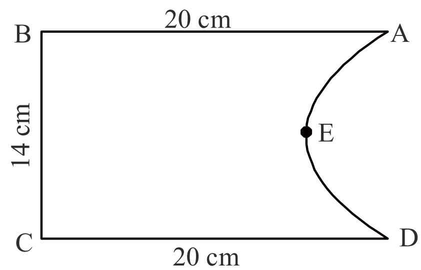
29. If the perimeter of a circle is equal to that of square, then find the ratio of their areas.

30. What is the ratio of the areas of a circle to an equilateral triangle whose diameter and a side are respectively equal?

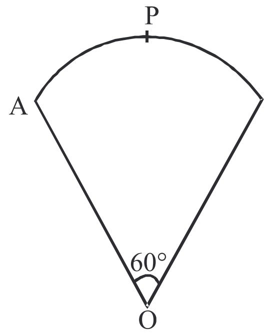
31. In figure, O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.



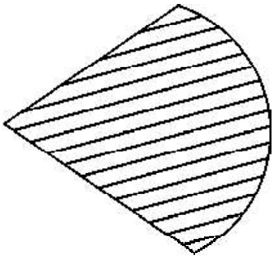
32. Find the perimeter of the given figure, where AED is a semicircle and ABCD is a rectangle. **(CBSE 2015)**



33. In figure, OAPBO is a sector of a circle of radius 10.5 cm. Find the perimeter of the sector.



34. A Japenese fan can be made by sliding open its 7 small sections, each of which is in the form of sector of a circle having central angle of 15°. If the radius of this fan is 24 cm, find the length of the lace that is required to cover its entire boundary. (Use $\pi = \frac{22}{7}$)



(CBSE 2014)

35. The perimeter of a sector of circle of radius 6.3 cm is 25.8 cm. Find the area of the sector.

36. Find the area of a circle in which a square of area 64 sq.cm is inscribed.

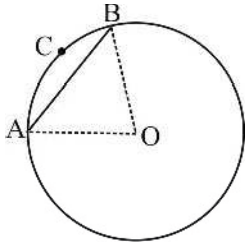
37. Find the area of a circle which is inscribed in a square of area 64 sq.cm.

SHORT ANSWER TYPE QUESTIONS-II

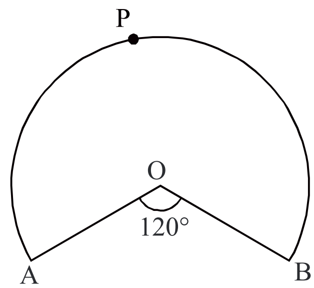
38. Area of a sector of a circle of radius 36 cm is 54π sq.cm. Find the length of the corresponding arc of the sector.

39. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am to 6:40 am.

40. Find the area of the segment bounded by a chord AB and the arc ACB of the circle with centre O having radius 7 cm and central angle equal to 90°, as shown in the figure.



41. In figure, OAPB is a sector of a circle of radius 3.5 cm with the centre at O and $\angle AOB = 120°$. Find the length of OAPBO.



42. Circular footpath of width 2 m is constructed at the rate of ₹ 20 per square meter, around a circular park of radius 1500 m. Find the total cost of construction of the foot path. (Take $\pi = 3.14$)

43. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm. Calculate the speed of cycle.

44. In a circle with centre O and radius 4 cm, and of angle 30°. Find the area of minor sector and major sector AOB. (Use $\pi = 3.14$)

45. Find the area of the largest triangle that can be inscribed in a semi circle of radius r unit.

46. In a square park of side 8 m two goats are tied at opposite vertices with a rope of length 1.4 m and a cow is tied at the centre with a rope of length 2.1m. Calculate the area of park which cannot be grazed by them.

47. A sector of 100° cut off from a circle contains area 70.65 sq.cm. Find the radius of the circle. (Use $\pi = 3.14$)

48. The hour and minute hand of a 12 hour clock are 3.5 cm and 7 cm long respectively. Find the sum of distance travelled by their tips in a day. $\left(\text{use } \pi = \frac{22}{7}\right)$

49. A square water tank has its each side equal to 40 m. There are four semi circular grassy plots all around it. Find the cost of turfing the plot at ₹1.25 per sq. m. (Use $\pi = 3.14$)

50. Length of a chord of a circle of a radius of 4 cm is 4 cm. Find the area of the sector and segment formed by the chord.

51. Find the area of the minor segment of a circle of radius 21 cm, when the angle of the corresponding sector is 120°.

52. A piece of wire 11 cm long is bent into the form of an arc of a circle subtending an angle of 45° at its centre. Find the radius of the circle.

53. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

54. A pendulum swings through an angle of 45° and describes an arc of 22 cm in length. Find the length of the pendulum. $\left(\text{use } \pi = \frac{22}{7}\right)$

LONG ANSWER TYPE QUESTIONS

55. Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

56. Find the number of revolutions made by a circular wheel of area 6.16 sq.m. in rolling a distance of 572 m.

57. Three horses are tied at the vertices of a triangular park of sides 35 m, 84 m and 91 m with the help of a rope of length 14 m each. Calculate the ratio of the area which can be grazed to the area which can't be grazed.

58. Two circle touch each other internally. The sum of their area is 116π sq.cm. and distance between their centres is 6 cm. Find the radii of the circles.

(CBSE = 2017)

ANSWERS AND HINTS

1. $\pi r + d = \frac{22}{7} \times 7 + 14 = 36$ cm

2. $2\pi r = \pi r^2 \Rightarrow$ diameter $= 4$ units

3. Side of the square is equal to diameter of the circle,

$$\pi r^2 = \pi \times \frac{a^2}{4} \text{ (side} = a\text{, radius} = \frac{a}{2}\text{)}$$

4. $l = \frac{\theta}{360°} \times 2\pi r,$ Area $= \frac{\theta}{360°} \times \pi r^2 = \frac{l \times \pi r^2}{2\pi r} = \frac{lr}{2}$ sq. units

5. $\dfrac{\text{distance}}{\text{circumference}} = \dfrac{11 \times 1000 \times 7 \times 100}{2 \times 22 \times 25} = 7000$

6. $\pi r^2 = 616 \quad \Rightarrow \quad r = 14$ cm, so, $2\pi r = 88$ cm

7. Side of the square is equal to the diameter of the circle

   $\Rightarrow \quad$ r = 3 cm or $\pi r^2 = \pi(3)^2 = 9\pi$ sq.cm.

8. $\pi R^2 = \pi r_1^2 + \pi r_2^2 \Rightarrow R = 25$ and diameter = 50 cm.

9. $2\pi r = 2 \times \dfrac{22}{7} \times 35 = 220$ cm , Side of square $= \dfrac{220}{4} = 55$ cm

   Area of square = 55 × 55 = 3025 sq.cm.

10. $l = \dfrac{\theta}{360^\circ} \times 2\pi r \quad \Rightarrow \quad 3\pi = \dfrac{\theta}{360^\circ} \times 2\pi \times 6 \quad \Rightarrow \quad \theta = 90^\circ$

11. $\dfrac{2\pi r_1}{2\pi r_2} = \dfrac{2}{3} \Rightarrow r_1 = \dfrac{2}{3} r_2$ or $\dfrac{\pi r_1^2}{\pi r_2^2} = \dfrac{\left(\dfrac{2}{3} r_2\right)^2}{r_2^2} = 4:9$

12. $(2\pi r - r) = 37$ or $r = 7$, $2\pi r = 2 \times \dfrac{22}{7} \times 7 = 44$ cm

13. 96%

14. $\dfrac{210^\circ \times 22 \times 6 \times 6}{360^\circ \times 7} = 66$ cm^2 $(\theta = 210^\circ)$ (11: 20 to 11: 55 = 35 minutes)

15. 280 sq.cm.

16. 124.74 sq.cm.

17. 10.27 sq.cm.

18. (b) 14:11

19. (d) 9 π sq.cm.

20. (d) Quadrupled

21. (d) 14 sq.cm.

22. (a) 154 sq.cm.

23. (b) 5

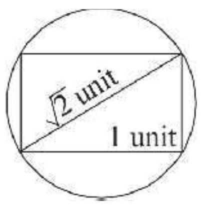
24. (a) 49 π sq.cm.

25. $2\pi r = 22,\ r = \dfrac{7}{2}$

Area of quadrant $= \dfrac{\pi r^2}{4} = \dfrac{22 \times 7 \times 7}{7 \times 4 \times 2 \times 2} = 9.625$ sq.cm.

26. $l = \dfrac{\theta}{360°} \times 2\pi r \Rightarrow 5\pi = \dfrac{\theta}{360°} \times 2\pi \times 10 \Rightarrow \theta = 90°$

27.



If side of square is 1 unit, by Pythagoras Theorem

Diameter $= \sqrt{2}$ unit.

Area of square = 1 × 1 = 1 sq units.

Area of Circle $= \pi r^2 = \pi \times \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{2}}{2} = \dfrac{\pi}{2} = \dfrac{11}{7}$

Required ratio = 11 : 7

28. 154 sq.cm.

29. $2\pi r = 4$ unit or $\dfrac{2\pi r}{4 \text{ unit}} = \dfrac{\text{Perimeter of circle}}{\text{Perimeter of square}}$ (Let side of square = 1 unit)

$$r = \frac{7}{11} \text{ unit}$$

$\dfrac{\pi r^2}{1} = \dfrac{22}{7} \times \dfrac{7}{11} \times \dfrac{7}{11} = \dfrac{14}{11}$ or 14 : 11

30. Area of equilateral triangle $= \dfrac{\sqrt{3}}{4} a^2$

Area of circle $= \pi \left(\dfrac{a}{2}\right)^2$

Required ratio $= \sqrt{3} : \pi$

31. $\dfrac{\theta}{360°} \pi r^2 = \pi r^2 \times \dfrac{5}{18}$

$\theta = 100°$

32. 20 cm + 14 cm + 20 cm + πr

$$20 + 14 + 20 + \frac{22}{7} \times 7 = 76 \text{ cm}$$

33. $\dfrac{\theta}{360^\circ} \times 2\pi r = \dfrac{60 \times 2 \times 22 \times 105}{360^\circ \times 7 \times 10} = 11 \text{ cm}$

Perimeter = 10.5 + 10.5 + 11 = 32 cm

34. $\theta = 7 \times 15^\circ = 105^\circ$

$$l = \frac{\theta}{360^\circ} 2\pi r = 44 \text{ cm}$$

Length of lace = $l + 2r$

= 44 + 48 = 92 cm

35. Perimeter of sector = $l + 2r$

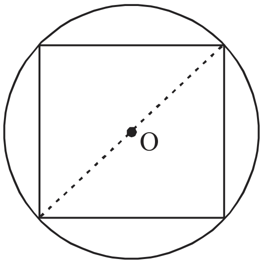
$l = 25.8 - 12.6 = 13.2$ cm

$$\frac{\theta}{360^\circ} \times 2\pi r = l \quad \Rightarrow \quad \theta = 120^\circ$$

Area of sector = $\dfrac{\theta}{360^\circ} \pi r^2$

Area of sector = 41.58 sq.cm.

36. d = Diagonal of square

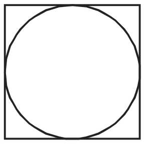


$d = \text{side}\sqrt{2} = 8\sqrt{2}$ cm

$r = 4\sqrt{2}$ cm

Area = $\pi R^2 = 32\pi$ sq.cm.

37. Diameter of circle = Side of square



$\therefore$ $r = 4$ cm

Area = 16π sq.cm.

38.
$$54\pi = \frac{\theta \times \pi \times 36 \times 36}{360^\circ}$$

$$\theta = 15^\circ$$

$$l = \frac{\theta}{360^\circ} \times 2\pi r = \frac{15^\circ \times 2 \times \pi \times 36}{360^\circ} = 3\pi \text{ cm}$$

39. Area $= \frac{\theta}{360°} \times \pi r^2 = \frac{210° \times 22 \times 5 \times 5}{360° \times 7} = \frac{1650}{36} = 45\frac{5}{6}$ sq.cm.

($\theta = 210°$ in 35 minutes)

40. Area of sector = area of sector – area of ΔAOB

$= \frac{77}{2} - \frac{49}{2}$

= 14 sq.cm.

41.

$$l = \frac{240° \times 2 \times 22 \times 35}{360° \times 7 \times 10}$$

$= 14.67$

Length of OAPBO $= 14.67 + 3.5 + 3.5$

$= 21.67$ cm.

42. Total Cost $= \pi[(1502)^2 - (1500)^2] \times 20$

$= 3.14\,[(1502)^2 - (1500)^2] \times 20$

= ₹ 377051.2

43. Circumference of cycle = $2\pi r$

$= 2 \times \frac{22}{7} \times 30$

$= 188.57$ cm

Speed of cycle $= \frac{18857 \times 140 \times 60}{100 \times 1000}$

$= 15.84$ km/h

44. Area of Minor sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30°}{360°} \times 3.14 \times 4 \times 4$ sq.cm.

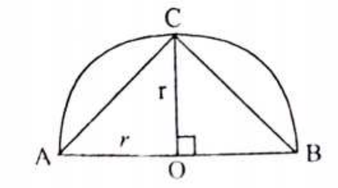
$= 4.19$ sq.cm. (approx.)

Area of major sector $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{330°}{360°} \times 3.14 \times 4 \times 4$$

$$= 46.1 \text{ sq.cm. (approx)}$$

45. Area of $\Delta = \frac{1}{2}$ base $\times$ height



$$= \frac{1}{2} AB \times OC$$

$$= \frac{1}{2} 2r \times r = r^2 \text{ square unit}$$

46. Grazing area of Goats = 2 × area of quadrants

$$= 2 \times \frac{22}{7} \times 1.4 \times 1.4 \times \frac{1}{4} = 3.08 \text{ sq.m}$$

Grazing area of cow = Ar. of circle

$$= \frac{22}{7} \times 2.1 \times 2.1 = 13.86 \text{ sq.m}$$

Area which can't be grazed = Area of square – total grazing area

= 64 – 16.94 = 43.06 sq.m

47.
$$\frac{7065}{100} = \frac{100° \times 314 \times r^2}{360° \times 100}$$

$$\frac{7065 \times 360}{100 \times 314} = r^2$$

$$9 = r$$

$$r = 9 \text{ cm.}$$

48. Distance by minute hand in 1 day = 24 × 2πR
Distance by hour hand in 1 day = 2 × 2πr
Total distance travelled by tips of both hands = 24 × 2πR + 2 × 2πR
= 1056 + 44
= 1100 cm

49. Four semicircluar means 2 circles ,

Area of 2 circles $= 2\pi r^2$

$= 2 \times 3.14 \times 20 \times 20$

$$= 2512 \text{ sq.m}$$

$$\text{Total cost} = 2512 \times 1.25$$

$$= ₹\ 3140$$

50. Length of chord = radius

$\therefore$ Angle of sector = 60°

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{8\pi}{3} \text{cm}^2 \quad \text{sq.cm}$$

Area of segment = Area of sector - Area of triangle

$$= \frac{8\pi}{3} - \frac{\sqrt{3}}{4} r^2$$

$$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right) \text{sq.cm}$$

51. Area of the segment = Area of sector – Area of Δ

$$\text{Area of sector} = \frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21 = 462 \text{ sq.cm}$$

$$\text{Area of } \Delta = \frac{441}{4}\sqrt{3} \text{ sq.cm}$$

$$\text{Area of segment} = \left(462 - \frac{441}{4}\sqrt{3}\right) \text{ sq.cm}$$

$$= \frac{21}{4}\left(88 - 21\sqrt{3}\right) \text{ sq.cm}$$

52.

$$l = \frac{\theta}{360°} \times 2\pi r$$

$$11 = \frac{45°}{360°} \times \frac{2 \times 22 \times r}{7}$$

$$14 = r$$

$$r = 14 \text{ cm}$$

53. $2\pi r = 2r + 16.8$

$$2\times\frac{22}{7}r - 2r = \frac{168}{10} \quad \text{or} \quad 2r\left(\frac{22}{7}-1\right)=\frac{168}{10}$$

or, $$2r\left(\frac{15}{7}\right) = \frac{168}{10} \quad \text{or} \quad \text{r} = \frac{168\times 7}{10\times 2\times 15} = \frac{1176}{300} = 3.92 \text{ cm}$$

54. $$l = \frac{\theta}{360°}\times(2\pi r)$$

$$22 = \frac{45°}{360°}\times 2\times\frac{22}{7}\times r$$

$$r = 28$$

Length of pendulum = 28 cm

55. $$\pi r_1^2 + \pi r_2^2 = 130\,\pi \Rightarrow r_1^2 + r_2^2 = 130 \qquad ...(1)$$

and $r_1 + r_2 = 14 \qquad \ldots(2)$

Substitute the value of r_1 from (2) in (1) and solve.

$$2r_2^{\,2} - 28\,r_2 + 66 = 0$$

$$r_2^2 - 14r_2 + 33 = 0 \qquad \text{(Neglecting – ve)}$$

$$r_2 = 11 \text{ cm and } r_1 = 3 \text{ cm}$$

56. $$\pi r^2 = \frac{616}{100} \quad \text{or} \quad r^2 = 1.96 \quad \text{or} \quad r = 1.4 \text{ m}$$

$$2\pi r = 2\times\frac{22}{7}\times\frac{14}{10} = \frac{616}{100} = 8.8 \text{ m}$$

Number of revolutions = $\dfrac{572}{8.8} = 65$

57. Grading area of Horses = $\dfrac{180°}{360°}\times\dfrac{22}{7}\times(14)^2 = 308\text{sq.m.}$

Area of triangular park = $\frac{1}{2}\times 35\times 84 = 1470\,\text{sq.m}$

Area which can't be grazed = $1162\,\text{sq.m}$

Grazing Area : Area can't be grazed = 308 : 1162

= 22 : 83

58. $R^2 + r^2 = 116$ ...(1)

$R - r = 6$ ...(2)

Squaring both sides and solving, we get

$2Rr = 80$ ...(3)

Adding and solving (1) and (3)

$R + r = 14$ ...(4)

Solving (2) and (4)

$R = 10$ cm, $r = 4$ cm

PRACTICE-TEST

AREAS RELATED TO CIRCLES

Time : 45 Minutes | *M.M.: 20*

SECTION-A

1. If the area of sector is $\frac{7}{18}$ of the area of the circle. Find the measure of central angle of the sector. **1**

2. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is: **1**

   (a) 48 cm (b) 31 cm (c) 25 cm (d) 17 cm

3. The area of sector whose perimeter is four times its radius of measure r units is_________. **1**

4. If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20π sq.cm, then find the radius of the circle.

SECTION-B

5. The perimeter of a sector of circle of radius 5.7 cm is 27.2 cm. Find the area of the sector. **2**

6. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand between 6:10 pm and 6:45 pm. **2**

7. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular cardboard of dimensions 16 cm × 8 cm. Find the area of the remaining cardboard. **2**

SECTION-C

8. The length of a rope by which a cow is tied is increased from 12m to 19m. How much more area can the cow graze now? (Use $\pi = \frac{22}{7}$) **3**

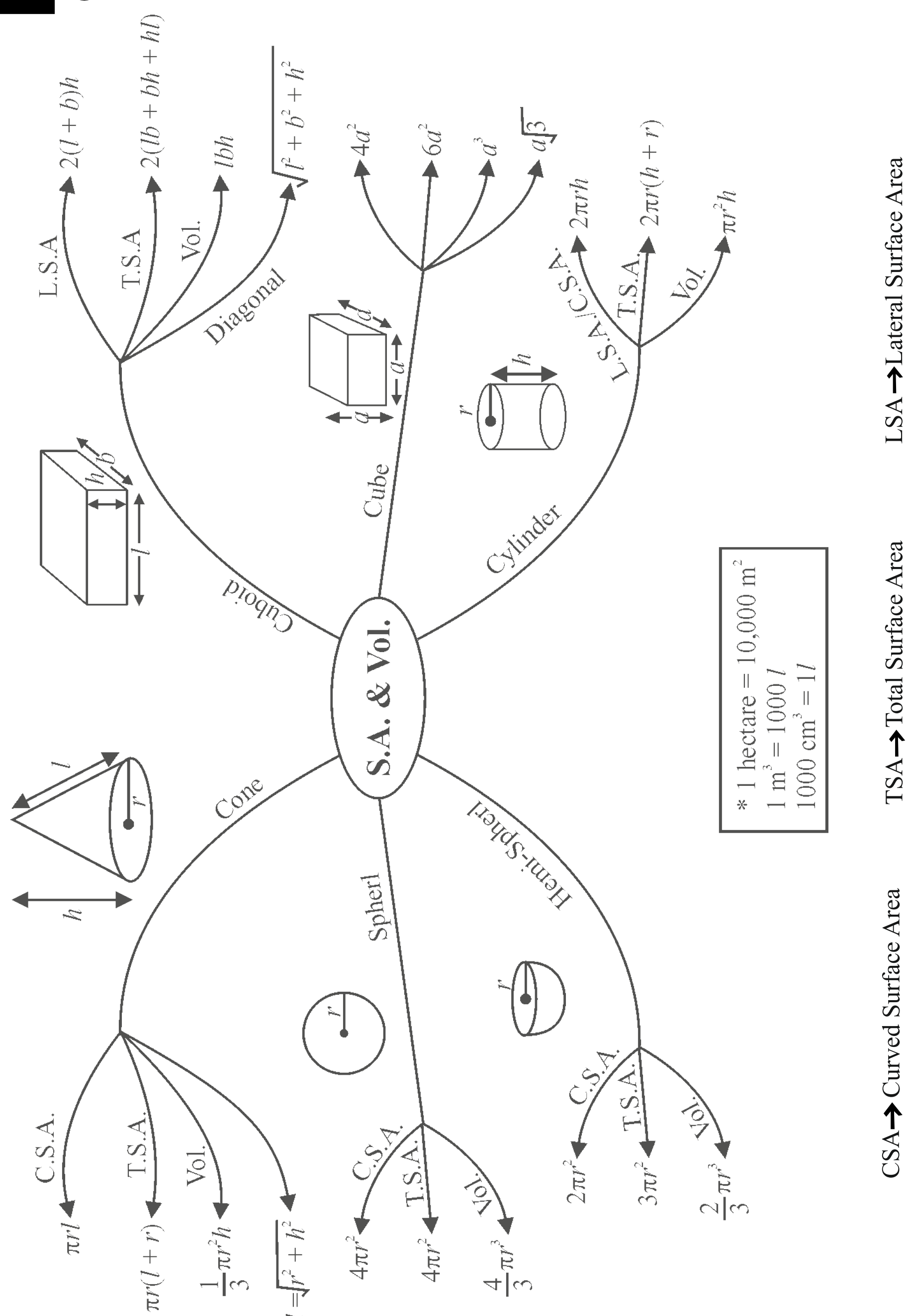
9. A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment. (Use $\pi = \frac{22}{7}$) **3**

SECTION-D

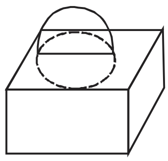
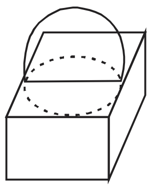
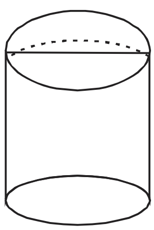
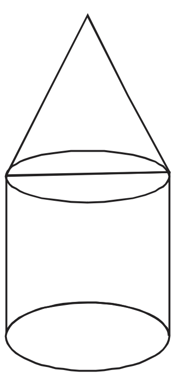
10. Find the area of minor and major segments of a circle of radius 42 cm, if the length of the arc is 88 cm. **4**

