DIRECTORATE OF EDUCATION

Govt. of NCT, Delhi

SUPPORT MATERIAL

2025-2026

Class : XI

MATHEMATICS

Under the Guidance of

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सचिव (शिक्षा)

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सत्यमेव जयते

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Date : 07/11/2025

MESSAGE

The Directorate of Education remains steadfast in its vision to achieve excellence in the academic domain and its commitment to develop meaningful, engaging, and child-friendly learning content.

Each year, the Directorate carefully reviews and updates the Support Material to ensure alignment with the latest CBSE guidelines and emerging academic developments.

The Support Material provides comprehensive academic support through well-structured practice questions and exercises that strengthen conceptual understanding and exam readiness and aims to nurture students' critical thinking, analytical abilities, and problem-solving skills. Through such sustained efforts, the Directorate of Education continues to guide students towards academic excellence and holistic growth.

This Support Material is intended to bridge classroom learning and examination preparation, enabling students to consolidate knowledge through systematic practice. It has been thoughtfully designed for students, with the belief that its effective use will strengthen their understanding and support them in achieving their learning goals with confidence.

I appreciates the dedication and collaborative effort of all those involved in the development of this material and extends my best wishes to all students —may this Support Material serve as an essential academic aid, enhancing students' confidence and preparedness for examinations.

Best Wishesh.

(Pandurang K. Pole)

VEDITHA REDDY, IAS
Director, Education & Sports

सत्यमेव जयते

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DE-5/228/Exam/Message/S-M/2018/402
dated - 09/05/25

MESSAGE

Education is the cornerstone of a progressive society, and providing students with the right learning resources is essential for their academic and personal growth. Keeping this in mind, the Directorate of Education, GNCT of Delhi, develops comprehensive Support Material every year for various subjects of Classes IX to XII.

The support material serves as an additional study resource to supplement textbooks by offering clear and easy-to-understand explanations of complex topics. Our dedicated team of expert faculty members has meticulously reviewed and updated this material, aligning it with the latest CBSE syllabus, question paper patterns and assessment guidelines Our effort is to simplify difficult concepts and make them more accessible to students, helping them save time and effort with ready references for effective preparation.

As Ruskin Bond beautifully said, "Education must Inspire the spirit of inquiry, Creativity and joy" True learning goes beyond memorisation-it encourages curiosity, fosters creativity, and makes the learning process meaningful and enjoyable.

In alignment with the vision of NEP 2020, the CBSE framework now places emphasis on competency-based assessments for 50% of the evaluation, highlighting the need for students to develop critical thinking and problem-solving skills. The Support Material is designed to help students analyse concepts deeply, think innovatively, and apply their knowledge effectively, ensuring they are well-prepared not only for exams but also for real-life challenges.

I appreciate the dedicated efforts of the entire team of subject experts in developing this valuable learning resource. I am confident that both teachers and students will make the best use of these materials to enhance learning and academic success.

Wishing all students great success in their exams and a bright, fulfilling future ahead.

Veditha

(