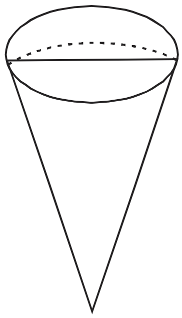
CHAPTER

12 Surface Areas and Volumes

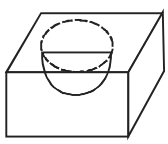
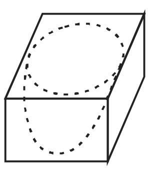
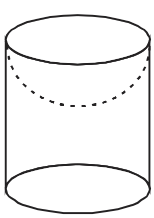


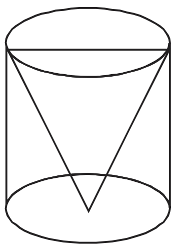
COMBINATION OF SOLIDS-I

Figure	Surface Area of Resultant Figure	Volume of Resultant Figure
 Cube & Hemisphere	$T.S.A_{cuboid} + C.S.A_{h.sphere}$ - Area of circle	$Vol._{cuboid} + Vol._{h.sphere}$
 Cubiod & Hemisphase	$T.S.A_{cube} + C.S.A_{h.sphere}$ - Area of circle	$Vol._{cube} + Vol._{h.sphere}$
 Cylinder and Hemisphese	Case I $\rightarrow$ when cylinder is hollow $C.S.A_{cyl.} + C.S.A_{h.sphere}$ Case II $\rightarrow$ when cylinder is solid $C.S.A_{cyl.} + C.S.A_{h.sphere}$ +Ar. of base	$Vol._{cyl.} + Vol._{h.sphere}$
 Cylinder & Cone	Case I $\rightarrow$ when cylinder is hollow $C.S.A_{cyl.} + C.S.A._{cone}$ Case II $\rightarrow$ when cylinder is solid $C.S.A_{cyl.} + C.S.A_{cone}$ +Ar. of base	$Vol._{cyl.} + Vol._{cone}$

 Cone & Hemisphase	$C.S.A_{cone} + C.S.A_{h.sphere}$	$Vol._{cone} + Vol._{h.sphere}$

COMBINATION OF SOLIDS - II
SURFACE AREA OF RESULTANT FIGURE

Figure	Surface Area of Resultant Fig.	Volume of Resultant Fig.
 Hemisphase curved out of cube	$T.S.A_{cuboid} + C.S.A_{h.sphere}$ -Area of circle	$Vol._{cuboid} – Vol._{h.sphere}$
 Hemisphase curved out of cubiod	$T.S.A_{cube} + C.S.A_{h.sphere}$ - Area of circle	$Vol._{cube} – Vol._{h.sphere}$
 Hemispherical depression in cylinder	Case I $\rightarrow$ hollow cylinder $C.S.A_{cyl.} + C.S.A_{h.sphere}$ Case II $\rightarrow$ Solid cylinder $C.S.A_{cyl.} + C.S.A_{h.sphere}$ + Ar. of circle	$Vol._{cyl.} – Vol._{h.sphere}$



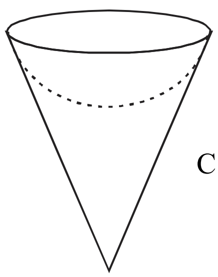
Case I $\rightarrow$ when cylinder is hollow

$C.S.A_{cyl.} + C.S.A._{cone}$ $\qquad$ $Vol._{cyl.} - Vol._{cone}$

Case II $\rightarrow$ when cylinder is solid

$C.S.A_{cyl.} + C.S.A_{cone} + Ar.\ of\ base$

Conical depression in cylinder



$C.S.A_{cone} + C.S.A_{h.sphere}$ $\qquad$ $Vol._{cone} - Vol._{h.sphere}$

Hemispherical depression in cone

VERY SHORT ANSWER TYPE QUESTIONS

1. The total surface area of a solid hemisphere of radius *r* is

(a) πr^2 (b) $2\pi r^2$ (c) $3\pi r^2$ (d) $4\pi r^2$

2. The volume and the surface area of a sphere are numerically equal, then the radius of sphere is

(a) 0 units (b) 1 unit (c) 2 units (d) 3 units

3. A cylinder, a cone and a hemisphere are of the same base and of the same height. The ratio of their volumes is

(a) 1:2:3 (b) 2:1:3 (c) 3:1:2 (d) 3:2:1

4. A solid sphere of radius '*r*' is melted and recast into the shape of a solid cone of height '*r*'. Then the radius of the base of cone is

(a) 2r (b) r (c) 4r (d) 3r

5. Three solid spheres of diameters 6 cm, 8 cm and 10 cm are melted to form a single solid sphere. The diameter of the new sphere is

(a) 6 cm (b) 4.5 cm (c) 3 cm (d) 12 cm

6. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is:

(a) 12 cm (b) 14 cm

(c) 15 cm (d) 18 cm

7. Find total surface area of a solid hemi-sphere of radius 7cm.

8. Volume of two spheres are in the ratio 64 : 125. Find the ratio of their surface areas.

9. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volumes of cylinder to that of the cone.

10. If the volume of a cube is 1331 cu.cm, then find the length of its edge.

11. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes? (CBSE 2020)

SHORT ANSWER TYPE QUESTION- (I)

12. How many cubes of side 2 cm can be cut from a cuboid measuring (16cm × 12cm × 10cm)?

13. Find the height of largest right circular cone that can be cut out of a cube whose volume is 729 cu.cm

14. Two identical cubes each of volume 216 cu.cm are joined together end to end. What is the surface area of the resulting cuboid?

15. Two cones with same base radius 8 cm and height 15 cm are joined together along with their bases. Find the surface area of the shape so formed.

16. The total surface area of a right circular cone is 90π sq.cm. If the radius of the base of the cone is 5 cm, find the height of the cone.

17. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cu.cm. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$

18. Find the volume of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

SHORT ANSWER TYPE QUESTION - (II)

19. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere. **(CBSE-2012)**

20. Find the depth of a cylindrical tank of radius 10.5 cm, if its capacity is equal to that of a rectangular tank of size 15 cm × 11 cm × 10.5 cm.

21. Volume of two spheres are in the ratio 64:27. Find the ratio of their surface areas. **(CBSE-2012)**

22. A petrol tank is a cylinder of base diameter 28 cm and length 24 cm fitted with conical ends each of 28 cm diameter and length 9 cm. Determine the capacity of the tank.

23. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

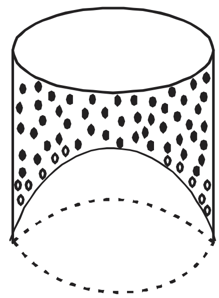
24. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE 2019)**

25. The diameter of a 120 cm long roller is 64 cm. If it takes 500 complete revolutions to level a playground, determine the cost of levelling it at the rate of 30 paise per square meter. **(CBSE 2013)**

26. The sum of the radius of base and height of a solid right circuler cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq.cm., then find the volume of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE-2016)**

27. A juice seller was serving his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass.

[Use $\pi = 3.14$]



28. The internal and external diameters of a hollow hemispherical vessel are 12 cm and 16 cm respectively. If the cost of painting 1sq.cm of the surface area is ₹ 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

29. Suresh decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is ₹ 40 per metre, find the amount by which Suresh helped the centre. **(CBSE 2017)**

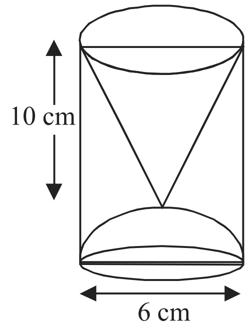
30. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

LONG ANSWER TYPE QUESTIONS

31. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cu.cm.of iron has approximately 8 gm mass. (Use $\pi = 3.14$)

32. A right cylindrical container of radius 6 cm and height 15 cm is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone. **(CBSE 2019)**

33. A wooden article as shown in the Figure was made from a cylinder by scooping out a hemisphere from one end and a cone from the other end. Find the total surface area of the article.



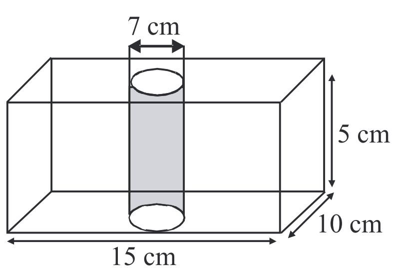
34. The height of a solid cylinder is 15 cm and its diameter is 7 cm. Two equal conical holes of radius 3 cm and height 4 cm are cut off. Find the volume and surface area of the solid.

35. If h, c and V respectively represent the height, curved surface area and volume of a cone, prove that

$$c^2 = \frac{3\pi V h^3 + 9V^2}{h^2}$$

36. A solid wooden toy is in the form of a hemi-sphere surmounted by a cone of same radius. The radius of hemi-sphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cu.cm. Find the height of the toy. Also, find the cost of painting the hemi-spherical part of the toy at the rate of ₹ 10 per sq.cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **(CBSE, 2015)**

37. In the given figure, from a cuboidal solid metalic block of dimensions 15 cm × 10 cm × 5 cm a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left(\text{Use } \pi = \frac{22}{7}\right)$



38. A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the toy.

39. A tent is in the shape of a right circular cylinder upto a height of 3 m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of ₹ 80 per sq.m.

40. The difference between outer and inner curved surface areas of a 14 cm long hollow right circular cylinder, is 88 sq.cm. If the volume of the metal used in making the cylinder is 176 cu.cm. Find the outer and inner diameters of the cylinder.

41. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid. **(CBSE 2020)**

42. A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to edge of the cube. Determine the volume of the remaining block. **(CBSE 2020)**

ANSWERS AND HINTS

1. (c) $3\pi r^2$

2. (d) 3 units

3. (c) 3 : 1 : 2

4. (a) 2r

5. (d) 12 cm

6. (b) 14 cm

7. 462 sq.cm

8. 16 : 25

9. 3 : 1

10. 11 cm

11. 3 : 1

12. No. of cubes $= \dfrac{16 \times 12 \times 10}{2 \times 2 \times 2} = 240$

13. Side of cube $= \sqrt[3]{729} = 9$cm

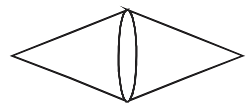
Height of largest cone = Side of cube = 9 cm

14. Side of cube $= \sqrt[3]{216} = 6$ cm

Length, breadth and height of new cuboid is 12 cm, 6 cm and 6 cm respectively.

Surface area of cuboid = 2[12 × 6 + 6 × 6 + 6 × 12] = 360 sq.cm.

15. $l = \sqrt{r^2 + h^2}$



$l = 17$ cm

Area = $2\pi rl$ = 854.85 sq.cm

16. $\pi r\,(l + r) = 90\pi$

$l = 13$ cm

$h = \sqrt{l^2 - r^2}$

$h = 12$ cm

17. Let the height and radius of cylinder be x cm and x cm respectively.

Volume of cylinder = $\dfrac{176}{7}$ cu.cm

$\dfrac{22}{7} \times (x)^2 \times x = \dfrac{176}{7}$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$ cm

18. d = 4.2 cm; r = 2.1 cm

h = 4.2 cm

Volume of cone = $\dfrac{1}{3}\pi r^2 h$

Volume of cone = 19.4 cu.cm (approx)

19. Radius of sphere = 3 cm

Volume of sphere = $\dfrac{4}{3}\pi r^3$

= 113.14 cu.cm

20. Capacity of cylindrical tank = Capacity of rectangular tank

$\dfrac{22}{7} \times (10.5)^2 \times \text{h} = 15 \times 11 \times 10.5$

h = 5 cm

21. $\dfrac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \dfrac{64}{27}$

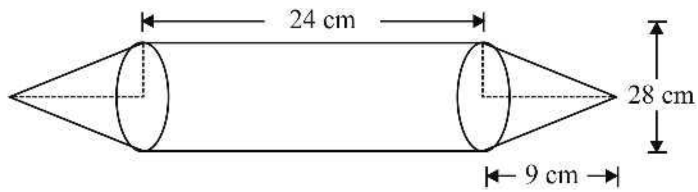
$\Rightarrow R^3 : r^3 = 64 : 27$

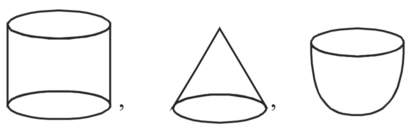
$\Rightarrow R : r = 4 : 3$

$\pi R^2 : 4\pi r^2 = R^2 : r^2 \Rightarrow 4^2 : 3^2 = 16 : 9$

22. Capacity of tank = Volume of cylindrical part + 2 × Volume of conical part

= 18480 sq.cm



23. 

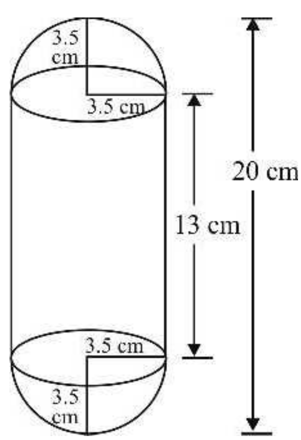
Radius = r, height = r

$\text{Volume}_{\text{cylinder}} : \text{Volume}_{\text{cone}} : \text{volume}_{\text{hemisphere}}$

Req. Ratio = $\pi r^3 : \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3$

$= 1 : \frac{1}{3} : \frac{2}{3}$

$= 3 : 1 : 2$



24. Height of cylinder = 20 – 3.5 – 3.5 = 13 cm

Volume of solid =Volume of cylindrical part + 2 × Volume of hemispherical part

$$= \frac{22}{7} \times (3.5)^2 \times 13 + 2 \times \frac{2}{3} \times \frac{22}{7}(3.5)^3$$

$$= 680\frac{1}{6} \text{ cu.cm.}$$

25. $r = 32$ cm; $h = 120$ cm

Area covered in 1 revolution

= C.S.A. of roller

$= 2\pi rh$

$= 24137.14$ sq.cm

Area covered in 500 rev. $= 1206.86$ sq.m

Cost of levelling = Area × Rate

$= 1206.86 \times 0.3$

$=$ ₹ 362.06

26. $r + h = 37$

$2\pi r(r + h) = 1628$

$r = 7$ cm

$h = 30$ cm

Volume $= \pi r^2 h$

Volume $= 4620$ cu.cm

27. Apparent capacity $= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10 = 196.25$ cu.cm

Actual capacity = Volume of cylindrical part – Volume of hemispherical part

$$= 196.25 - \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3$$

$$= 163.54 \text{ cu.cm. approx}$$

28. $r = 6$cm; R = 8 cm

S.A. of vessel $= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi \times 228 = 715.92$ sq.cm

Total cost = S.A. × Rate

$=$ ₹ 3579.60

29. $r = 7$cm; $h = 24$m

$l = 25$ m

S.A. of tent $= \pi rl$

$= 550$sq.m

Area of 10 tents = 5500 sq.m

Total cost = Area × Rate

$= 5500 \times \frac{40}{2}$

$=$ ₹ 1,10,000

30. $r = 7$cm; $h = 14$ cm

$l = \sqrt{245} = 15.65$sq.cm

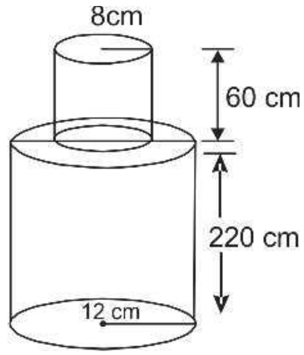
S.A. of remainging solid

= T.S.A. of cube + C.S.A. of cone – Area of circle

$= 6a^2 + \pi rl - \pi r^2$

= 1366.3 sq.cm.

31.



Volume of solid $= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$

$= 111532.8$ cu.cm

Mass of the pole $= 111532.8 \times \frac{8}{1000}$

$= 892.2624$ kg

32. Let radius of conical section be r cm.

$\therefore$ Height of conical section be 4r cm.

According to the question

$10 \times$ Volume of ice-cream in 1 cone = Volume of cylindrical container

$$10 \times \left[\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right] = \pi(6)^2 \times 15$$

$r = 3$ cm

33. $r = 3$ cm

S.A. of article = C.S.A.$_{\text{cylinder}}$ + C.S.A.$_{\text{sphere}}$ + C.S.A.$_{\text{cone}}$

S.A. $= 2\pi r\text{H} + 2\pi r^2 + \pi rl$

$= \pi r(2\text{H} + 2r + l)$

$= 3\pi(20 + 6 + \sqrt{58})$

$= \pi(78 + 3\sqrt{58})$sq.cm

34. Surface area of solid = C.S.A.$_{\text{cyl.}}$ + 2 Area of Ring + 2C.S.A.$_{\text{cone}}$

$$= 2\pi\left[\frac{7}{2}\times 15 + 6.5\times 0.5 + 15\right]$$

$$= 2\times\frac{22}{7}\times 70.75 = \frac{3113}{7}$$

= 444.7sq.cm (approx.)

35. $\text{V} = \dfrac{1}{3}\pi R^2 h$

$$\Rightarrow R^2 = \frac{3\text{V}}{\pi h} \qquad ...(1)$$

Now, $c = \pi\text{R}l$

$$c^2 = \pi^2 R^2 l^2$$

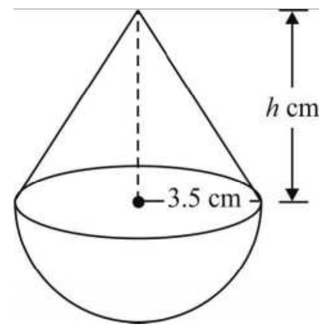
$$c^2 = \pi^2 R^2 (h^2 + R^2)$$

$$c^2 = \pi^2 \frac{3\text{V}}{\pi h}\left(h^2 + \frac{3\text{V}}{\pi h}\right)$$

$$c^2 = \frac{3\pi^2 V(\pi h^3 + 3V)}{\pi^2 h^2}$$

$$c^2 = \frac{3\pi\text{V}h^3 + 9\text{V}^2}{h^2}$$

36. Volume of toy $= \frac{1001}{6}$ cu.cm



$$\frac{2}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3+\frac{1}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times h = \frac{1001}{6}$$

$$h = 6 \text{ cm}$$

Area of hemispherical part of toy

$$= 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 = 77 \text{ sq.cm}$$

Cost of painting = 77 × 10 = ₹ 770

37. Surface of the remaining block = TSA of cuboidal block + CSA of cylinder – Area of two circular bases

$$= 2(15\times 10 + 10\times 5 + 15\times 5) + 2\times\frac{22}{7}\times\frac{7}{2}\times 5 - 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2$$

$= 583$ sq.cm

38. Volume of toy = Volume of cylindrical part + Volume of hemispherical part + Volume of conical part

$$= \frac{22}{7}\times(2.1)^2\times 12+\frac{1}{3}\times\frac{22}{7}\times(2.1)^2\times 7+\frac{2}{3}\times\frac{22}{7}\times(2.1)^3$$

$= 218.064$ cu.cm

39. Slant height $= \sqrt{(14)^2+(10.5)^2} = 17.5$ m

Surface area of tent $= 2\times\frac{22}{7}\times 3\times 14+\frac{22}{7}\times 14\times 17.5$

$= 1034$ sq.m

Cost of cloth = 1034 × 80 = ₹ 82720

40. Let inner and outer radius of hallow cylinder be r cm and R cm respectively.

Difference between Outer and Inner CSA = 88 sq.cm

$$2\times\frac{22}{7}\times 14\times[R-r] = 88$$

$$R - r = 1 \qquad ...(1)$$

Volume of hollow cylinder = 176 cu.cm

$$\frac{22}{7} \times 14 \times [R^2 - r^2] = 176$$

$$R^2 - r^2 = 4$$

$$(R - r)(R + r) = 4$$

$$R + r = 4 \qquad ...(2) \;[\because \text{ from } (1)]$$

From (1) and (2), we get

$$R = 2.5 \text{ cm and } r = 1.5 \text{ cm}$$

$\therefore$ Outer and inner diameter are 5 cm and 3 cm respectively.

41. Height of cone =9.5 – 3.5 = 6 cm

$$\text{Volume of solid} = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 + \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6$$

$$= 166.83 \text{ cu.cm approx}$$

42. Radius of hemisphere $= \frac{21}{2} = 10.5$ cm

$$\text{Volume of remaining block} = (21)^3 - \frac{2}{3} \times \frac{22}{7} \times (10.5)^3$$

$$= 6835.5 \text{ cu.cm}$$

PRACTICE-TEST

SURFACE AREAS AND VOLUMES

Time : 45 Minutes | *M.M.: 20*

SECTION-A

1. The total surface area ofa hemisphere of radius $2r$ is ________. **1**

2. The radius of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is **1**

(a) 4.2 cm (b) 8.4 cm

(c) 2.1 cm (d) 1.05 cm

3. The volume of a cube is $1l$. Find the length of the side of the cube. **1**

4. Volume of two cubes are in the ratio 27 : 125. The ratio of their surface areas is_______. **1**

SECTION-B

5. A cube and a sphere have equal total surface area. Find the ratio of the volume of sphere and cube. **2**

6. Two cubes, each of side 8cm are joined end to end. Find the surface area of the resulting figure. **2**

7. The volume of a hemi-sphere is 2156 cu.cm. Find its curved surface area. **2**

SECTION-C

8. A circus tent is in the shape of a cylinder surmounted by a conical roof. If the common diameter is 56 m, the height of the cylindrical portion is 6 m and the height of the roof from the ground is 30 m, then find the area of the canvas used for the tent. **3**

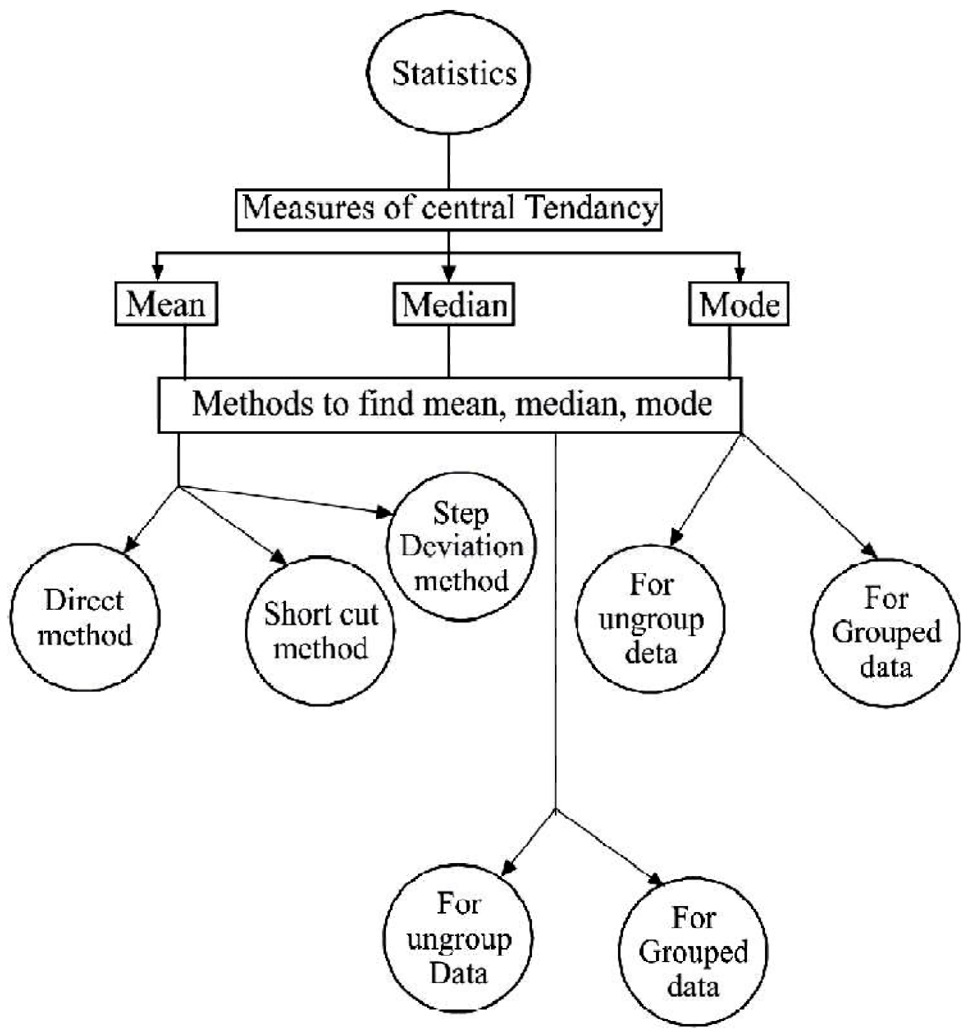
9. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole of radius $\frac{3}{2}$ cm and depth $\frac{8}{9}$ cm is drilled in the cylinder. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape. **3**

SECTION-D

10. A decorative block is made up by joining a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 4 cm. Find the cost of painting it at a price of ₹ 2.5 per sq.cm. **4**

CHAPTER

13 Statistics



KEY POINTS:

1. **Mean $(\bar{x})$**

(a) For raw data, $\bar{x} = \frac{\Sigma x_i}{n} = \frac{x_1 + x_2 + ... + x_n}{n}$

i.e. $\bar{x} = \frac{\text{sum of observations}}{\text{number of observations}}$

(b) For Grouped data

(i) For small calculation, we apply Direct method

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

(ii) If calculations are tedius or observations are large, then we apply short cut/ Assumed Mean method or step Deviation method

Short cut/Assumed Mean Method

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}, \quad a \rightarrow \text{assumed mean}$$

$$d_i = x_i - a$$

Step Deviation Method

$$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h, \quad u_i = \frac{d_i}{h}, \ h \rightarrow \text{class size}$$

2. **Median**

(a) For ungrouped data, we first arrange data in ascending or descending order.

Count number of times say 'n'. If n is odd, then Median = $\left(\frac{n+1}{2}\right)^{th}$ observation

If n is even, then Median = $\frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$ obsevation

(b) For grouped data

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

(3) Mode $= l + \dfrac{(f_1 - f_o)}{(2f_1 - f_o - f_2)} \times h$ (For grouped data)

For ungrouped data mode is the most frequent observation.

NOTES:

1. Empirical relationship between three measures of central tendency:

mode = 3 median – 2 mean.

2. If class interval is discontinuous, then make it continuous by subtracting 0.5 from Lower Limit and adding 0.5 to upper limit.

3. x_i = class mark $= \dfrac{\textit{Upper Limit} + \textit{Lower Limit}}{2}$

4. h = class size = Upper Limit – Lower limit

5. Modal class $\rightarrow$ A class interval having maximum frequency.

6. Median class $\rightarrow$ A class interval is which cumulative frequency is greater than and nearest to $\dfrac{n}{2}$ $(n = \Sigma f_i)$

8. If mean of $x_1, x_2, ..., x_n$ is $\overline{x}$ then

(a) Mean of $kx_1, kx_2, ..., kx_n$ is $k\overline{x}$

(b) Mean of $\dfrac{x_1}{k}, \dfrac{x_2}{k}, ..., \dfrac{x_n}{k}$ is $\dfrac{\overline{x}}{k}$

(c) Mean of $x_1 + k, x_2 + k, ..., x_n + k$ is $\overline{x} + k$

(d) Mean of $x_1 - k, x_2 - k, ..., x_n - k$ is $\overline{x} - k$

9. If mean of n_1 observation is $\overline{x}_1$ and mean of n_2 observation is $\overline{x}_2$ then their combined

$$\text{Mean} = \frac{n_1 \overline{x}_1 + n_2 \overline{x}_2}{n_1 + n_2}$$

10. $\Sigma x_i = n\overline{x}$

11. Range = Highest observation – Lowest observation

VERY SHORT ANSWER TYPE QUESTIONS

1. What is the mean of first 12 prime numbers?

2. The mean of 20 numbers is 18. If 2 is added to each number, then what is the new mean?

3. The mean of 5 observations 3, 5, 7, x and 11 is 7, find the value of x.

4. What is the median of first 5 natural numbers?

5. What is the value of x, if the median of the following data is 27.5?
24, 25, 26, $x + 2$, $x + 3$, 30, 33, 37

6. What is the mode of the observations 5, 7, 8, 5, 7, 6, 9, 5, 10, 6?

7. The mean and mode of a data are 24 and 12 respectively. Find the median.

8. Write the class mark of the class 19.5 – 29.5.

9. Multiple Choice Question

(i) If the class intervals of a frequency distribution are 1 – 10, 11 – 20, 21 – 30, ..., 51 – 60, then the size of each class is:

(a) 9 (b) 10 (c) 11 (d) 5.5

(ii) If the class intervals of a frequency distribution are 1 – 10, 11 – 20, 21 – 30 ...., 61 – 70, Then the upper limit of 21 – 30 is:

(a) 21 (b) 30

(c) 30.5 (d) 20.5

(iii) Consider the frequency distribution.

Class	0 – 5	6 – 11	12 – 17	18 – 23	24 – 29
Frequency	13	10	15	8	11

The upper limit of median class is :

(a) 17 (b) 17.5 (c) 18 (d) 18.5

(iv) Daily wages of a factory workers are recorded as:

Daily wages (in ₹)	121 – 126	127 – 132	133– 138	139 – 144	145 – 150
No. of workers	5	27	20	18	12

The lower limit of Modal class is:

(a) ₹ 127 (b) ₹ 126 (c) ₹ 126.50 (d) ₹ 133

(v) For the following distribution

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

The sum of Lower limits of the median class and modal class is: **(CBSE 2020)**

(a) 15 (b) 25 (c) 30 (d) 35

(vi) The median and mode respectively of a frequency distribution are 26 and 29. Then, its mean is **(CBSE 2020)**

(a) 27.5 (b) 24.5 (c) 28.4 (d) 25.8

10. Find the class-marks of the classes 10-25 and 35-55. **(CBSE 2020)**

SHORT ANSWER TYPE QUESTIONS (I)

11. The mean of 11 observation is 50. If the mean of first Six observations is 49 and that of last six observation is 52, then find sixth observation.

12. Find the mean of following distribution:

x	12	16	20	24	28	32
f	5	7	8	5	3	2

13. Find the median of the following distribution:

x	10	12	14	16	18	20
f	3	5	6	4	4	3

14. Find the mode of the following frequency distribution:

Class	0–5	5–10	10 –15	15–20	20–25	25–30
Frequency	2	7	18	10	8	5

15. Convert the following deistribution in frequency distribution:

Marks	**No. of students**
Less than 20	0
Less than 30	4
Less than 40	16
Less than 50	30
Less than 60	46
Less than 70	66
Less than 80	82
Less than 90	92
Less than 100	100

16. Write the following data into less than cummulative frequency distribution table :

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	7	9	6	8	10

17. Find mode of the following frequency distribution :

Class Interval	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
Frequency	25	34	50	42	38	14

(CBSE 2018 - 19)

18. What is the median of the following data? **(CBSE 2011)**

x	10	20	30	40	50
f	2	3	2	3	1

19. Mean of a frequency distribution ($\bar{x}$) is 45. If $\Sigma f_i = 20$ then find $\Sigma f_i x_i$

(CBSE 2011)

20. Find the mean of the following distribution : **(CBSE 2020)**

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

21. Find the mode of the following data : **(CBSE 2020)**

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120-140
Frequency	6	8	10	12	6	5	3

22. Compute the mode for the following frequency distribution: **(CBSE 2020)**

Size of items (in cm)	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28
Frequency	5	7	9	17	12	10	6

SHORT ANSWER TYPE QUESTIONS (II)

23. If the mean of the following distribution is 54, then find the value of P.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	7	P	10	9	13

24. Find the median of the following frequency distribution :

C.I.	0–10	10–20	20–30	30–40	40–50	50–60
f	5	3	10	6	4	2

25. The median of following frequency distribution is 24 years. Find the missing frequency x.

Age (In years)	0–10	10–20	20–30	30–40	40–50
No. of persons	5	25	x	18	7

26. Find the median of the following data:

Marks	Below 10	Below 20	Below 30	Below 40	below 50	Below 60
No. of student	0	12	20	28	33	40

27. Find the mean weight of the following data:

Weight (In kg.)	30–35	35–40	40–45	45–50	50–55	55–60
No. of Students	2	4	10	15	6	3

28. Find the mode of the following data:

Height (In cm)	Above 30	Above 40	Above 50	Above 60	Above 70	Above 80
No. of plants	34	30	27	19	8	2

29. The following table represent marks obtained by 100 students in a test:

Marks obtained	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
No. of students	14	16	28	23	18	8	3

Find mean marks of the students. **(CBSE 2018 -19)**

30. The following table represent pocket allowance of children of a colony. The mean pocket allowance is ₹ 18. Find the missing frequency.

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
No. of children	3	6	9	13	k	5	4

(CBSE – 2018)

31. Find mode of the following frequency distribution:

Class Interval	0–20	20–40	40–60	60–80	80–100
No. of Students	15	18	21	29	17

The mean of above distribution is 53. Use Empirical formula to find approximate value of median.

LONG ANSWER TYPE QUESTIONS

32. The mean of the following data is 53, Find the values of f_1 and f_2.

C.I	0–20	20–40	40–60	60–80	80–100	Total
f	15	f_1	21	f_2	17	100

33. If the median of the distribution given below is 28.5, find the values of x and y.

C.I	0–10	10–20	20–30	30–40	40–50	50–60	Total
f	5	8	x	15	y	5	60

34. The median of the following distribution is 35, find the values of a and b.

C.I	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Total
f	10	20	a	40	b	25	15	170

35. Find the mean, median and mode of the following data:

C.I	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	2	3	6	7	14	12	4	2

36. The rainfall recorded in a city for 60 days is given in the following table:

Raifall (in cm)	0–10	10–20	20–30	30–40	40–50	50–60
No. of Days	16	10	8	15	5	6

Calulate the median rainfall.

37. Find the mean of the following distribution by step- deviation method:

Daily Expenditure (in ₹)	100–150	150–200	200–250	250–300	300–350
No. of Households	4	5	12	2	2

38. The distribution given below show the marks of 100 students of a class:

Marks	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
No. of Students	4	6	10	10	25	22	18	5

Find the median marks of the above distribution.

39. The annual profit earned by 30 factories in an industrial area is given below:

Profit (₹ in lakh)	**No. of Factories**
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3
More than or equal to 40	0

Find the median of the above data.

40. Find the mean and median of the following distribution:

(CBSE 2018 -19)

Class Interval	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	7	5	8	10	6	6	8

41. If mean of the given distribution is 65.6, then find the missing frequencies.

(CBSE 2017)

Class Interval	10 – 30	30 – 50	50 – 70	70 – 90	90 – 110	110 – 130	Total
Frequency	5	8	f_1	20	f_2	2	50

42. The mode of the frequency distribution is 36. Find the missing frequency (f).

(CBSE 2020)

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	8	10	f	16	12	6	7

43. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f. **(CBSE 2020)**

Class Interval	11–13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	3	6	9	13	f	5	4

44. The following table gives production yield per hectare of wheat of 100 farms of a village :

Production Yield	40 – 45	45 – 50	50 – 55	55 – 60	60 – 55	65 – 70
Frequency	4	6	16	20	30	24

Find the mode of the above data.

45. Find the unknown entries a, b, c, d, e, f in the following distribution of heights of students in a class:

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Frequency	12	b	10	d	e	2
Cummulative Frequency	a	25	c	43	48	f

Find the mode of the above data.

ANSWERS AND HINTS

1. 16.4 approx.

2. 20

3. 9

4. 3

5. $x = 25$

6. 5

7. Median = 20

8. 24.5

9. (i) b (First make intervals continuous, Then find class size)

(ii) c

(iii) b

(iv) c

(v) b $\begin{bmatrix} \text{Modal class } 15-20 \\ \text{Median class } 10-15 \end{bmatrix}$

(vi) b

10. 17.5 and 45

11. 56 **12.** 20

13. 14 **14.** 12.89 approx.

15.

Marks	No. of students
20-30	4
30-40	12
40-50	14
50-60	16
60-70	20
70-80	16
80-90	10
90-100	8

16.

Marks	No. of students
less than 10	7
less than 20	16
less than 30	22
less than 40	30
less than 50	40

17.

Class Interval	Frequency
25 – 30	25
30 – 35	$34 = f_0$
35 – 40	$50 = f_1$
40 – 45	$42 = f_2$
45 – 50	38
50 – 55	14

Mode = $l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h = 35 + \frac{(50 - 34)}{(100 - 34 - 42)} \times 5 = 35 + \frac{16 \times 5}{24}$

= 35 + 3.33 = 38.33 approx.

18.

x_i	f_i	cf
10	2	2
20	3	5
30	2	7
40	3	10
50	1	11
Total	11	

N = 11 (odd)

Median = $\left(\frac{N+1}{2}\right)^{th}$ observation = 6th observation = 30

19. $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 45 = \frac{\Sigma f_i x_i}{20} \Rightarrow \Sigma f_i x_i = 900$

20. 8.15

21. 65

22. 14.46 cm

23. 11

24. 27

25. 25

26. 30

27. 46

28. 63.75 cm

29.

Mark	x_i	d_i	u_i	f_i	$f_i u_i$
30 – 35	32.5	– 15	– 3	14	– 42
35 – 40	37.5	– 10	– 2	16	– 32
40 – 45	42.5	– 5	– 1	28	– 28
45 – 50	47.5 = a	0	0	23	0
50 – 55	52.5	5	1	18	18
55 – 60	57.5	10	2	8	16
60 – 65	62.5	15	3	3	9
				110	–59

$$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 47.5 - \frac{59}{110} \times 5 = 47.5 - 2.68 = 44.82$$

30. (Make Table just like Q. 29)

$$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$18 = 18 + \frac{(k-8)}{40+k} \times 2$$

$$2k - 16 = 0$$

$$k = 8$$

31. Mode $= l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$

$$= 60 + \frac{(29-21)}{(2 \times 29 - 21 - 17)} \times 20 = 68$$

Mode = 3 Median – 2 mean

68 = 3 Median – 2 × 53

$$\frac{68+106}{3} = \text{Median}$$

Median = 58

32. $f_1 = 18, f_2 = 29$

33. $x = 20, y = 7$

34. $a = 35, b = 25$

35. Mean = 32, median = 33, mode = 34.39 approx.

36. Median = 25 cm

37. Mean = ₹ 211

38. Median = 24

39. Median = ₹ 17.5 lakhs.

40. Mean = 65.6, Median = 65

41.

C.I	f_i	x_i	$f_i x_i$
10 – 30	5	20	100
30 – 50	8	40	320
50 – 70	f_1	60	$60f_1$
70 – 90	20	80	1600
90 – 110	f_2	100	$100f_2$
110 – 130	2	120	240
	$\mathbf{35 + f_1 + f_2}$		$\mathbf{2260 + 60\,f_1 + 100\,f_2}$

$35 + f_1 + f_2 = 50 \quad \Rightarrow \quad f_1 + f_2 = 15$ ....(1)

$$\bar{x} = \frac{\Sigma fixi}{\Sigma fi}$$

$$65.6 = \frac{2260 + 60\,f_1 + 100\,f_2}{50}$$

$$\Rightarrow 3f_1 + 5f_2 = 51 \qquad ...(2)$$

Solve (1) & (2) $\quad f_1 = 12, f_2 = 3$

42. $f = 10$

43. $f = 8$

44. Mode = 63.125

45. $a = 12, b = 13, c = 35, d = 8, e = 5, f = 50$

PRACTICE-TEST

Statistics

Time : 45 Minutes *M.M. : 20*

SECTION-A

1. Find the mean of first 10 natural numbers. **1**

2. The range of the data 14, 27, 29, 61,45,15, 9,18 is **1**

(a) 61 (b) 52

(c) 47 (d) 53

3. In a continuous frequency distribution, the median of the data is 24. If each item is increased by 2, then find the new median. **1**

4. For a frequency distribution, mean, median and mode are connected by the relation. **1**

(a) mode = 3 mean – 2median (b) mode = 2 median – 3mean

(c) mode = 3 median – 2mean (d) mode = 3 median + 2 mean

SECTION-B

5. The mean of 10 observations is 42. If each observation in the data is decreased by 12, then find the new mean of the data. **2**

6. The mean of 10 numbers is 15 and that of another 20 number is 24. Find the mean of all 30 observations. 2

7. The mileage (km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below: **2**

Mileage (in km/*l*)	10 – 12	12 – 14	14 – 16	16 – 18
No. of cars	7	12	18	13

Find the mean mileage.

SECTION-C

8. Life time of 400 fans are given in the following frequency distribution table:

Life time	2000-2400	2400-2800	2800-3200	3200-3600	3600-4000
No. of fans	5	15	20	23	17

Find the median number of fans. **3**

9. The mode of the following data is 36. Find the value of x. **3**

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	x	16	12	6	7

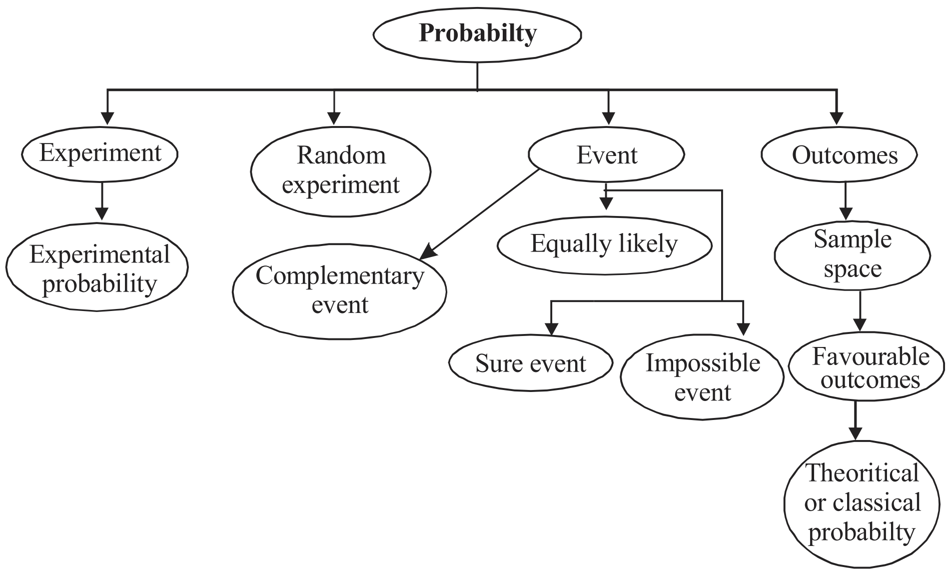
SECTION-D

10. The median of the following data is 28. Find the values of x and y, if the total frequency is 50. **4**

Marks	0-7	7-14	14-21	21-28	28-35	35-42	42-49
No. of	3	x	7	11	y	16	9

CHAPTER

14 Probability



KEY POINTS:

1. Probability is a quantitative measure of likelihood of occurrence of an event.
2. Probability of an event $(E) = \dfrac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}} = \dfrac{N(E)}{N(S)}$
3. $0 \leq P(E) \leq 1$
4. If $P(E) = 0$, then it is an impossible event.
5. If $P(E) = 1$, then it is sure event.
6. If E is an event, then not $E(\overline{E})$ is called complementary event.
7. $P(\overline{E}) = 1 - P(E) \Rightarrow P(E) + P(\overline{E}) = 1$
8. Probability of an event is never negative.
9. Sample space (S) : The collection of all possible outcomes of random experiment.

Examples of Sample space

1. When one coin is tossed, then S = {H, T}
2. When two coins are tossed, then S = {HH, TT, HT, TH}
3. When three coins are tossed, then S = {HHH, TTT, HTT, THT, TTH, THH, HTH, HHT}
4. When four coins are tossed, then S = {HHHH, TTTT, HTTT, THTT, TTHT, TTTH, HHHT, HHTH, HTHH, THHH, HTHT, THTH, TTHH, HHTT, THHT, HTTH}.

1 coin	2coins	3 coins	4 coins
2 outcomes	2×2 outcomes	$2 \times 2 \times 2 = 8$ outcomes	$2 \times 2 \times 2 \times 2 = 16$ outcomes

1. When a die is thrown once, then S = {1, 2, 3, 4, 5, 6}, n(S) = 6
2. When two dice are thrown together or A die is thrown twice, then

$$
\begin{aligned}
S = \; & \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\
& (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\
& (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\
& (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\
& (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\
& (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\} \\
n(S) = \; & 6 \times 6 = 36
\end{aligned}
$$

3. When 3 dice are thrown or a die is thrown thrice then

$$
\begin{aligned}
n(S) = \; & 6 \times 6 \times 6 = 216, \\
n(S) \rightarrow \; & \text{no. of outcomes in sample space}
\end{aligned}
$$

Playing cards n(s) = 52

Red Cards (26) — Black Cards (26)

Heart (13) — Diamond (13) — Spade (13) — Club (13)

Each suit contains 1 ace, 1 king, 1 Queen, 1 jack and nine number cards 2, 3, 4, 5, 6, 7, 8, 9, 10

Face card 12
4 king, 4 Queen & 4 Jack

Non face card 40
36 number cards + 4 aces

VERY SHORT ANSWER TYPE QUESTIONS

1. Multiple Choice Questions

(i) Which of the following cannot be the probability of an event?

(a) 0.7 (b) $\frac{2}{3}$ (d) -1.5 (d) 15%

(ii) Which of the following can be the probability of an event?

(a) -0.04 (b) 1.004 (c) $\frac{18}{23}$ (d) $\frac{8}{7}$

(iii) An event is very unlikely to happen, its probability is closest to

(a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

(iv) Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is:

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{4}{9}$ (d) $\frac{2}{5}$

(v) When a die is thrown, the probability of getting an odd number less than 3 is:

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0

(vi) Rashmi has a die whose six faces show the letters as given below:

A	B	C	D	A	C

If she throws the die once, then the probability of getting C is:

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

(vii) A card is drawn from a well shuffled pack of 52 playing cards. The event E is that the card drawn is not a face card. The number of outcomes favourable to the event E is:

(a) 51 (b) 40 (c) 36 (d) 12

2. **Choose the correct answer from the given four options**

(i) If the probability of an even is 'p' then probability of its complementary event will be:

(a) p – 1 (b) p (c) 1 – p (d) $1-\frac{1}{p}$

(ii) If P (Winning) = $\frac{x}{12}$, P(Losing) = $\frac{1}{3}$ then the value of 'x' is :

(a) 6 (b) 8 (c) 7 (d) 9

(iii) The probability of a number selected at random from the numbers 1, 2, 3, ... 15 is a multiple of 4 is: **(CBSE 2020)**

(a) $\frac{4}{15}$ (b) $\frac{2}{15}$ (c) $\frac{1}{15}$ (d) $\frac{1}{5}$

(iv) The probability that a non-leap year selected at random will contains 53 Mondays is:

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

(v) A bag contains 6 red and 5 blue balls. One ball is drawn at random. The probability that the ball is blue is:

(a) $\frac{2}{11}$ (b) $\frac{5}{6}$ (c) $\frac{5}{11}$ (d) $\frac{6}{11}$

(vi) One alphabet is chosen from the word MATHEMATICS. The probability of getting a vowel is:

(a) $\frac{6}{11}$ (b) $\frac{5}{11}$ (c) $\frac{3}{11}$ (d) $\frac{4}{11}$

(vii) Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

3. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king.

4. Out of 250 bulbs in a box, 35 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.

5. Non Occurance of any event is 3:4. What is the probability of Occurance of this event?

6. If 29 is removed from (1, 4, 9, 16, 25, 29), then find the probability of getting a prime number.

7. A card is drawn at random from a deck of playing cards. Find the probability of getting a face card.

8. In 1000 lottery tickets, there are 5 prize winning tickets. Find the probability of winning a prize if a person buys one ticket.

9. One card is drawn at random from a pack of cards. Find the probability that it is a black king. **(CBSE 2020)**

10. A die is thrown once. Find the probability of getting a perfect square.

11. Two dice are rolled simultaneously. Find the probability that the sum of the two numbers appearing on the top is more than or equal to 10.

12. Find the probability of multiples of 7 in 1, 2, 3, ...,33, 34, 35.

13. If a pair of dice is thrown once, then what is the probability of getting a sum of 8? **(CBSE 2020)**

14. A letter of English alphabet is chosen at random. Determine the probability that chosen letter is a consonant. **(CBSE 2020)**

15. If the probability of winning a game is 0.07, what is the probability of losing it? **(CBSE 2020)**

SHORT ANSWER TYPE QUESTIONS-I

16. Two unbiased coins are tossed simultaneously. If the probability of getting no head is $\frac{a}{b}$ then find $(a + b)^2$.

17. Two different dice are rolled together. Find the probability
(a) of getting a doublet,
(b) of getting a sum of 10, of the numbers on the two dice.

18. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the box. **[CBSE 2018]**

19. An integer is chosen random between 1 and 100. Find the probability that (i) it is divisible by 8, (ii) Not divisible by 8. **[CBSE 2018]**

20. Three different coins are tossed together. Find the probability of getting (i) exactly two heads, (ii) at least two heads.

21. Card from 11 to 30 are put in a box and mixed thoroughly. A card is then drawn from the box at random. Find the probability that the number on the drawn card is a prime number.

22. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

23. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5.

24. Find the probability that 5 sundays occurs in the month of November of a randomly selected year.

25. In a family of three children. Find the probability of having at least two boys.

26. In a family of two children. Find the probability of having at most one girl.

27. Two dice are thrown at the same time. Find the probability of getting different numbers on the two dice.

28. If a number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

SHORT ANSWER TYPE QUESTIONS-II

29. A number x is selected at random from the numbers 1, 2, 3. Another number y is selected at random from the numbers 1, 4, 9. Find the probability that the product of x and y is less than 9.

30. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

31. An integer is chosen between 0 and 100. What is the probability that it is

(i) divisible by 7?

(ii) not divisible by 7?

32. Two dice are rolled once. Find the probability of getting such numbers on the two dice,

(a) whose product is 12.

(b) Sum of numbers on the two dice is atmost 5.

33. Card with number 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has (i) an even number (ii) a square number.

34. In a lottery, there are 10 prizes and 25 are empty. Find the probability of getting a prize. Also verify $P(E) + P(\overline{E}) = 1$ for this event. **[CBSE 2020]**

35. If P(winning) = $\frac{x}{12}$, and P(Losing) = $\frac{1}{3}$ .then find x.

LONG ANSWER TYPE QUESTIONS

36. Cards marked with numbers 3, 4, 5, ...,50 are placed in a box and mixed thoroughly. One card is drawn at random from the box, find the probability that the number on the drawn card is

(i) divisible by 7 (ii) a two digit number

37. A bag contains 5 white balls, 7 red balls, 4 black balls and 2 blue balls. One ball is drawn at random from the bag. Find the probability that the balls drawn is

(i) White or blue (ii) red or black

(iii) not white (iv) neither white nor black

38. The king, queen and jack of diamonds are removed from a pack of 52 playing cards and the pack is well shuffled. A card is drawn from the remaining cards.

Find the probability of getting a card of

(i) diamond (ii) a jack

39. The probability of a defective egg in a lot of 400 eggs is 0.035. Calculate the number of defective eggs in the lot. Also calculate the probability of taking out a non defective egg from the lot.

40. Slips marked with numbers 3,3,5,7,7,7,9,9,9,11 are placed in a box at a game stall in a fair. A person wins if the mean of numbers are written on the slip. What is the probabilty of his losing the game?

41. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two digit number (ii) a perfect square number

(iii) a number divisible by 5.

42. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a card of spade or an ace (ii) a red king

(iii) neither a king nor a queen (iv) either a king or a queen

43. A card is drawn from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a face card (ii) red colour face card

(iii) black colour face card

44. Ramesh got ₹ 24000 as festival Bonus. He donated ₹ 5000 to temple. He gave ₹ 12000 to his wife, ₹ 2000 to his servant and gave rest of the amount to his daughter. Calculate the probability of

(i) wife's share (ii) Servant's Share

(iii) daughter's share.

45. 240 students reside in a hostel. Out of which 50% go for the yoga classes early in the morning, 25% go for the Gym club and 15% of them go for the morning walk. Rest of the students have joined the laughing club. What is the probability of students who have joined laughing club?

46. A box contains cards numbered from 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is:

(i) A square number (ii) a multiple of 7.

47. A die is thrown twice. Find the probability that:

(i) 5 will come up at least once

(ii) 5 will not come up either time

48. Cards marked 1, 3, 5 .... 49 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is : **[CBSE 2017]**

(i) divisible by 3 (ii) a composite number

(iii) not a perfect square (iv) multiple of 3 and 5

49. 50 A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

(i) triangle (ii) square

(iii) square of blue colour (iv) triangle of red colour

50. A box contain 24 balls of which x are red, 2x are white and 3x are blue. A ball is selected at random. What is the probability that it is

(i) not red?

(ii) White?

ANSWERS AND HINTS

1. (i) (c)-1.5 (ii) (c) $\frac{18}{23}$

(iii) (a) 0.0001 (iv) (b) $\frac{1}{4}$

(v) (a) $\frac{1}{6}$ (vi) (a) $\frac{1}{3}$

(vii) (b) 40

2. (i) (c) 1-P

(ii) (b) 8

(iii) (d) (Probability $= \frac{1}{5}$)

(iv) (a) $\frac{1}{7}$ (Total weeks $=52$, Remaining day $=1$, sample space = {S, M, Tu, W, Th, F, Sat})

(v) (c) $\frac{5}{11}$

(vi) (d) (vowels A, A, E, I) $\frac{4}{11}$

(vii) (d) $\frac{3}{4}$

3. Total Cards = 52

No. of Aces = 4

No. of kings = 4

P (neither ace nor king) = $\frac{44}{52} = \frac{11}{13}$

4. P (not defective bulb) = $1 - \frac{35}{250} = \frac{43}{50}$

5. Total case = 3 + 4 = 7

P(occurrence) = $\frac{4}{7}$

6. P(prime no.) = 0

7. No. of face card = 12

P(face card) = $\frac{12}{52} = \frac{3}{13}$

8. Probability of winning = $\frac{5}{1000} = 0.005$

9. Total black king = 2

P(Black King) = $\frac{2}{52} = \frac{1}{26}$

10. Sample space : {1, 2, 3, 4, 5, 6}

Perfect square : 1, 4

P(perfect square) = $\frac{2}{6} = \frac{1}{3}$

11. Total cases = 36

Favourable cases: {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

P(sum of two numbers is ≥ 10) = $\frac{6}{36} = \frac{1}{6}$

12. Multiples of 7 are 7, 14, 21, 28, 35

P (multiple of 7) = $\frac{5}{35} = \frac{1}{7}$

13. P(sum of 8) = $\frac{5}{36}$

14. P (consonant) = $\frac{21}{26}$

15. P (losing) = 1 – 0.07 = 0.93

16. $(a + b)^2 = 25$

17. (i) Doublets are {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

Required probability = $\frac{6}{36} = \frac{1}{6}$

(ii) Sum 10 cases : {(4, 6), (5, 5), (6, 4)}

Required probability = $\frac{3}{36} = \frac{1}{12}$

18. $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$

19. Total outcomes between 1 and 100 = 98

(i) Nos. divisible by 8 : {8, 16, 24, ..., 96}

favourable cases = 12

Required probability = $\frac{12}{98} = \frac{6}{49}$

(ii) Probability (integer is not divisible by 8) = $1 - \frac{6}{49} = \frac{43}{49}$

20. Sample space : {HHH, TTT, HTT, THT, TTH, THH, HTH, HHT}

(i) P(exactly 2 heads) = $\frac{3}{8}$

(ii) P(atleast 2 heads) = $\frac{4}{8} = \frac{1}{2}$

21. Total cards = 20

Prime Nos. are {11, 13, 17, 19, 23, 29}

Required probability = $\frac{6}{20} = \frac{3}{10}$

22. Let the number of blue balls = x

Total balls = $(5 + x)$

P (Blue ball) = 3 × P (Red ball)

$$\frac{x}{5+x} = 3\times\left(\frac{5}{5+x}\right)$$

$\Rightarrow x = 15$

23. Favourable outcomes : {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}

P (sum less than 5) = $\frac{6}{36} = \frac{1}{6}$

24. Number of total days in the month of November = 30

i.e. 4 complete weeks and 2 days.

$\therefore$ P (5 Sundays) = $\frac{2}{7}$

25. P (atleast two boys) = $\frac{4}{8} = \frac{1}{2}$

26. P (atmost one girls) = $\frac{3}{4}$

27. P (Different numbers) = $\frac{30}{36} = \frac{5}{6}$

28. Favourable outcomes : {–2, –1, 0, 1, 2}

P ($x^2 \leq 4$) = $\frac{5}{7}$

29. Sample space = {(1, 1), (1, 4), (1, 9)
(2, 1), (2, 4), (2, 9)
(3, 1), (3, 4), (3, 9)}

Favourable cases : $xy < 9$ {(1,1) (1,4) (2,1) (2,4) (3,1)}

Required probability = $\frac{5}{9}$

30. Total outcomes = 36

Favourable outcomes = {(1, 3), (2, 4), (3, 5), (4, 2), (4, 6), (5, 3), (6, 4), (3,1)}

Required probability $\frac{8}{36} = \frac{2}{9}$

31. Total number of integers = 101

(a) Favourable out conmes = {7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98}

Required probability = $\frac{14}{101}$

(b) $1-\frac{14}{101}=\frac{87}{101}$

32. (a) $S = \begin{Bmatrix} (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6) \\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6) \\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6) \\ (4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6) \\ (5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6) \end{Bmatrix}$

Favourable outcomes: {(2, 6), (3, 4), (4, 3), (6, 2)}

Required probability = $\frac{4}{36}=\frac{1}{9}$

(b) Favourable outcomes (sum ≤ 5)

= {(1, 1), (1, 2), (1, 3) (1, 4) (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (4, 1)}

Required probability = $\frac{10}{36}=\frac{5}{18}$

33. (i) Total cards = 101 – 2 + 1 = 100, Even numbers = 2, 4, ..., 100 = 50

Required probability = $\frac{50}{100}=\frac{1}{2}$

(ii) Square number = {4, 9, 16, 25, 36 ,49, 64, 81, 100}

Required probability = $\frac{9}{100}=0.09$

34. Total tickets = 35

P(E) = P(getting a prize) = $\frac{10}{35}=\frac{2}{7}$

$P(\bar{E})$ = P(not getting a prize) = $\frac{25}{35}=\frac{5}{7}$

$$P(E) + P(\bar{E}) = \frac{2}{7}+\frac{5}{7}=\frac{7}{7}=1$$

35. P(winning) + P(losing) = 1

$$\frac{x}{12}+\frac{1}{3}=1 \Rightarrow x = 8$$

36. Total cards = 50 – 3 + 1 = 48

(i) No. divisible by 7 are 7, 14, 21, 28, 35, 42, 49

Required probability = $\frac{7}{48}$

(ii) Two digit no. are 10, 11, 12, ... 50
No. of favourable outcomes = 50 – 10 + 1 = 41

Required probability = $\frac{41}{48}$

37. (i) $\frac{5+2}{18}=\frac{7}{18}$ (ii) $\frac{7+4}{18}=\frac{11}{18}$

(iii) $\frac{7+4+2}{18}=\frac{13}{18}$ (iv) $\frac{7+2}{18}=\frac{9}{18}=\frac{1}{2}$

38. (i) Remaining cards = 52 – 3 = 49
Remaining diamonds = 13 – 3 = 10

Required probability = $\frac{10}{49}$

(ii) P(jack) = $\frac{3}{49}$ (as 1 jack has been removed)

39. Total eggs = 400
P(defective eggs) = 0.035
Let defective eggs = x

$$\frac{x}{400} = 0.035$$

$$x = 400 \times 0.035$$

$$x = 14$$

P(non defective eggs) = 1 – 0.035 = 0.965

40. Mean = $\frac{3+3+5+7+7+7+9+9+9+11}{10}=\frac{70}{10}=7$

$$P(\text{loses}) = 1-\frac{3}{10}=\frac{7}{10}$$

41. Total no. = 90

(i) Two digit no.s are 10, 11, 12, ..., 90

No. of favourable cases = 90 – 10 + 1 = 81

Required probability = $\frac{81}{90} = \frac{9}{10}$

(ii) Perfect square no. are 1, 4, 9, 16, 25, 36, 49, 64, 81

Required probability = $\frac{9}{90} = \frac{1}{10}$

(iv) No.s divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90

Required probability = $\frac{18}{90} = \frac{1}{5}$

42. (i) P(a card of spade or an ace) = $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

(ii) P(red king) = $\frac{2}{52} = \frac{1}{26}$

(iii)P(neither a king nor a queen) = $1 - \frac{8}{52} = 1 - \frac{2}{13} = \frac{11}{13}$

(iv)P(either a king or a queen) = $\frac{8}{52} = \frac{2}{13}$

43. (i) $\frac{12}{52} = \frac{3}{13}$ (ii) $\frac{6}{52} = \frac{3}{26}$ (iii) $\frac{6}{52} = \frac{3}{26}$

44. (i) P(wife's share) = $\frac{12000}{24000} = \frac{1}{2}$

(ii) P(servant's share) = $\frac{2000}{24000} = \frac{1}{12}$

(iii)P(Daughter's share) = $\frac{5000}{24000} = \frac{5}{24}$

45. 10% students joined laughing club

P(students who have joined laughing clubs) = $\frac{10}{100} = \frac{1}{10}$

46. Total cards = 123 – 11 + 1 = 113

(i) Square numbers : 16, 25, 36, 49, 64, 81, 100, 121

Required probability = $\frac{8}{113}$

(ii) Multiple of 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119.

Required Probality = $\frac{16}{113}$

47. Total outcomes = 36

(i) P(5 will come up at least once) = $\frac{11}{36}$

Favourable cases {(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)}

(ii) P(5 will not come up either time) = $1-\frac{11}{36}=\frac{25}{36}$

48. S = 1, 3, 5, ..., 49. Total outcome = 25

(i) No. divisible by 3 are 3, 9, 15, 21, 27, 33, 39, 45

Required probability = $\frac{8}{25}$

(ii) Composite Nos are 9, 15, 21, 25, 27, 33, 35, 39, 45, 49

Required probability = $\frac{10}{25}=\frac{2}{5}$

(iii)P(not a perfect square) = 1 – P(perfect square) {Perfect square no. are 1, 9, 25, 49}

$$= 1-\frac{4}{25} = \frac{21}{25}$$

(iv) Multiple of 3 and 5

$\Rightarrow$ Multiple of 15 = 15, 45

Required probability = $\frac{2}{25}$

49. (i) $\frac{8}{18} = \frac{4}{9}$ (ii) $\frac{10}{18} = \frac{5}{9}$

(iii) $\frac{6}{18} = \frac{1}{3}$ (iv) $\frac{5}{18}$

50. (i) P(not red) = $\frac{20}{24} = \frac{5}{6}$

(ii) P(white) = $\frac{8}{24} = \frac{1}{3}$

r r r

PRACTICE-TEST

Probabiltiy

Time : 45 Minutes *M.M. : 20*

SECTION-A

1. When a die is thrown once, the probability of getting an odd number less than 3 is: **1**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 0

2. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting neither green ball nor red ball. **1**

3. One card is drawn at random from the well shuffled pack of 52 cards. Find the probability of getting a non face card. **1**

4. Cards are marked with numbers 5, 6, 7,.......50 are placed in the box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting a two digit number? **1**

SECTION-B

5. A letter is chosen at random from 26 alphabets. Find the probability that the letter chosen is from the word 'ASSASSINATION'. **2**

6. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective. **2**

7. Find the probability of getting 53 Fridays or 53 Saturdays in a leap year. **2**

SECTION - C

8. Daksh and Moksh are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year). **3**

9. Two dice are thrown together. Find the probability that sum of two numbers will be a multiple of 4. **3**

SECTION - D

10. Five cards—the ten, jack, queen, king and ace of diamonds, are removed from the well-shuffled 52 playing cards. One card is then picked up at random. Find the probability of getting:

(a) neither a heart nor a king

(b) either a heart or a spade card

(c) neither a red card nor a queen card

(d) a black card or an ace. **4**

CASE STUDY BASED QUESTIONS
REAL NUMBERS

1. During a health check-up camp, three types of patients registered themselves. 60 were suffering from joint problem, 84 were suffering from some type of fever and 108 were diabetic. The organisers want to call doctors for this camp.

Based on the above information answer the following questions:

(i) What is the maximum number of doctors required if each doctor treats same number of patients of each type of problem?

(a) 64 (b) 14 (c) 16 (d) 12

(ii) How many patients each doctor will treat?

(a) 7 (b) 12 (c) 21 (d) 9

(iii) At the end of the day when the total count was done the number of patients with joint problems were 48, suffering from fever were 60 and diabetic patients were 72 only. How many patients each doctor treated?

(iv) If HCF (48,60, 72) = 7m – 2, what is the value of m?

2. Deepika wants to organize her birthday party. She was happy on her birthday. She is very health conscious. Thus she decided to serve fruits only. She has 36 apples and 60 bananas at home and decided to serve them. She wants to distribute fruits among guests. She does not want to discriminate among guests so she decided to distribute equally among all.

Based on the above information answer the following questions:

(i) How many maximum guests Deepika can invite?

(a) 6 (b) 12 (c) 18 (d) 24

(ii) How many apples and bananas will each guest get?

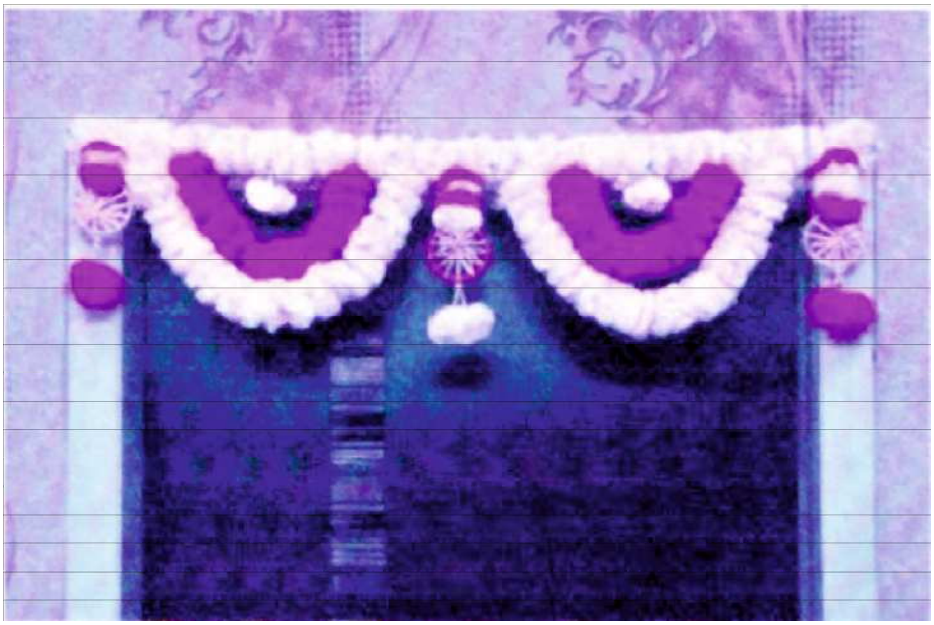
(a) 3 apples and 5 bananas (b) 5 apples and 3 bananas

(c) 2 apples and 4 bananas (d) 4 apples and 2 bananas

(iii) Deepika decides to distribute 42 mangoes also. In this case how many maximum guests Deepika can invite?

(iv) How many total fruits will each guest get now?

3. Radha decorated the door of her house with garlands on the occasion of Diwali. Each garland forms the shape of a parabola.



Based on the above information answer the following questions:

(i) Suppose the quadratic polynomial for the given curve is $ax^2 + bx + c$, then 'a' is always

(a) >0 (b) <0 (c) ≥ 0 (d) ≤ 0

(ii) A quadratic polynomial with the sum and product of its zeroes as –1 and –2 respectively, is:

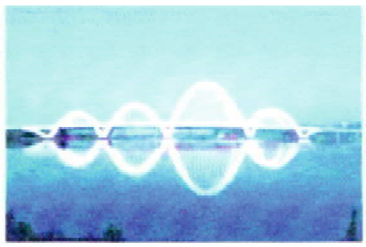
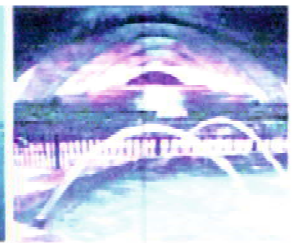
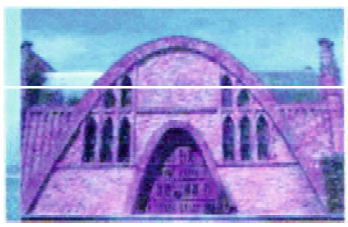
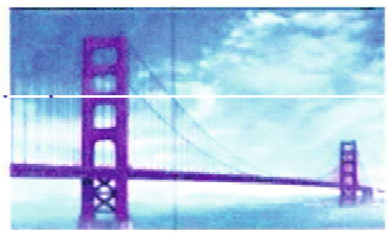
(a) $x^2 + x + 2$ (b) $x^2 - x - 2$

(c) $x^2 + x - 2$ (d) $x^2 - x + 2$

(iii) For what value of 'k', –1 is one of the zeroes of the quadratic polynomial $(k - 2) x^2 - 2x - 5$.

(iv) If α, β are the zeroes of the polynomial $f(x) = x^2 - 7x + 12$, then find the value of: $\frac{1}{\alpha} + \frac{1}{\beta}$

4. The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.

Based on the above information answer the following questions:

(i) In the standard form of quadratic polynomial $ax^2 + bx + c$, a, b and c are

(a) All are real numbers.

(b) All are rational numbers.

(c) 'a' is a non-zero real number and b and c are any real numbers.

(d) All are integers

(ii) The quadratic polynomial whose zeroes are - 4 and - 5 is

(a) $x^2 - 9x - 20$

(c) $x^2 - 9x - 20$

(b) $x^2 + 9x - 20$

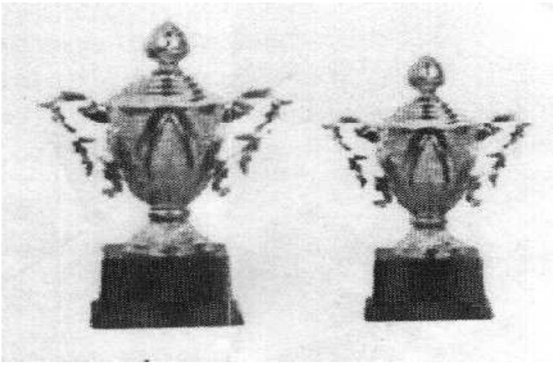
(d) $x^2 + 9x + 20$

(iii) If α and $\dfrac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - 8x + k$, then find 'k'.

(iv) Form a quadratic polynomial whose sum of zeroes is '$-p$' and product of zeroes is $\dfrac{-1}{p}$.

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

5. Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹ x per students and Cricket ₹ y per student. School 'P' decided to award a total of ₹9500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award ₹ 7,370 for the two games to 4 and 3 students respectively.



Based on the above information, answer the following questions:

(i) Represent the following information algeraically (in terms of x and y).

(ii) (a) What is the prize amount for hockey?

OR

(b) Prize amount on which game is more and by how much?

(iii) What will be the total prize amount if there are 2 students each from two games?

QUADRATIC EQUATIONS

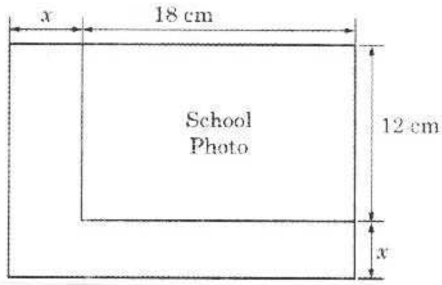
6. While designing the school year book, a teacher asked the student that the length and width of a particular photo is increased by x units each to double the area of the photo. The original photo is 18cm long and 12cm wide .

Based on the above information , answer the following questions:

(I) Write an algebraic equation depicting the above information.

(II) Write the corresponding quadratic equation in standard form.

(III) What should be the new dimension of the enlarged photo?



OR

Can any rational value of x make the new area equal to 220 square cm?

7. Nikhil and Niharika are very close friends. Both the famihes decide to go for a picnic to Palampur in their own cars, Niharika's car travels 5 km/h more than Nikhil's car. Nikhil's car took 4 hours more than Niharika's car in covering 400 km. Assume that Nikhil's car was travelling at a speed of 'y' km/h.

Based on the above information, answer the following questions:

(i) What will be the distance covered by Niharika's car in two hours?

(a) $2(y + 5)$ km (b) $(y - 5)$ km

(c) $2(y + 10)$ km (d) $(2y + 5)$ km

(ii) Which of the following quadratic equations describes the speed of Nikhil's car?

(a) $y^2 - 5y - 500 = 0$ (b) $y^2 + 4y - 400 = 0$

(c) $y^2 + 5y - 500 = 0$ (d) $y^2 - 4y + 400 = 0$

(iii) What is the speed of Nikhil's car?

(iv) How much time it took for Niharika's family to complete the journey?

8. A farmer wants to make a rectangular pen for his sheep in the garden near his house. To make the pen the farmer planned to make it with wooden fencing to cover the three sides. He has 60 m fencing material to cover three sides and the other side being a brick wall.

Based on the above information, answer the following questions:

(i) If the width be x, then the length of the pen

(a) $60 - 2x$ (b) $2x + 6$ (c) $6x + 20$ (d) $20 - 6x$

(ii) According to the given conditions area of the pen using length as calculated in (i) is

(a) $60x^2 - 2x$ (b) $60x + 2x^2$ (c) $6x - 20x^2$ (d) $60x - 2x^2$

(iii) Form a quadratic equation if the area of the pen is 250 square metre.

(iv) What could be the possible width if area of the pen is 400 square metre?

9. With the increasing demand and supply pressure worldwide, India has emerged as a competitive manufacturing location due to the low cost of manpower and strong engineering capabilities. The production in a factory increased uniformly by a fixed number every year. If the production in the factory was 4100 units in the fifth year which was then increased to 7600 units in the 10th year.

Based on the above information, answer the following questions:

(i) Find the production during 1st year.

(a) 500 units (b) 400 units (c) 1300 units (d) 700 units

(ii) Find the difference in production during 9th year and 7th year.

(a) 700 units (b) 1400 units (c) 350 units (d) 2100 units

(iii) Find the general term representing the number of units produced during a particular year.

(iv) Calculate the total number of units produced from 4th year to 10th year.

10. As we know a tree or a plant needs both soil and water along with sunlight to grow. It will have the necessary nourishment from both water and sun to make its leaves green and fruit to grow. A group of people planted 20 trees at equal distances of 10 m in a line with a water tank placed at a distance of 15 m from the tree at one end. Everyday a member of the group waters all the trees separately starting from the water tank and returns to the tank after watering each tree to get water for the next tree from the tank.

Based on the above information, answer the following questions:

(i) Distance travelled by the member to water nearest tree and back to the tank is;

(a) 15m (b) 30m (c) 7.5m (d) 40m

(ii) A.P. formed in the above condition is :

(a) 15. 25, 35, 45,................. (b) 30, 40, 50, 60,.................

(c) 30, 50, 70, 90,................. (d) 15, 35, 55, 75,.................

(iii) Calculate the distance travelled by the member to water the last tree.

(iv) Calculate the total distance travelled by the member in a day in order to water all the trees.

11. Burj Khalifa is the tallest tower in the world which is located in Dubai, United Arab Emirates. The height of Burj Khalifa is about 828 m. It has the highest observation deck open to the public in the world. A person walking on the deck observed the shadows of Burj Khalifa and the buildings in the proximity. At an instance, he found the length of shadow of Burj Khalifa and that of a building 'P' as 207m and 46m respectively.

Based on the above information, answer the following questions:

(i) Name the property which can be used to find out the length of the building 'P'.

(ii) At the same instance when the length of the shadow of Burj Khalifa was 207 m, what will be the length of the shadow of building 'Q' of height 108 m?

(a) 108 m (b) 54 m (c) 216 m (d) 27 m

(iii) Calculate the height of building 'P'.

(iv) What is the length of shadow of Burj Khalifa when the length of shadow of building 'Q' is 81 metres?

12. Walking regularly is a good habit to keep us healthy and stress free. After dinner, some people were walking in the society park. A person noticed the dynamic

shadows of walking people formed due to light from the lamp posts and started observing them. He observed that as people were moving away from the lamp post, the length of the shadow gradually increases. In the same group there, was Neha of height 180 cm, who was talking to Yamini and moving away from a 5.4m high lamp post at a speed of 0.6 m per second.

Based on the above information, answer the following questions:

(i) How far Neha was from the lamppost after 4 seconds?

(a) 240 cm (b) 24 cm (c) 120 cm (d) 60 cm

(ii) What would be the length of Neha's shadow after 3 seconds?

(a) 0.6 m (b) 0.9 m (c) 1.08 m (d) 1.8 m

(iii) After how much time the length of Neha shadow will be 1.8 m?

(iv) At an instance the shadow of Neha was 1.5 times her height. How far was she from the lamp post?

CO-ORDINATE GEOMETRY

13. Birla Science Museum is the first Science and Technology Museum of the country, established in 1954. It hosts exhibits and displays on science and technology where visitors can interact with the exhibits to make the understanding of science and technology easy and entertaining.

BirIa Science Museum has set aside a children's room having planets and stars painted on the ceiling. Suppose an imaginary coordinate system is placed on the ceiling in the room with the centre of the ceiling at (0, 0). Three particular stars are located space S(–8, 3), T(5, –10) and R (–5, – 7), where the coordinates represent the distance in metre from the centre of the room.

Based on the above information, answer the following questions:

(i) What is the distance between the star 'S' and 'T'.

(a) $4\sqrt{29}$ m (b) $2\sqrt{29}$ m (c) $13\sqrt{2}$ m (d) $16\sqrt{3}$ m

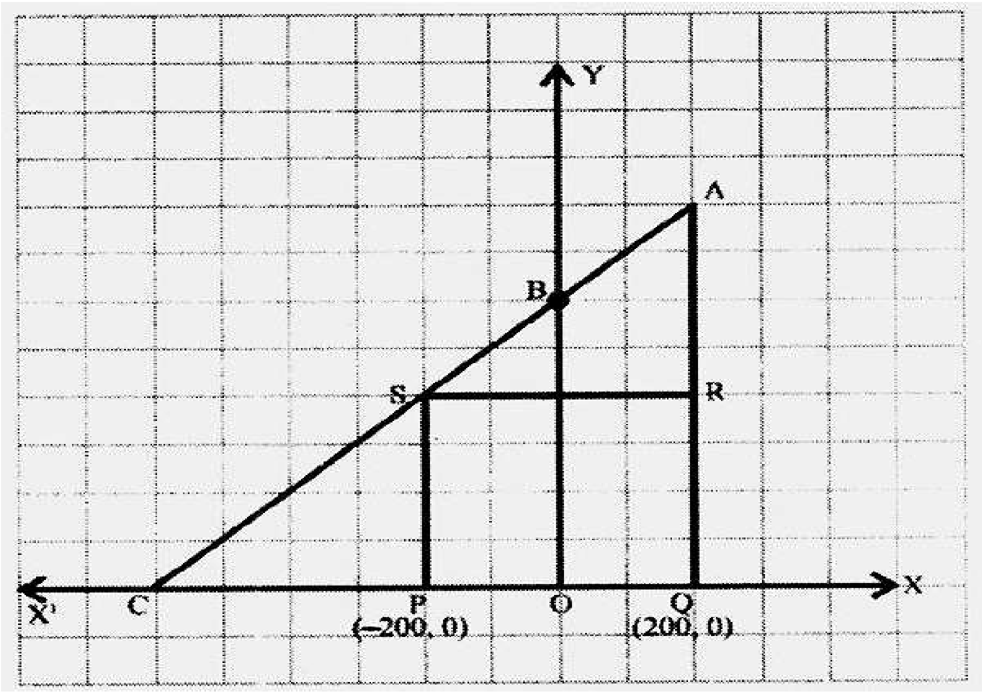
(ii) If a star M is at mid point of stars 'S' and 'R'. Its coordinate are:

(a) (3,–2) (b) $(\frac{-13}{2}, -2)$ (c) (–7,3) (d) $(\frac{13}{2}, 2)$

(iii) Which star is farthest from the centre of the room?

(iv) What is the distance between R (–5, –7)and T(5, –10)?

14. Jagdish has a field which is in the shape of a right angled tringle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetable (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P and Q are (-200,0) and (200,0) respectively. PQRS being a square,what are the coordinates of R and S?

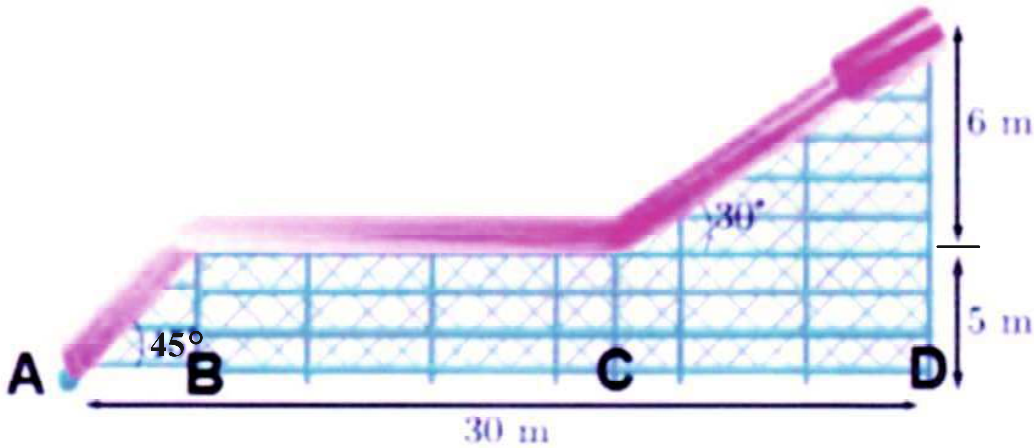
(ii) (a) What is the area of square PQRS?

OR

(b) What is the length of diagonal PR in square PQRS?

(iii) If S divides CA in the ratio K:1, what is the value of K, where point A is (200,800)?

15. Water Slide Design: Slide shown in the figure is part of a design for a water slide.



Based on the above information, answer the following questions:

(i) What is the length of flat part of slide?

(a) 44.69 m (b) 22.16 m (c) 14.62 m (d) 34.18 m

(ii) What is the total length of the slide?

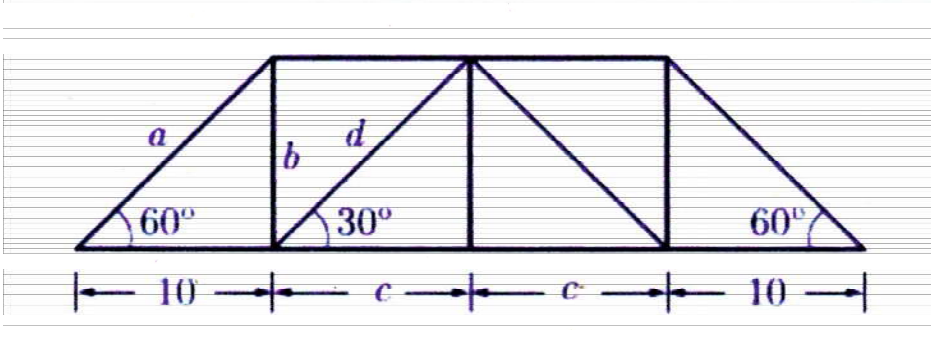
(a) 5.4 m (b) 21.6 m (c) 33.7 m (d) 43.69 m

(iii) Find the total slant height of the slide.

(iv) Find the distance of CD.

16. A truss is a structure that consists of members organised into connected triangles so that the overall assembly behaves as a single object. Trusses are most commonly used in bridges, roofs and towers.

A line diagram of a truss is shown below:



Based on the above information, answer the following questions:

(i) What is the length *a*?

(a) 30 m (b) 20 m (c) 34.6 m (d) 17.32 m

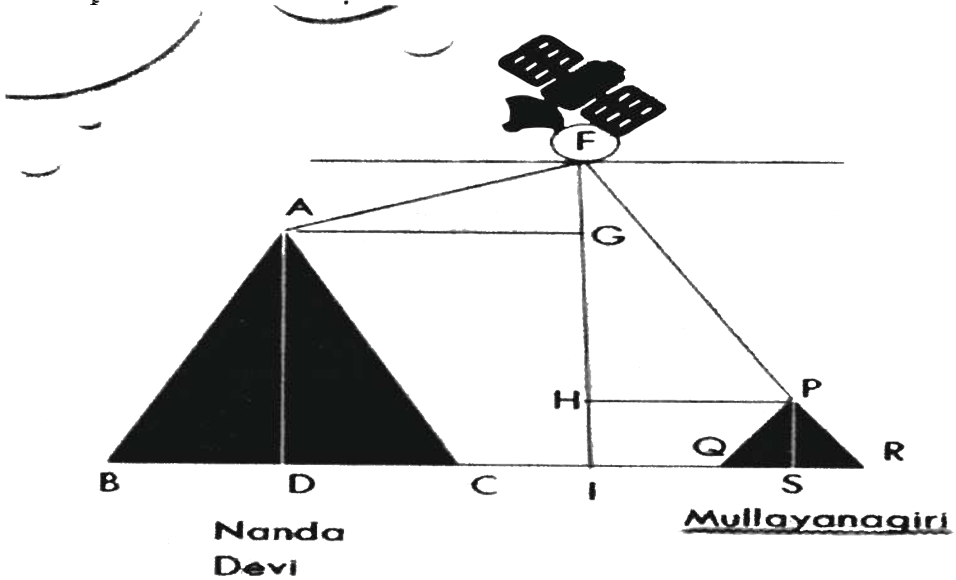
(ii) What is the length *b*?

(a) 30 m (b) 20 m (c) 34.6 m (d) 17.32 m

(iii) Find the length 'c'.

(iv) Find the value of (b+d).

17. A Satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Kamataka, them being Nanda Devi (height 7816m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. The distance between two mountains is 1937 km and the satellite is vertically above the midpoint of the distance between the two mountains.



Based on the above information, answer the following questions:

(i) The distance of the satellite from the top of Nanda Devi is

(a) 1136.4 km

(b) 577.52 km

(c) 1937 km

(d) 1025.36 km

(ii) The distance of the satellite from the top of Millayanagiri is

(a) 1136.4 km

(b) 577.52 km

(c) 1937 km

(d) 1025.36 km

(iii) Find the height of the satellite from the ground.

(iv) Find the angle of elevation of the Nanda Devi, if Rahul is standing at a distance of 7816 m from the base of Nanda Devi.

18. Statue of Unity. It is a colossal statue of Indian statesman, an independent activist Sardar Vallabh Bhai Patel, who was the first Deputy Prime Minister and first Home Minister of Independent India. Patel was highly respected for a leadership in uniting the 562 princely states of India to form a single Union of India. It is located in the state of Gujarat and it is the world tallest statue.

Based on the above information, answer the following questions:

(i) For a person, standing 120 m from the centre of the base of the statue, the angle of elevation from the base of statue is 45°. Find the height of the statue.

(a) 110 m

(b) 240 m

(c) $120\sqrt{3}$ m

(d) 120 m

(ii) For a person, standing *x m* from the centre of the base of the statue, the angle of elevation from the base of statue is 30°. Find the value of x if the height of the statue is 182 metre.

(a) $182\sqrt{3}$ m

(b) $364\sqrt{3}$ m

(c) $91\sqrt{3}$ m

(d) $\frac{182}{\sqrt{3}}$ m

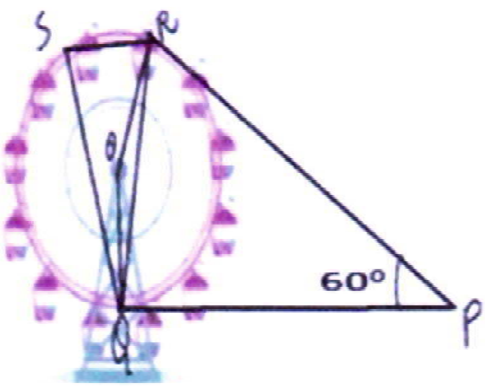
(iii) A cop in the helicopter near the top of the statue (height of statue is 182 metre) notices a car at some distance from the statue. The angle of the depression from the cop's eyes to the car is 60°. How far is the car from the centre of the base of the statue?

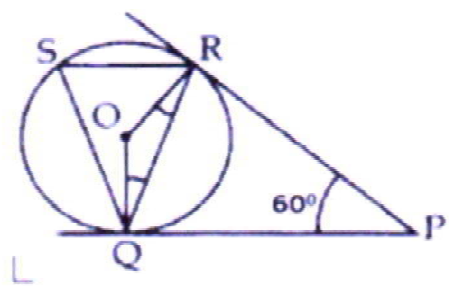
(iv) A cop in the helicopter near the top of the statue (height of statue is 182 metre) notices a car at some distance from the statue. The angle of the depression from the cop's eyes to the car is 60°. Find the distance between car and helicopter.

TANGENTS TO CIRCLE

19. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride . She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





Based on the above information, answer the following questions:

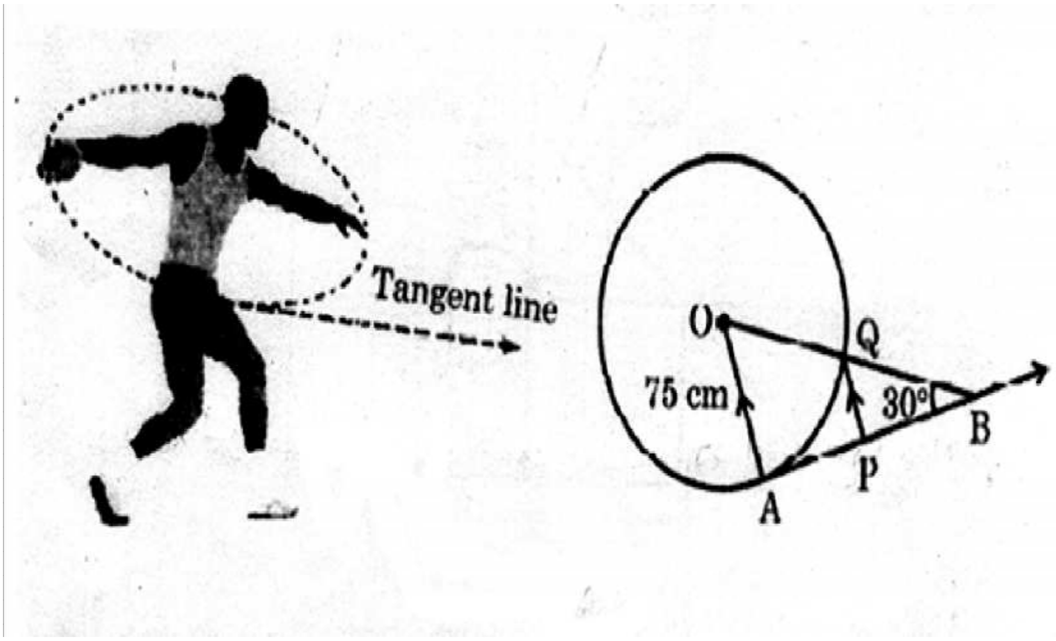
(i) In the given figure find $\angle ROQ$.

(ii) Find the value of $\angle RQP$

(iii) Find the value of $\angle RSQ$

(iv) Find the value of $\angle ORP$

20. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, and then releases the throw. When released, the discus travels along tangent to the circular spin orbit. In the given figure, AB is one such tangent to a circle of radius 75 cm and point O is centre of the circle, $\angle ABO = 30°$ and PQ is parallel to OA.



Based on above information; answer the following questions:

(a) Find the length of AB.

(b) Find the length of OB.

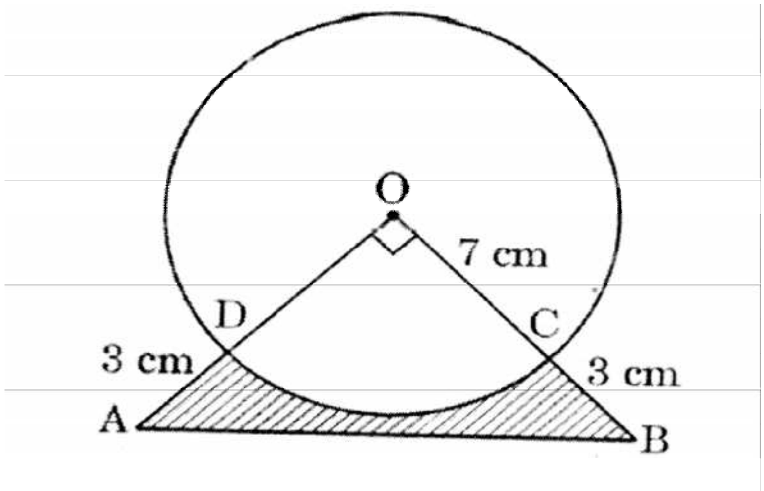
(c) Find the length of AP.

OR

Find the length of PQ.

AREA RELATED TO CIRCLES

21. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is ₹ 20 per square cm.



Based on the above information ,answer the following questions:

(i) What is the area of quadrant ODCO?

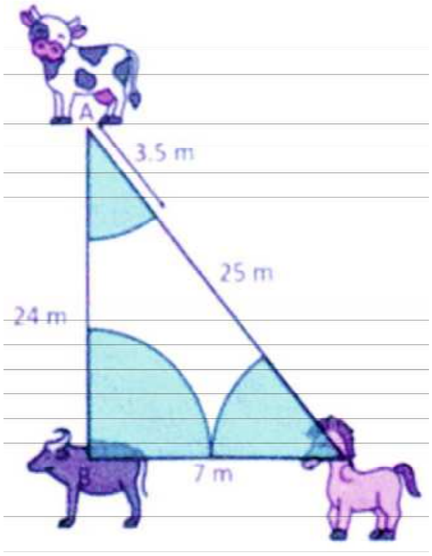
(ii) Find the area of ΔAOB.

(iii) (a) What is the total cost of silver plating the shaded part ABCD?

OR

(b) What is the length of arc CD?

22. A buffalo, a cow and a horse are tied to pegs at the corners of a right triangular field of sides 24 m, 7 m and 25 m by means of a 3.5 m long rope as shown in the figure. Use $\pi = \frac{22}{7}$.



Based on the above information, answer the following questions:

(i) What is the area of right triangular grass field?

(a) 84 sq.m (b) 168 sq.m (c) 175 sq.m (d) 87.5 sq.m

(ii) The combined angle made by the grazing area of horse and cow is:

(a) 45° (b) 90° (c) 60° (d) Cannot be determined

(iii) The area of that part of field in which buffalo can graze.

(iv) Calculate the decrease in the grazing area, if the ropes were 3 m instead of 3.5 m.

SURFACE AREA AND VOLUMES

23. A committee has decided to celebrate Durga Puja in a circular park of radius 35 m. The committee has given the contract to a tent house to set up the tent. The architect has designed a canvas tent in the shape of a semi cylindrical roof surmounted on an open cuboidal shape as shown in the figure. The dimensions of the rectangular base is 50 m × 21 m and the total height of the tent is 19 m.

Based on the above information, answer the following questions:

(i) The height of the cuboidal part of the tent is :

(a) 19 m (b) 8.5 m (c) 11.5 m (d) 15 m

(ii) Area of the park outside the tent is:

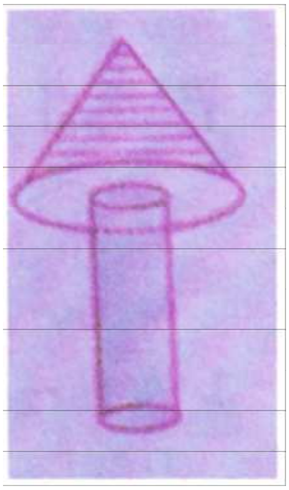
(a) 2800 sq.m (b) 3850 sq.m

(c) 1050 sq.m (d) 1570 sq.m

(iii) Find the total cost of canvas if it is purchased at the rate of ₹ 4 per square metre.

(iv) Find the volume of air present in the tent.

24. In a toys store wooden parts are assembled and painted to prepare a toy full stop. One such specific toy is in the shape of a cone mounted on a cylinder.



For the wood processing activity centre, the wood is taken out of storage to be saved, after which it undergoes rough polishing, then it is cut, drilled and has holes punched in it. It is then fine polished using sandpaper and then decorated using paint.

The total height of the toy is 26 cm and the height of the conical part is 6 cm. The diameters of the base of the conical part is 5 cm and that of the cylindrical part is 4 cm.

Based on the above information, answer the following questions:

(i) If the cylindrical part is to be painted yellow the surface area need to be painted is:

(a) 80π sq.cm (b) 82π sq.cm

(c) 84π sq.cm (d) 88π sq cm

(ii) The volume of the wood used in making this toy is:

(a) 92.5π cu.cm (b) 89.5π cu.cm

(c) 85.5π cu.cm (d) 72.5π cu.cm

(iii) Find the cost of painting the toy at 3 paisa per square cm.

(iv) Find the cost of painting 200 toys, if the paint company gives the discount of 5%.

STATISTICS

25. India meteorological department observes seasonal and annual rainfall every year in different sub-divisions of our country. It helps them to compare and analyse the result. The table given below shows sub –division wise seasonal (monsoon rainfall in mm) in 2018:

Rainfall (in mm)	Number of Sub-divisions
200 – 400	2
400 – 600	4
600 – 800	7
800 – 1000	4
1000 – 1200	2
1200 – 1400	3
1400 – 1600	1
1600 – 1800	1

Based on the above information answer the following questions:

(i) Write the modal class.

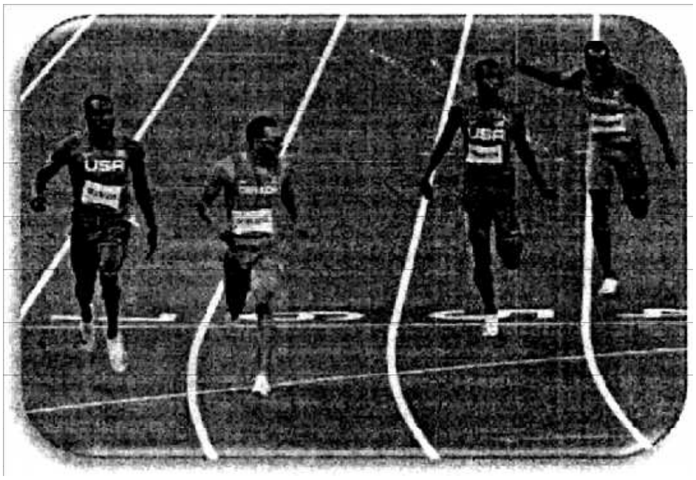
(ii) Find the median of the given data.

(iii) Find the average rainfall in this season

OR

If sub-division having atleast 1000mm rainfall during monsoon season, is considered good rainfall sub division-, then how many sub divisions had good rainfall?

26. The men's 200 m race event at the 2020 Tokyo Olympic took place on 3rd and 4th August. A stopwatch was used to find the time taken by a group of Athletes to run 200 m.



Time (in seconds)	0-20	20-40	40-60	60-80	80-100
No. of Athletes	8	10	13	6	3

Based on the above information answer the following questions:

(i) Number of athletes who finished the race within 1 min:

(a) 10 (b) 8 (c) 31 (d) 13

(ii) Average of lower limits of median class and modal class is :

(a) 30 (b) 50 (c) 60 (d) 40

(iii) Find the mean time taken by a athlete to finish the race.

(iv) Find the mode of the above data.

PROBABILITY

27. Aisha took a pack of 52 cards. She kept aside all the face cards and shuffled the remaining cards well.

Based on the above information answer the following questions:

(i) The number of favourable outcomes for the event a club card or a '4' is

(a) 13 (b) 17 (c) 14 (d) 12

(ii) She drew a card from the well-shuffled pack of remaining cards. The probability that the card drawn is a red card is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{4}{13}$ (d) $\frac{2}{13}$

(iii) Find the probability of drawing a black queen.

(iv) Find the probability of getting neither a black card nor an ace card.

28. Akriti and Sukriti have to start the game of ludo. They are fighting for who will start the game. They found three coins and decided to toss them simultaneously to know who will start the game.

Based on the above information answer the following questions:

(i) The possible number of outcomes:

(a) 8 (b) 6 (c) 2 (d) 4

(ii) The probability of getting 3 tails on tossing three coins simultaneously:

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{6}$

(iii) Akriti says, if I get atleast one head, I will win and start the game. Find the probability that Akriti will start the game.

(iv) Sukriti says, if I get atmost one tail, I will start the game. Find the probability that Sukriti will start the game.

ANSWERS

1. (i) (d) 12

   (ii) (c) 21

   (iii) 15 patients

   (iv) m = 2

2. (i) (b) HCF (36, 60) = 12. Thus fruits will be equally distributed among 12 guests.

   (ii) (a) each guest will get (36 ÷ 12) = 3 apples and (60 ÷ 12) = 5 bananas.

   (iii) HCF (36, 42, 60) = 6. Thus fruits will be equally distributed among 6 guests.

   (iv) Each guest will get (36 ÷ 6) = 6 apples, (42 ÷ 6) = 7 mangoes, and (60 ÷ 6) = 10 bananas. Thus each guest will get 6 + 7 + 10 = 23 fruits.

3. (i) (a) > 0

   (ii) (c) $x^2 + x - 2$

   (iii) Put $x = -1$ to get 'k' = 5

   (iv) $\alpha + \beta = 7$ and $\alpha\beta = 12$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7}{12}$$

4. (i) (c) 'a' is a non-zero number and b and c are any real numbers.

   (ii) (d) $x^2 + 9x + 20$

   (iii) 'k' = $\frac{1}{4}$

   (iv) $k\left(x^2 + px - \frac{1}{p}\right)$

5. (i) $5x + 4y = 9500$, $4x+3y = 7370$

(ii) (a) Prize for hockey (x) = ₹980

OR

Cricket by ₹170

(iii) $2x+2y$ = ₹4260

6. (i) $(18+x)(12+x) = 2 \times 18 \times 12$

(ii) $x^2+30x-216 = 0$

(iii) 24 cm, 18 cm

OR

No,

7. (i) (a) $2(y + 5)$ km

(ii) (c) $y^2 + 5y - 500 = 0$

(iii) speed = 20 km/h

(iv) time = 16 hours

8. (i) (a) $60 - 2x$

(ii) (d) $60x - 2x^2$

(iii) $x^2 - 30x + 125 = 0$

(iv) width could be 10 m or 20 m

9. (i) (c) 1300 units

(ii) (b) 1400 units

(iii) $a_n = 600 + 700\, n$

(iv) 38500 units

10. (i) (b) 30 m

(ii) (c) 30, 50, 70, 90, ...

(iii) 410 m

(iv) 4400 m

11. (i) Similarily of triangles

(ii) (d) 27 m

(iii) 184 m

(iv) 621 m

12. (i) (a) 240 cm

(ii) (b) 0.9 m

(iii) 6 seconds

(iv) 5.4 m

13. (i) (c) $13\sqrt{2}$ m

(ii) (b) $\left(\frac{-13}{2}, -2\right)$

(iii) T

(iv) $\sqrt{109}$ m

14. (i) R (200,400), S (–200, 400)

(ii) (a) 1600 sq units

OR

(b) $400\sqrt{2}$ units

(iii) K= 1

15. (i) (c) 14.62 m

(ii) (d) 43.69 m

(iii) 19.07 m

(iv) 10.38 m

16. (i) (b) 20 m

(ii) (d) 17.32 m

(iii) 30 m (approx)

(iv) 51.96 m

17. (i) (a) 1136.4 km

(ii) (c) 1937 km

(iii) 8385.7 km

(iv) 45°

18. (i) (d) 120 m

(ii) (a) $182\sqrt{3}$ m

(iii) 107 m approx

(iv) 214 m approx

19. (i) (b) 120°

(ii) 60°

(iii) 60°

(iv) 60°

20. (i) $75\sqrt{3}$ cm

(ii) 150 cm

(iii) $\frac{75}{2}\sqrt{3}$ cm

OR

37.5 cm

21. (i) 38.5 sq.cm

(ii) 50 sq.cm

(iii)(a)₹230

OR

(b) 11 cm

22. (i) (a) 84 sq.m

(ii) (b) 90°

(iii) 9.625 sq.m

(iv) 5.11 sq.m approx

23. (i) (b) 8.5m

(ii) (a) 2800 sq.m

(iii) Rs. 11428

(iv) 17587.5 cubic m

24. (i) (c) 84π square cm

(ii) (a) 92.5 π cu. cm

(iii) Rs. 9.66 approx.

(iv) Rs. 1835.40 approx.

25. (i) 600-800

(ii) $771\frac{3}{7}$ mm

OR

850 mm

(iii) 7

26. (i) (c) 31

(ii) (d) 40

(iii) 43 second

(iv) 46 seconds

27. (i) (a) 13

(ii) (a) $\frac{1}{4}$

(iii) 0

(iv) $\frac{18}{40}$ or $\frac{9}{20}$

28. (i) (a) 8

(ii) (b) $\frac{1}{8}$

(iii) $\frac{7}{8}$

(iv) $\frac{4}{8}$ or $\frac{1}{2}$

ASSERTION AND REASON BASED QUESTIONS

The following Questions are Assertion and Reason based questions. Two statements are given , one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a),(b),(c)and(d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

---

1. **Assertion (A):** $\frac{\text{HCF}(a,b)\times\text{LCM}(a,b)}{a\times b}=1$

   Reason (R): HCF(a,b) ×LCM(a,b)=a×b

2. **Assertion (A):** If HCF (26,169) =13 then LCM (26,169) =338

   Reason (R): HCF(a,b)×LCM(a,b) =a×b

3. **Assertion (A):** HCF of two coprime number is 1.

   Reason (R): Two numbers having only 1 as the common factor is known as co prime number.

4. **Assertion (A):** Every composite number can be expressed as product of primes.

   Reason (R): 11×4×3×2+4 is a composite number.

5. **Assertion (A):** The LCM of two numbers is 1200. 500 cannot be their HCF.

   Reason (R): LCM of two or more numbers is always divisible by their HCF.

6. **Assertion (A):** If the sum of the zeroes of the quadratic polynomial x^2-2kx+8 is 2 then value of k is 1.

   Reason (R): Sum of zeroes of a quadratic polynomial ax^2+bx+c is $-\frac{b}{a}$.

7. **Assertion (A)**: If the product of the zeroes of the quadratic polynomial $x^2+3x+5k$ is –10 then value of k is –2.

Reason (R): Sum of zeroes of a quadratic polynomial ax^2+bx+c is $-\frac{b}{a}$.

8. **Assertion (A):** –1 and –4 are the zeroes of polynomial x^2-3x-4.

Reason (R): A real number k is said to be a zero of polynomial p(x) if p(k) = 0.

9. **Assertion (A):** The graph of quadratic polynomial p (x) intersect x-axis at two points.

Reason (R): Degree of quadratic polynomial is 2.

10. **Assertion (A) :** The pair of equations x+2y+5=0 and –4x-8y+20=0 has infinitely many solutions.

Reason (R) : If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of equations has infinitely many solutions.

11. **Assertion (A):** The pair of equations x+2y+5=0 and –3x-6y+1= 0 has unique solutions.

Reason (R): If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then given pair of equations has no solution.

12. **Assertion (A):** $(x-2)^2 + 1 = 2x - 3$ is a quadratic equation.

Reason (R): It is not in the form of ax^2 + bx + c = 0, a ≠ 0.

13. **Assertion (A):** The discriminant 'D' of the quadratic equation $2x^2-4x+3=0$, is –8 and hence its roots are not real.

Reason (R): If . $b^2 - 4ac < 0$, then roots are not real.

14. **Assertion (A):** The roots of the equation $7x^2+x-1=0$ are real and distinct.

Reason (R): If $b^2 - 4ac > 0$, then roots are real and distinct.

15. **Assertion (A):** The equation $9x^2 + 3kx + 4 = 0$ has equal roots for $k = 9$.

Reason (R): If discriminant 'D' of a quadratic equation is equal to zero, then roots of equation are real and equal.

16. **Assertion (A):** a,b,c are in A.P .if and only if 2b =a+c.

Reason (R): The sum of first n odd natural number is n^2.

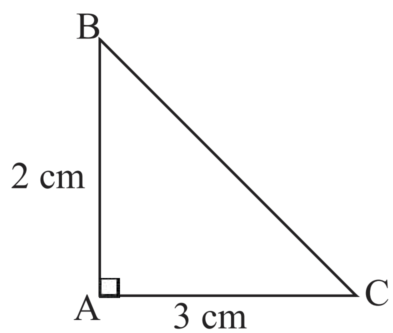
17. **Assertion (A):** If sum of first n terms of an A.P is given by $S_n = 5n^2 + 3n$, then n^{th} term of A.P is $a_n = 10n - 2$.

Reason (R): The n^{th} term of an A.P may be written as $S_n - S_{(n-1)}$.

18. **Assertion (A):** If 12 , a , b and –3 are in A.P ,then a+b=9.

Reason (R): If first term of an A.P is 'a' and the n^{th} term of A.P is 'b', then its common difference is $\dfrac{b-a}{n-1}$.

19. **Assertion (A):** The perimeter of ΔABC is a rational number.



Reason (R): The sum of squares of two rational numbers is always rational.

20. **Assertion (A):** In a ΔABC, a line DE || BC, intersects AB in D and AC in E, then $\dfrac{AB}{AD} = \dfrac{AC}{AE}$.

Reason (R): If a line is drawn parallel to one side of a triangle intersecting the two side, then the other two sides are divided in the same ratio.

21. **Assertion (A):** The line segment joining the mid points of any two sides of a triangle is parallel to the third side.

Reason (R): A line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side.

22. **Assertion (A):** All congruent triangles are similar but the similar triangles need not to be congruent .

Reason (R): If the corresponding sides of two triangles are proportional , then they are similar.

23. **Assertion (R):** If the corresponding sides of two triangles are proportional then their corresponding angles are equal ,and hence the two triangles are similar.

Reason (R): If the bisector of an angle of a triangle bisects the opposite side, then the triangle is isosceles.

24. **Assertion (R):** Point P (0,2) is the point of intersection of y-axis with the line $3x+2y = 4$.

Reason (R): The distance of point P (0,2) from x-axis is 2 units.

25. **Assertion (R):** If the points A(4,3) and B(x,5) lie on a circle with centre O (2,3), then the value of x is 2 .

Reason (R): Centre of a circle is the midpoint of each chord of the circle.

26. **Assertion (A):** The value of p is 4, for which the distance between the points M (2,–4) and N (10, p) is 11.

Reason (R): Three points A, B and C are collinear if AB+BC=AC.

27. **Assertion (A):** For $0 < \theta \leq 90°$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.

Reason (R): $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

28. **Assertion (A):** $(\cos^4 A - \sin^4 A)$ is equal to $2\cos^2 A - 1$.

Reason (R): tan A is the product of tan and A .

29. **Assertion (A):** In a ΔPQR, right angled at P, of $\cos R = \frac{5}{13}$, then $\cot Q = \frac{5}{12}$.

Reason (R): The value of $\cos\theta$ decreases with the increase in value of θ; $0 \le \theta \le 90°$

30. **Assertion (A):** If $\cos\theta + \cos^2\theta = 1$, then $\sin^2\theta + \sin^4\theta = 1$.
    Reason(R): $\sin^2\theta + \cos^2\theta = 1$, for all values of θ.

31. **Assertion (A):** The length of the ladder leaning against a window 18 m above the ground at an angle of 60° is 9 m.

    Reason (R): According to Pythagoras theorem, $h^2 = p^2 + b^2$; where h is hypotenuse, p is perpendicular and b is base.

32. **Assertion (A):** If at an instance height of a building is equal to length of its shadow, then the angle of elevation of sum is 45° .

    Reason (R): The value of tan 45° is 1.

33. **Assertion (A):** A tangent to a circle is perpendicular to the radius through the point of contact.

    Reason (R): The lengths of tangents drawn from an external point to a circle are equal.

34. **Assertion (A):** If PA and PB tangent drawn from an external point P to a Circle with the centre O , then the quadrilateral AOBP is cyclic.

    Reason (R): The angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the lines segments joining the points of contact at the centre.

35. **Assertion (A):** The angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the lines segments joining the points of contact at the centre.

    Reason (R): The tangent to a circle is perpendicular to the radius through the point of contact.

36. **Assertion (A):** From a point P, 10 cm away from the centre of a circle, if a tangent PT of length 8cm is drawn then the radius of a circle is 5cm.

    Reason (R): A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

37. Assertion (A): If the circumference of a circle is 176cm,then its radius is 28cm.

Reason (R): Circumference of a circle is $2\pi r$.

38. Assertion (A): In a circle of a radius 6cm, the angle of a sector is 60°, then the area of sector is $18\frac{6}{7}cm^2$.

Reason (R): Area of the circle with radius r is πr^2.

39. Assertion (A): If a wire of a length 22 is bent in the shape of a circle, then area of circle so formed is $38.5cm^2$

Reason (R): Area of the circle = Length of wire.

40. Assertion (A): Length of arc of a circle is 2π cm, if radius of a circle is 4 cm and angle subtended by arc at the centre of circle is 90°.

Reason (R): Length of arc $= \frac{\pi r \theta}{360°}$

41. Assertion (A): The surface area of largest sphere that can be inscribed in a hollow cube of side 'a' cm is πa^2 cm^2.

Reason (R): The surface area of a sphere of radius r is $4\pi r^2$.

42. Assertion (A): The diameter of a sphere, whose surface area is 616 cm^2, is 7 cm.

Reason (R): The surface area of a sphere of radius r is $4\pi r^2$.

43. Assertion (A): Length of diagonal of a cube is $11\sqrt{3}$ cm, if its volume is 1331 cm^3.

Reason (R): Volume of a cube is equal to a^3, where a is the side of cube.

44. Assertion (A): Height of largest right circular cone that can be cut out of a cube whose volume is 729 cm^3, is 9 cm.

Reason (R): Volume of right circular cone be $\frac{1}{3}\pi r^2 h$, where r be the radius and h be the height of the cone.

45. **Assertion (A):** If the mean and the median of a distribution are 169 and 170 respectively, then its mode is 172.

Reason (R): Mode = 3Median – 2 Mean

46. **Assertion (A):** Median of first 11prime natural number be 13.

Reason (R): Median $\left(\frac{n+1}{2}\right)^{th}$ observation, if number of observations (n) is odd.

47. **Assertion (A):** Difference between mode and median is 12 , if the difference of median and mean be 6.

Reason (R): 3 Median= Mode +2 Mean.

48. **Assertion (A):** Mean of 12 prime number is $16\frac{5}{12}$.

Reason (R): Mean $= \frac{sum\,of\ the\,observations}{number\,of\ \ observations}$

49. **Assertion (A):** The probability of getting a prime number when a die is thrown once is $\frac{2}{3}$.

Reason (R): On the faces of a die , prime numbers are 2,3 and 5.

50. **Assertion (A):** The probability of getting a Card of red or black King from a pack of playing card is $\frac{7}{13}$.

Reason (R): Total number of playing card is 52.

51. **Assertion (A):** When two coins are tossed together, the probability of getting no tail is $\frac{1}{4}$.

Reason (R): The probability P(E) of an event E satisfies $0 \le \text{P(E)} \le 1$.

52. Assertion (A): The probability of randomly drawing a Card with an even number from a box containing cards numbers 1 to 100 is $\frac{1}{2}$.

Reason (R): $\text{P(Event)} = \dfrac{\textit{number of favourable outcomes}}{\textit{total number of possible outcomes}}$

Answer

1.	(a)	2.	(a)
3.	(a)	4.	(b)
5.	(a)	6.	(a)
7.	(b)	8.	(a)
9.	(d)	10.	(a)
11.	(d)	12.	(c)
13.	(a)	14.	(a)
15.	(d)	16.	(b)
17.	(a)	18.	(a)
19.	(d)	20.	(a)
21.	(b)	22.	(b)
23.	(b)	24.	(b)
25.	(c)	26.	(d)
27.	(a)	28.	(c)
29.	(b)	30.	(a)
31.	(c)	32.	(a)
33.	(b)	34.	(a)
35.	(a)	36.	(d)
37.	(a)	38.	(b)
39.	(c)	40	(c)
41.	(a)	42.	(d)
43.	(b)	44.	(b)
45.	(a)	46.	(a)
47.	(a)	48.	(a)
49.	(a)	50.	(b)
51.	(b)	52.	(a)

Practice Paper –I

Time : 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions . All questions are compulsory.

(ii) This question paper is divided into five Sections A,B,C,D and E.

(iii) In Section A, Question no . 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion - Reason based questions of 1 mark each.

(iv) In Section B, Question no . 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Question no . 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In Section D , Question no . 32 to 35 are Long answer (LA) type questions, carrying 5 marks each.

(vii) In Section E, Question no . 36 to 38 are case study based questions, carrying 4 marks each. Internal choice is provided in 2 marks questions in each case- study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.

(x) Use of calculators is not allowed.

Section – A

This section has 20 Multiple Choice Questions. Each question carries 1 mark.

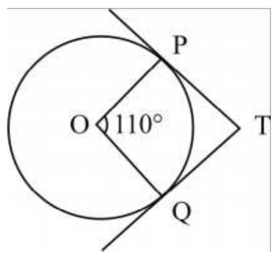
1. If two positive integers 'a' and 'b' are written as $a= x^3y^2$ and $b=xy^3$, where x,y are prime numbers,then the result obtained by dividing the product of the positive integers by the LCM(a,b) is:

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

2. If p-1, p+1 and 2p+3 are three consecutive terms of an A.P., then the value of 'p' is:

(a) –2 (b) 4 (c) 0 (d)2

3. In figure,if TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to:



(a) 60° (b) 70° (c) 80° (d) 90°

4. $\left[\frac{3}{4}\tan^2 30° - Sec^2 45° + Sin^2 60°\right]$ equal to:

(a) –1 (b) $\frac{5}{6}$ (c) $-\frac{3}{2}$ (d) $\frac{1}{6}$

5. Which of the following is a quadratic polynomial having zeroes $\frac{-2}{3}$ and $\frac{2}{3}$?

(a) $4x^2 - 9$ (b) $\frac{4}{9}(9x^2 + 4)$ (c) $x^2 + \frac{9}{4}$ (d) $5(9x^2 - 4)$

6. In what ratio , does x-axis divide the line segment joining the points A (3,6) and B(-12,–3)?

(a) 1:2 (b) 1:4 (c) 4:1 (d) 2:1

7. The value of 'k' for which the pair of equations $kx = y + 2$ and $6x = 2y + 3$ has infinitely many solutions ,is:

(a) k = 3 (b) does not exist. (c) k = –3 (d) k = 4

8. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the sun is__________.

(a) 30° (b) 45° (c) 60° (d) 90°

9. What is the area of a semi-circle of diameter 'd'?

(a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$ (c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$

10. Sec θ when expressed in terms of cot θ is equal to:

(a) $\frac{1+\cot^2\theta}{\cot\theta}$ (b) $\sqrt{1+\cot^2\theta}$ (c) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ (d) $\frac{\sqrt{1-\cot^2\theta}}{\cot\theta}$

11. If three coins are tossed simultaneously, what is the probability of getting at most one tail?

(a) $\frac{3}{8}$ (b) $\frac{4}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{8}$

12. which of the following quadratic equation has sum of its roots as 4?

(a) $2x^2-4x+8=0$ (b) $-x^2+4x+4=0$

(c) $\sqrt{2}x^2-\frac{4}{\sqrt{2}}x+1=0$ (d) $4x^2-4x+4=0$

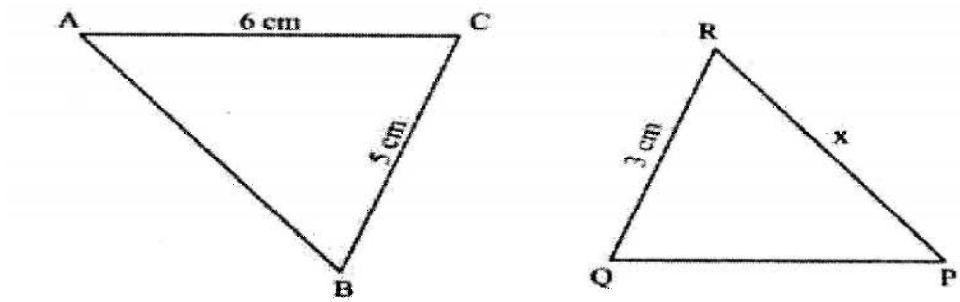
13. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is:

(a) 10 m (b) 15 m (c) 20 m (d) 24 m

14. A bag contains 100 cards numbered from 1 to 100. One card is drawn at random from this bag. What is the probability that the number on the card is a perfect cube?

(a) $\frac{1}{20}$ (b) $\frac{3}{50}$ (c) $\frac{1}{25}$ (d) $\frac{7}{100}$

15. In the given figure, $\Delta ABC \sim \Delta QPR$. If AC = 6 cm. BC = 5 cm, QR= 3 cm and PR = x, then the value of x is:



(a) 3.6 cm (b) 2.5 cm (c) 10 cm (d) 3.2 cm

16. The distribution below gives the marks obtained by 80 students on a test:

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less then 60
Number of Students	3	12	27	57	75	80

The modal class of this distribution is:

(a) 10–20 (b) 20–30 (c) 30–40 (d) 50–60

17. The distance between the points $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$ is:

(a) $2\sqrt{10}$ *units* (b) $4\sqrt{10}$ *units* (c) $2\sqrt{20}$ *units* (d) 0 *unit*

18. A quadrilateral PQRS is drawn to circumscribe a circle. If PQ = 12 cm, QR = 15 cm and RS = 14 cm, then the length of SP is:

(a) 15 cm (b) 14 cm (c) 12 cm (d) 11 cm

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

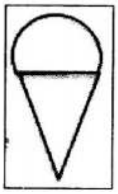
(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** a, b, c, are in A.P. if and only if 2b = a + c.

    Reason (R): The sum of first 'n' odd natural numbers is n^2.

20. **Assertion (A):** Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.



Reason (R): Top is obtained by fixing the plane surfaces of the hemisphere and cone together.

Section–B

This section comprises Very Short Answer (VSA) type question. Each question carries 2 marks.

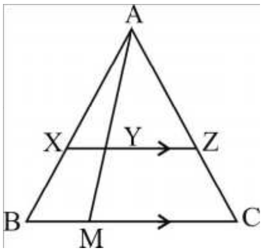
21. Find the greatest number which divides 85 and 72 leaving reminders 1 and 2 respectively.

22. If $\sin\theta + \cos\theta = \sqrt{3}$, then find the value of $\sin\theta \cos\theta$.

OR

If $4\cot^2 45° - sec^2 60° + \sin^2 60° + p = \frac{3}{4}$, then find the value of 'p'.

23. In the given figure, XZ is parallel to BC. If AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm, then find the length of XY.

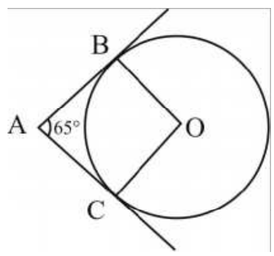


24. With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle.

OR

What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm?

25. In the given figure, O is the centre of the circle. AB and AC are tangents drawn to the circle from point A. If $\angle BAC = 65°$, then find the measure of $\angle BOC$.



Section –C

This section comprises Short Answer (SA) type question. Each question carries 3 marks.

26. Half of the difference of two numbers is 2. The sum of the greater number and twice the smaller numbers is 13.Find the numbers.

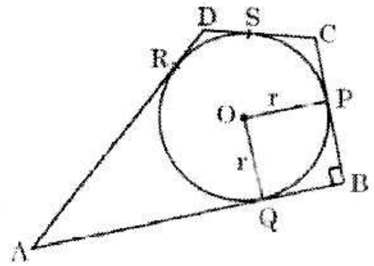
OR

If the system of linear equation $2x + 3y = 7$ and $2ax + (a+b)\,y = 28$ have infinite number of solutions, then find the values of 'a' and 'b'.

27. Find the LCM of the numbers 18180 and 7575 by prime factorization . Also, find the HCF of the two numbers.

28. Prove that: $\left(\frac{1}{Cos\theta} - Cos\theta\right)\left(\frac{1}{Sin\theta} - sin\theta\right) = \frac{1}{\tan\theta + \cot\theta}$

29. In the given figure, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90°$. If AD = 17 cm. AB = 20 cm and DS = 3 cm, then find the radius of the circle.



OR

Two concentric circle are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

30. Find the zeroes of the quadric polynomial $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients.

31. The mean of the following frequency distribution is 25. Find the value of 'a' and also find the mode of the data.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	18	15	a	6

Section – D

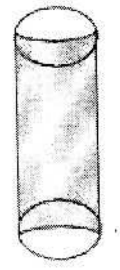
This section comprises Long Answer (LA) type questions. Each question carries 5 marks.

32. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

A motor boat whose speed is 18 km/h in still water take 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

33. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



34. The median of the following data is 50. Find the values of 'p' and 'q' , if the sum of all frequencies is 90. Also find the mode.

Marks obtained	Number of Students
20-30	p
30-40	15
40-50	25
50-60	20
60-70	q
70-80	8
80-90	10

OR

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mean and median of the following data.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency (periods)	7	14	13	12	20	11	15	8

35. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.

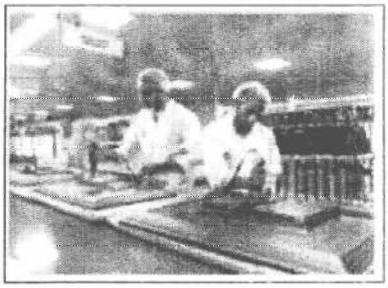
Section–E

In this section , there are 3 case study based units of assessment of 4 marks each.

36.

Case Study–1

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production run. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6^{th} year and 22600 in 9^{th} year.



(i) In which year, the production is 29,200.

(ii) Find the production during 8^{th} year.

OR

Find the production during first 3 years.

(iii) Find the difference of the production during 7^{th} and 4^{th} year.

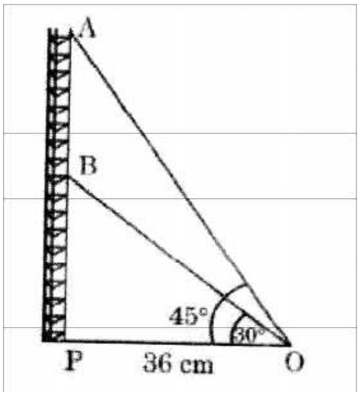
37.

Case Study–2

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure.

On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of Section B is 30° and the angle of elevation of the top of Section A is 45°.



Based on the above information, answer the following questions:

(i) Find the length of the wire from the point O to the top of Section B.

(ii) Find the distance AB.

OR

Find the area of ΔOPB

(iii) Find the height of the Section A from the base of the tower.

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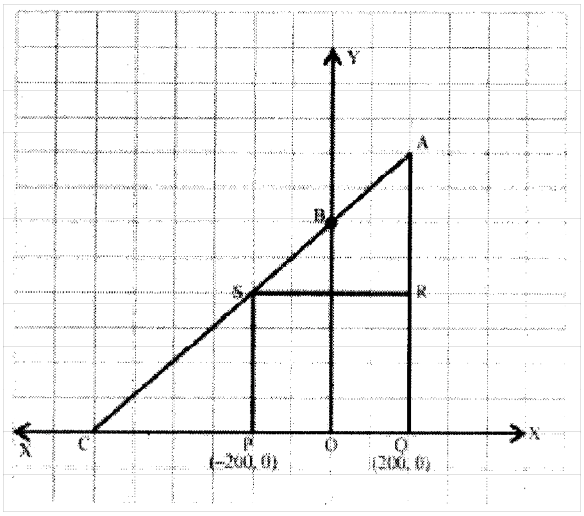
38. **Case Study–3**

Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200,0) PQRS being a square, what are the coordinates of R and S?

(ii) What is the area of square PQRS?

OR

What is the length of PR?

(iii) If S divides CA in the ratio k:l, what is the value of k, where point A is (200,800)?

Answer with solution

Section – A

1. (b) xy^2
2. (c) 0
3. (b) 70°
4. (a) – 1
5. (d) $5(9x^2 - 4)$
6. (d) 2:1
7. (b) does not exist.
8. (b) 45°
9. (c) $\frac{1}{8}\pi d^2$
10. (c) $\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
11. (b) $\frac{4}{8}$
12. (b) $2x^2 - 4x + 8 = 0$

13. (a) 10 m

14. (c) $\frac{1}{25}$

15. (b) 2.5 cm

16. (c) 30–40

17. (a) $2\sqrt{10}$ units

18. (d) 11 cm

19. (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

20. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).

Section –B

21. $85 - 1 = 84$

$72 - 2 = 70$

HCF (84, 70) = 14

$\therefore$ required number is 14.

22. $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow \sin\theta\cos\theta = 1$

OR

$$4(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + p = \frac{3}{4}$$

$\Rightarrow p = 0$

23. $\Delta AYZ \sim \Delta AMC$

$\therefore \frac{AZ}{ZC} = \frac{AY}{YM}$ ......(1)

$\Delta AXY \sim \Delta ABM$

$\therefore \frac{AY}{YM} = \frac{XY}{BM}$ ....(2)

from (1) and (2), we get

$\frac{AZ}{ZC} = \frac{XY}{BM}$

$\Rightarrow \frac{3}{2} = \frac{XY}{3}$

$\Rightarrow XY = 4.5cm$

24. Required area $\frac{22}{7} \times \frac{(14)^2 \times 180°}{360°} = 308cm^2$

OR

$\pi\left(\frac{d}{2}\right)^2 = (40)^2 + (9)^2$

$\Rightarrow d = 82$ cm

25. $\angle BOC = 180° - 65° = 115°$

Section –C

26. Let two numbers be x & y such that x > y

A.T.Q.

$\frac{1}{2}(x-y) = 2 \Rightarrow x - y = 4$ ....(1)

$x + 2y = 12$ ....(2)

solving (1) and (2), we get x = 7 and y = 3

OR

For infinite number of solutions

$$\frac{2}{2a}=\frac{3}{a+b}=\frac{7}{28}$$

Solving it, we get a = 4 and b = 8

27. $18180 = 2^2 \times 3^2 \times 5 \times 101$

$7575 = 3 \times 5^2 \times 101$

$\text{LCM} = 2^2 \times 3^2 \times 5^2 \times 101 = 90900$

$\text{HCF} = 3 \times 5 \times 101 = 1515$

28. $$\text{LHS}=\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\times\left(\frac{1-\sin^2\theta}{\sin\theta}\right)=\frac{\sin^2\theta\times\cos^2\theta}{\cos\theta\times\sin\theta}=\sin\theta\times\cos\theta$$

$$\text{RHS}=\frac{1}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}=\frac{\sin\theta\times\cos\theta}{\sin^2\theta+\cos^2\theta}=\sin\theta\times\cos\theta$$

$\therefore$ LHS = RHS

29. AQ = AR = AD – DR = AD – DS = 17 – 3 = 14 cm

QB = AB – AQ = 20 – 14 = 6 cm

OPBQ is a square.

$\therefore$ r = QB = 6 cm

OR

$$AP=\sqrt{(5)^2-(3)^2}=4\,\text{cm}$$

AB = 2AP = 2 × 4 = 8 cm

30. $4s^2 - 4s + 1$

$= (2s - 1)(2s - 1)$

Zeroes are $\frac{1}{2}$ and $\frac{1}{2}$.

$$\text{Sum of the zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-\text{coefficent of s}}{\text{coefficent of s}^2}$$

$$\text{Product of the zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{constant term}}{\text{coefficent of s}^2}$$

31.

C.I.	f_i	x_i	f_ix_i
0-10	5	5	25
10-20	18	15	270
20-30	15	25	375
30-40	a	35	35a
40-50	6	45	270
Total	44 + a		940 + 35a

$$25 = \frac{940 + 35a}{44 + a}$$

$\Rightarrow a = 16$

Modal Class: 10 – 20

$$\text{Mode} = 10 + \left(\frac{18-5}{2 \times 18 - 5 - 15}\right) \times 10$$

$= 18.125$

Section –D

32. Let the speed of the train be x km/h

ATQ

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$\Rightarrow$ $x^2 + 5x - 1800 = 0$

$\Rightarrow$ $(x + 45)(x - 40) = 0$

$\therefore$ $x = -45$ or $x = 40$

But speed is always positive.

So, speed of the train is 40 km/h

OR

Let the speed of the stream be x km/h

ATQ

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ $(x + 54)(x - 6) = 0$

$\therefore$ $x = -54$ or $x = 6$

But speed is always positive.

So, speed of the train is 6 km/h

33. $\text{TSA} = \left(\frac{22}{7} \times (3.5)^2 \times 10\right) + \left(2 \times 2 \times \frac{22}{7} \times (3.5)^2\right)$

$= 539 \text{ cm}^2$

34.

Marks obtained	**Number of students (f_i)**	***cf***
20-30	p	p
30-40	15	p + 15
40-50	25	p + 40
50-60	20	p + 60
60-70	q	p + q + 60
70-80	8	p + q + 68
80-90	10	p + q + 78

Median class: 50 – 60

$$50 = 50 + \left(\frac{\left(\frac{90}{2}\right) - (p + 40)}{20}\right) \times 10$$

$\Rightarrow$ $p = 5$

$p + q + 78 = 90$

$\Rightarrow\ 5 + q + 78 = 90$

$\Rightarrow\ q = 7$

Modal Class: 40 – 50

$$\text{Mode} = 40 + \left(\frac{25-15}{2\times 25-15-20}\right)\times 10$$

$$= 46\frac{2}{3}$$

OR

Number of cars	f_i	x_i	u_i	f_iu_i	**Cf**
0 – 10	7	5	–3	–21	7
10 – 20	14	15	–2	–28	21
20 – 30	13	25	–1	–13	34
30 – 40	12	35 = a	0	0	46
40 – 50	20	45	1	20	66
50– 60	11	55	2	22	77
60 –70	15	65	3	45	92
70 – 80	8	75	4	32	100
Total	100			57	

$$\text{Mean} = 35 + \frac{57}{100}\times 10 = 40.7$$

Median class : 40 – 50

$$\text{Median} = 40 + \left(\frac{\left(\frac{100}{2}\right)-46}{20}\right)\times 10$$

$$= 42$$

35. Correct figure, given, to prove, construction and proof.

Section –E

36. $a_6 = a + 5d = 16000$ and $a_9 = a + 8d = 22600$

$\therefore$ $a = 5000$ and $d = 2200$

(i) $29200 = 5000 + (n - 1) \times 2200$

$\Rightarrow$ $n = 12$

(ii) $a_8 = 5000 + 7 \times 2200 = 20400$

OR

$$S_3 = \frac{3}{2} \times [2 \times 5000 + 2 \times 2200] = 21600$$

(iii) $a_7 - a_4 = (a + 6d) - (a + 3d) = 3d = 3 \times 2200 = 6600$

37. (i) BO = $24\sqrt{3}$ cm

(ii) BP = $12\sqrt{3}$ cm cm and AP = 36 cm

OR

AB=AP–BP = $(36 - 12\sqrt{3})$ cm

(iii) AP = 36 cm

38. (i) R $\leftrightarrow$ (200,400) and S $\leftrightarrow$ (–200,400)

(ii) PQ = 400 units

ar (PQRS) = 160000 square units

OR

PQ = 400 units

$\therefore$ PR = $400\sqrt{2}$ units

(iii) C $\leftrightarrow$ (– 600, 0) and A $\leftrightarrow$ (200, 800)

$$400 = \frac{0 \times 1 + 800 \times k}{k+1} \Rightarrow k = 1$$

Practice Paper –II

Time : 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions . All questions are compulsory.

(ii) This question paper is divided into five Sections A,B,C,D and E.

(iii) In Section A, Question no . 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion - Reason based questions of 1 mark each.

(iv) In Section B, Question no . 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Question no . 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In Section D , Question no . 32 to 35 are Long answer (LA) type questions, carrying 5 marks each.

(vii) In Section E, Question no . 36 to 38 are case study based questions, carrying 4 marks each. Internal choice is provided in 2 marks questions in each case- study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.

(x) Use of calculators is not allowed.

Section – A

This section has 20 Multiple Choice Questions. Each question carries 1 mark.

1. If the line represented by the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ coincide, then the value of ‘r’ is:

   (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) 2

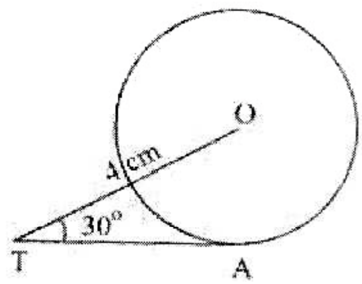
2. If $\Delta ABC \sim \Delta PQR$ with $\angle A = 32°$ and $\angle R = 65°$, then the measure of $\angle B$ is:

(a) $32°$ (b) $65°$ (c) $83°$ (d) $97°$

3. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a,b) is:

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

4. In the given figure, TA is a tangent to the circle with centre O such that $OT = 4$ cm, $\angle OTA = 30°$, then length of TA is:



(a) $2\sqrt{3}$ cm (b) 2 cm (c) $2\sqrt{2}$ cm (d) $\sqrt{3}$ cm

5. $(\sec A + \tan A)(1 - \sin A) =$

(a) sec A (b) sin A (c) cosec A (d) cos A

6. The least positive value of k, for which the quadratic equation $2x^2 + kx - 4 = 0$ has rational roots, is:

(a) $\pm 2\sqrt{2}$ (b) 2 (c) ± 2 (d) $\sqrt{2}$

7. The hour-hand of a clock is 6 cm long. The angle swept by it between 7:20 a.m. and 7:55 a.m. is:

(a) $\left(\frac{35}{4}\right)^\circ$ (b) $\left(\frac{35}{2}\right)^\circ$ (c) $35°$ (d) $70°$

8. If a pole 6 cm high casts a shadow $2\sqrt{3}$ m long the ground, then sun's elevation is:

(a) $60°$ (b) $45°$ (c) $30°$ (d) $90°$

9. The ratio of HCF to LCM of the least composite number and the least prime number is:

(a) 1:2 (b) 2:1 (c) 1:1 (d) 1:3

10. The coordinates of the vertex A of a rectangle ABCD whose three vertices are given as B (0,0), C(3,0) and D(0,4) are:

(a) (4,0) (b) (0,3) (c) (–3,4) (d) (4,3)

11. The radius of a circle is same as the as the side of a square. Their perimeters are in the ratio:

(a) 1:1 (b) 2: π (c) π: 2 (d) $\sqrt{\pi} : 2$

12. The empirical relation between the mode, median and mean of a distribution is:

(a) Mode = 3 Median –2 Mean (b) Mode = 3 Mean – 2 Median

(c) Mode = 2 Median –3 Mean (d Mode = 2 Mean – 3 Median

13. A girl calculates that the probability of her winning the first prize in the lottery is 0.08. If 6000 tickets were sold in all, how many tickets did the girl buy?

(a) 40 (b) 240 (c) 480 (d) 750

14. If 2 tan A = 3, then the value of $\frac{4\sin A + 3Cos A}{4\sin A - 3Cos A}$ is:

(a) $\frac{7}{\sqrt{13}}$ (b) $\frac{1}{\sqrt{13}}$ (c) 3 (d) does not exits

15. Find the upper limit of the modal class form the given distribution.

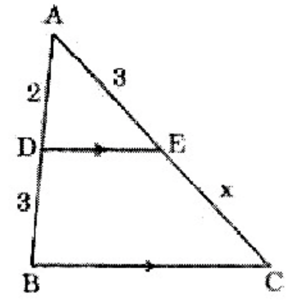
Height [in cm]	Below 140	Below 145	Below 150	Below 155	Below 160	Below 165
Number of girls	4	11	29	40	46	51

(a) 165 cm (b) 160 cm (c) 155 cm (d) 150 cm

16. Curved surface area of a cylinder of height 5 cm is 94.2 square cm Radius of this cylinder is: (Take π = 3.14)

(a) 2 cm (b) 3 cm (c) 2.9 cm (d) 6 cm

17. In the given figure, DE || BC. If AD = 2 units, DB = AE = 3 units and EC = x units, then the value of x is:



(a) 2 (b) 3 (c) 5 (d) $\frac{9}{2}$

18. A quadratic equation whose roots are $(2+\sqrt{3})$ and $(2-\sqrt{3})$ is:

(a) $x^2 - 4x + 1 = 0$ (b) $x^2 + 4x + 1 = 0$

(c) $4x^2 - 3 = 0$ (d) $x^2 - 1 = 0$

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

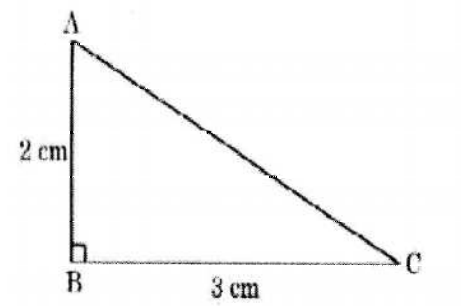
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** The perimeter of ΔABC given in the figure is a rational number.



Reason (R): The sum of the squares of two rational numbers is always a rational number.

20. **Assertion (A):** Point P (0,2) is the point of intersection of y-axis with the line $3x + 2y = 4$.

    Reason (R): The distance of point P (0, 2) from x-axis is 2 units.

Section –B

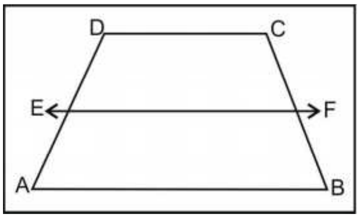
This section comprises Very Short Answer (VSA) type questions. Each question carries 2 marks.

21. Find whether the following pair of linear equations is consistent or inconsistent:

$$3x + 2y = 8$$
$$6x - 4y = 9$$

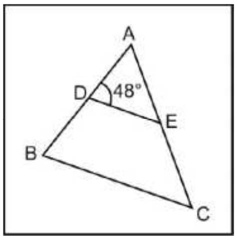
22. In the given figure, if ABCD is a trapezium in which AB|| CD||EF, then prove that

$$\frac{AE}{ED} = \frac{BF}{FC}$$



OR

In figure, if AD = 6 cm, DB = 9 cm, AE = 8 cm and EC = 12 cm and $\angle ADE = 48°$. Find $\angle ABC$.

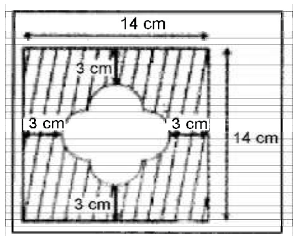


23. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$.

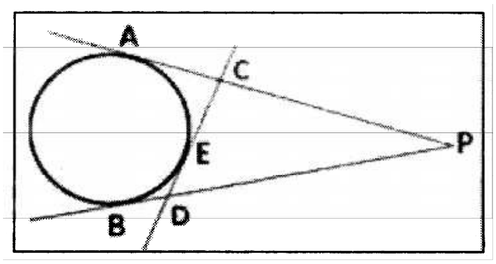
24. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. (Use $\pi = 3.14$)

OR

Find the area of the unshaded region shown in the given figure.



25. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of ΔPCD.



Section –C

This section comprises Short Answer (SA) type questions. Each question carries 3 marks.

26. If α and β are the zeroes of the polynomial $3x^2 + 5x + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = \frac{19}{9}$, then find the value of k.

27. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

OR

Solve:

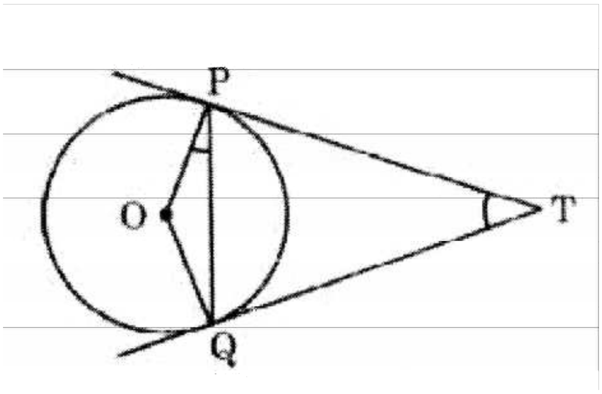
$$\frac{ax}{b} - \frac{by}{a} = a + b; ax - by = 2ab$$

28. A bag contains 6 red, 4 black and some white balls.

    (i) Find the number of white balls in the bag if the probability of drawing a white bell is $\frac{1}{3}$.

    (ii) How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$?

29. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



30. Prove that: sec A (1 – sin A) (sec A + tan A) = 1

OR

Prove that: $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

31. Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 a.m., when will they ring together again?

Section –D

This section comprises Long Answer (LA) type questions. Each question carries 5 marks.

32. Sides AB, BC and median AD of a triangle ABC are respectively proportional to sides PQ, QR and median PM of triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

33. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

OR

Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

34. The following table gives the distribution of the life time of 400 neon lamps:

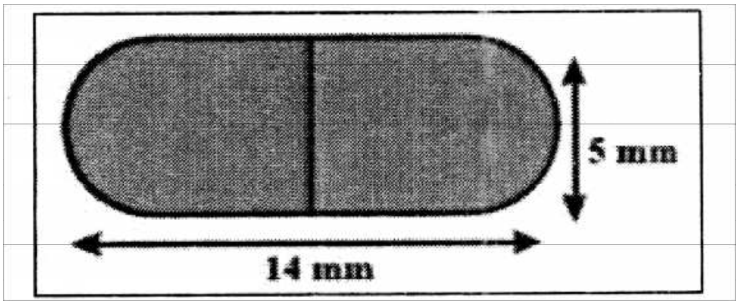
Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the average life of a lamp.

35. A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹500 per m^2.

OR

A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.

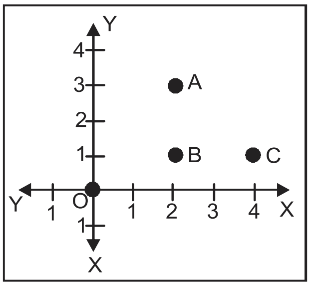


Section – E

In this section, there are 3 case study based units of assessment of 4 marks each.

Case Study – 1

36. Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A (2,3), Shagun's house is at B (2,1) and library is at C (4,1) Based on the above information, answer the following questions.



(i) How far is Alia's house from Shagun's house?

(ii) How far is the library from Shagun's house?

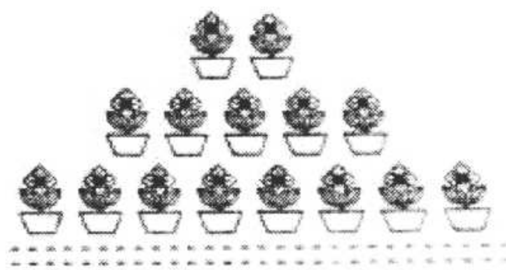
(iii) Which distance is more - Distance between Shagun's house and school or Distance between Alia's house and library?

OR

Show that Alia's house, Shagun's house and library form an isosceles right triangle.

Case Study – 2

37. Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed pots in such a way that number of pots in the first row is 2, second row in 5, third row is 8 and so on.



Based on the above information, answer the following questions:

(i) Find the number of pots placed in the 10^{th} row.

(ii) Find the difference in the number of pots placed in 5^{th} row and 2^{nd} row.

(iii) If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.

OR

If Aahana has sufficient space for 12 rows, then how many total numbers of pots are placed by her with the same arrangement?

Case Study – 3

38. A flagstaff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flagstaff is 60° and from the same point the angle of elevation of the top of tower is 45°.

Based on the above, answer the following questions:

(i) Draw a neat labelled diagram to represent the given situation.

(ii) What is the height of the flagstaff?

(iii) If at some other point, the top of tower's angle of elevation is 30°, then find the distance of this new point from the foot of the tower.

OR

Find the distance between the top of the tower and the point which the angle of elevation of the top of tower is 30°.

Answer

Section – A

1. (d) 2

2. (c) 83°

3. (b) xy^2

4. (a) $2\sqrt{3}cm$

5. (d) cos A

6. (c) ± 2

7. (b) $\left(\frac{35}{2}\right)^{\circ}$

8. (a) 60°

9. (a) 1:2

10. (c) (–3, 4)

11. (c) π:2

12. (a) Mode = 3 Median – 2 Mean

13. (c) 480

14. (c) 3

15. (d) 150 cm

16. (b) 3 cm

17. (b) $\frac{9}{2}$

18. (a) $x^2 - 4x + 1 = 0$

19. (d) Assertion (A) is false, but Reason (R) is true,

20. (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section –B

21. Consistent

22. Correct proof

OR

48°

23. 1

24. 28.5 cm²

OR

$41\frac{1}{7}$ cm²

25. 20 cm

Section –C

26. k = 2

27. 42 or 24 (two)

OR

x = b amd y = – a

28. (i) 5 (ii) 5

29. Correct proof

30. Correct proof

OR

Correct proof.

31. 6.36 a.m.

Section –D

32. Correct proof.

33. 25 hours, 15 hours

OR

9, 10, 11

34. 3410 hours

35. 1060 m^2, ₹530000

OR

220 mm^2

Section –E

36. (i) 2 units (ii) 2 units

(iii) Distance between Alia's house and library OR Correct proof.

37. (i) 29 (ii) 9 (iii) 8th **OR** 222

38. (i) Correct figure

(ii) $5(\sqrt{3}-1)m$

(iii) $5\sqrt{3}$m **OR** 10 m

Practice Paper –III

Time : 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions . All questions are compulsory.

(ii) This question paper is divided into five Sections A,B,C,D and E.

(iii) In Section A, Question no . 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion - Reason based questions of 1 mark each.

(iv) In Section B, Question no . 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Question no . 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In Section D , Question no . 32 to 35 are Long answer (LA) type questions, carrying 5 marks each.

(vii) In Section E, Question no . 36 to 38 are case study based questions, carrying 4 marks each. Internal choice is provided in 2 marks questions in each case- study.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.

(x) Use of calculators is not allowed.

Section – A

This section has 20 Multiple Choice Questions. Each question carries 1 mark.

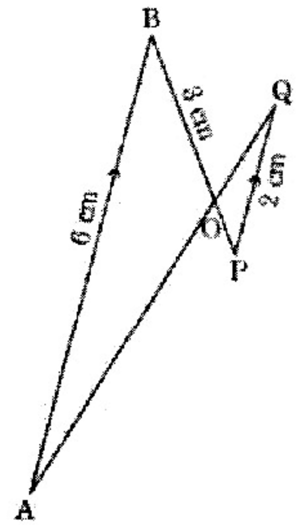
1. The roots of the equation $x^2 + 3x - 10 = 0$ are:

   (a) 2,–5 (b) –2, 5 (c) 2,5 (d) –2,–5

2. If 'p' and 'q' are natural numbers and 'p' is the multiple of 'q', then what is the HCF of 'p' and 'q'?

(a) pq (b) p (c) q (d) p + q

3. In the given figure, AB || PQ. If AB = 6 cm, PQ = 2 cm and OB = 3 cm, then the length of OP is:



(a) 9 cm (b) 3 cm (c) 4 cm (d) 1 cm

4. If $\cos A = \frac{4}{5}$ then the value of tan A is:

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{1}{8}$

5. What is the length of the are corresponding to a sector of a circle of radius 14 cm whose central angle is 90°?

(a) 22 cm (b) 44 cm (c) 88 cm (d) 11 cm

6. If the angle of elevation of the top of a tower from a point at a distance of 75 m from its foot is 60°, then the height of the tower is:

(a) $75\sqrt{2}m$ (b) $50\sqrt{3}m$ (c) $25\sqrt{3}m$ (d) $75\sqrt{3}m$

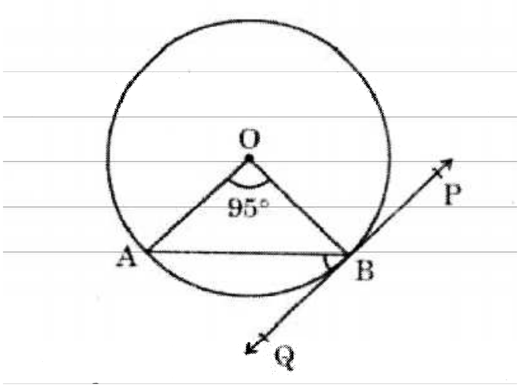
7. If α and β are the zeroes of a polynomial $p(x) = x^2 + x - 1$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ equals to:

(a) 1 (b) 2 (c) -1 (d) $-\frac{1}{2}$

8. In a group of 20 persons, 5 persons cannot swim. If a person is chosen at random, then the probability that he/she can swim is:

(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{4}$

9. In the given figure, PQ is tangent to the circle centred at O. If $\angle AOB = 95°$, then the measure of $\angle ABQ$ will be:



(a) 47.5° (b) 42.5° (c) 85° (d) 95°

10. The value of t for which the pair of linear equations $(t + 3)x - 3y = t$; $tx + ty + 12 = 0$ have infinitely many solutions, is:

(a) 6 (b) 0 (c) -6 (d) 12

11. The curved surface area of a cone having height 24 cm and radius 7 cm, is

(a) 528 cm^2 (b) 1056 cm^2 (c) 550 cm^2 (d) 500 cm^2

12. The ratio in which the x-axis divides the line segment joining the points A(6,5) and B(4,1) is:

(a) 1:5 (b) 1:7 (c) 5:1 (d) 7:1

13. The next term of the A.P. : $\sqrt{6}, \sqrt{24}, \sqrt{54}, \ldots$ is:

(a) $\sqrt{60}$ (b) $\sqrt{96}$ (c) $\sqrt{72}$ (d) $\sqrt{216}$

14. If 'p' is the probability that an event will occur and 'q' is the probability that it will not occur, then the relation between 'p' and 'q' is:

(a) $p + q = 1$ (b) $p = 1, q = 1$ (c) $p = q - 1$ (d) $p + q + 1 = 0$

15. If the value of each observation of a statistical data is increased by 3, then the mean of the data:

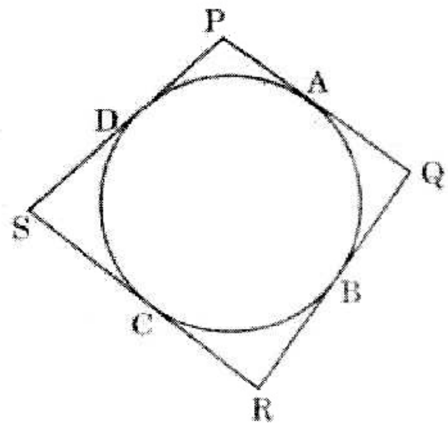
(a) remains unchanged. (b) increase by 3.

(c) increase by 6. (d) increase by 3n.

16. The area of the circle is 154 square cm. The radius of the circle is:

(a) 7 cm (b) 14 cm (c) 3.5 cm (d) 17.5 cm

17. In the given figure, the quadrilateral PQRS circumscribes a circle. Here PA + CS is equal to:



(a) QR (b) PR (c) PS (d) PQ

18. $1 - \cos^2 A$ is equal to:

(a) $\sin^2 A$ (b) $\tan^2 A$ (c) $1 - \sin^2 A$ (d) $\sec^2 A$

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** The surface area of largest sphere that can be inscribed in a hollow cube of side 'a' cm is πa^2 square cm.

    Reason (R): The surface area of a sphere of radius 'r' is $\frac{4}{3}\pi r^3$

20. **Assertion (A):** $-5, -\frac{5}{2}, 0, \frac{5}{2}$, ... is in Arithmetic Progression.

    Reason (R): The terms of an Arithmetic Progression cannot have both positive and negative rational numbers.

Section – B

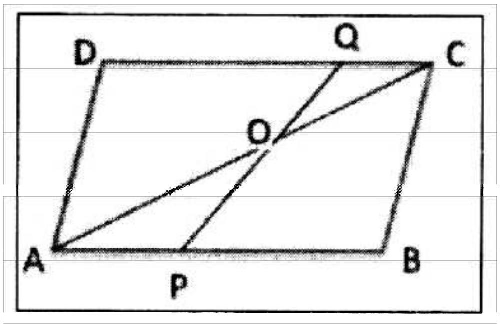
This section comprises Very Short Answer (VSA) type questions. Each question carries 2 marks.

21. Prove that $2+\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

    OR

    Two numbers are in the ratio 2:3 and their LCM is 180. What is the HCF of these numbers?

22. ABCD is a parallelogram. Point P divides AB in the ratio 2:3 and point Q divides DC in the ratio 4:1. Prove that OC $=\frac{1}{2}$OA.



23. If $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$, then find the value of cosecα + cosec β.

OR

Find the value of $2\sec^2\theta + 3\ \text{cosec}^2\theta - 2\sin\theta\cos\theta$ if $\theta = 45°$.

24. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the total area cleaned at each sweep of the two blades.

25. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Section – C

This section comprises Short Answer (SA) type questions. Each question carries 3 marks.

26. If 217x + 131y = 913 and 131x + 217y = 827, then solve the equations to find the values of x and y.

27. Prove that: $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$

OR

Prove that: $\frac{\sin A - \sin^3 A}{\cos^3 A - \cos A} = \tan A$

28. Prove that $\sqrt{5}$ is an irrational number.

29. The length of 40 leaves of a plant are measured correct to nearest millimeter, and the data obtained is represented in the following table.

Length (in mm)	Number of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

Find the median length of the leaves.

OR

Find the mean of the following data:

Class	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	12	15	11	20	16	6

30. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the centre of the circle bisects the angle between the two tangents.

31. If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^{-1} + \beta^{-1}$

Section – D

This section comprises Long Answer (LA) type questions. Each question carries 5 marks.

32. The ratio of the 11^{th} term to 17^{th} term of an A.P. is 3:4. Find the ratio of the 5^{th} term to 21^{st} term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms.

OR

How many terms of the Arithmetic Progression 45, 39, 33, ……. must be taken so that their sum is 180? Explain the double answer.

33. The monthly expenditure on milk in 200 families of a Housing Society is given below:

Monthly Expenditure (in ₹)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	x	30	22	16	7

Find the value of x and also, find the median and mean expenditure on milk.

34. A line BM is drawn from the mid-point M of the side CD of a parallelogram ABCD to intersect the diagonal AC at the point L and the side AD produced at the point E. Prove the EL= 2BL.

OR

In ΔPQR, S and T are points on PQ and PR respectively.

$\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

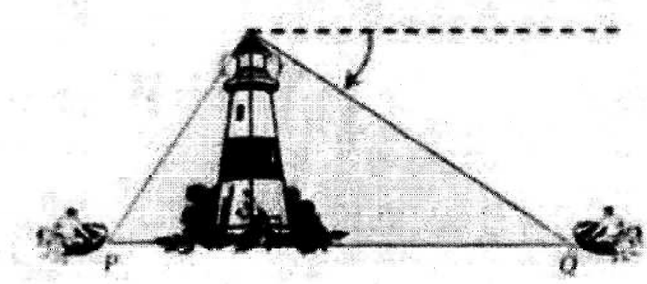
35. The mid-point D,E, F of the sides of a triangle ABC are (3,4), (8,9) and (6,7). Find the coordinates of the vertices of the triangle.

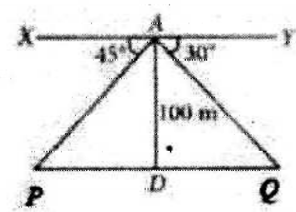
Section – E

In this section, there are 3 case study based units of assessment of 4 marks each.

Case Study – 1

36. A boy is standing on the top of light house. He observed that boat P and boat Q are approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.





Based on the above information, answer the following questions.

(i) What is the measure of ∠APD?

(ii) If ∠YAQ = 30°, then ∠AQD is also 30°, Why?

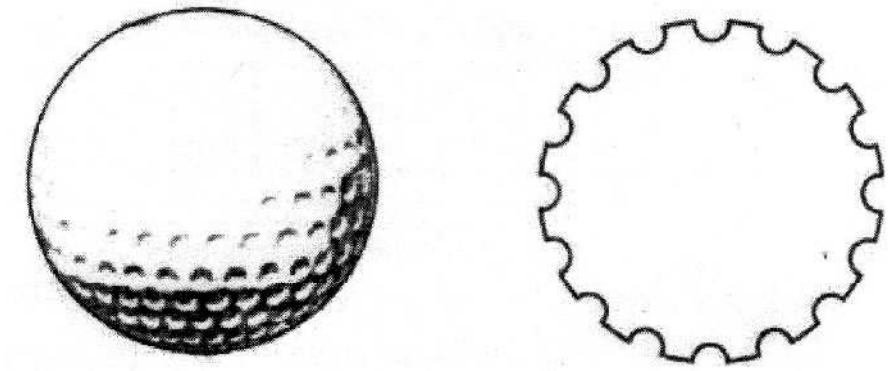
(iii) How far is boat P from the light house?

OR

How far is the boat Q from the light house?

Case Study – 2

37. A spherical golf ball has hemi-spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.



Based on the above, answer the following questions:

(i) Find the surface area of one such dimple.

(ii) Find the volume of the material dug out to make one dimple.

(iii) Find the total surface area exposed to the surroundings.

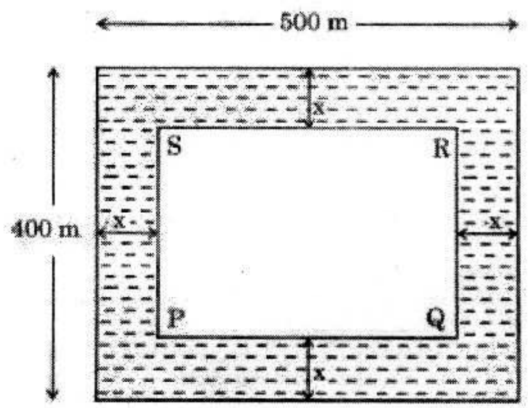
OR

Find the volume of the golf ball.

Case Study – 3

38. Social work aims at fulfillment of human needs. Social workers aim to open the doors of access and opportunity for those who are in greatest need. Free education is a great social work. By doing so, we can remove illiteracy from our society.

Rohan, being a social worker, wants to donate his land to the Village Panchayat for opening of a school.



Rohan's land is in the form of a rectangle of dimensions 500 x 400m. The village Panchayat decides to leave the area on all the four sides of the land for grass and flowers. If width of x m land is kept for grass and flowers on all the four sides (as shown is figure), then answer the following questions:

(i) Write a quadratic equation if area of grass and flowers region surrounding PQRS is 120000 square cm.

(ii) Find the value of x.

OR

Find the lengths PQ and QR.

(iii) Find the perimeter of the rectangle PQRS.

Answer

Section – A

1. (a) 2, –5

2. (c) q

3. (d) 1 cm

4. (b) $\frac{3}{4}$

5. (a) 22 cm

6. (d) $75\sqrt{3}m$

7. (a) 1

8. (a) $\frac{3}{4}$

9. (a) 47.5°

10. (c) – 6

11. (c) 550 square cm.

12. (c) 5:1

13. (b) $\sqrt{96}$

14. (a) p + q = 1

15. (b) increase by 3.

16. (c) 7 cm

17. (c) PS

18. (a) sin^2A

19. (c) Assertion (A) is true, but Reason (R) is false,

20. (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section –B

21. Correct proof.

OR

30

22. Correct proof.

23. $2+\sqrt{2}$

OR

9

24. 924 square cm.

25. 3 cm

Section –C

26. x = 3 and y = 2

27. Correct proof.

OR

Correct proof.

28. Correct proof.

29. 146.75 mm

OR

43.3125

30. Correct proof.

31. (i) $\frac{3}{5}$ (ii) $\frac{-5}{2}$

Section –D

32. 3:7, 25:189

OR

10 or 6 (as 'd' is negative)

33. x = 28, Median = ₹2553.57 approx. & Mean = ₹2662.50

34. Correct proof.

OR

Correct proof.

35. $(\frac{1}{2},1),(\frac{5}{2},3),(\frac{11}{2},6)$

Section –E

36. (i) 45° (ii) Alternate interior angles

(iii) 100 m **OR** $100\sqrt{3}m$

37. (i) 8π square mm (ii) $\frac{16}{3}\pi$ cubic mm

(iii) $3024\,\pi$ square mm **OR** $10668\,\pi$ cubic mm

38. (i) $x^2 - 450x + 20000 = 0$

(ii) $x = 50$ m **OR** PQ = 400 m and QR = 300 m

(iii) 1400 m

Note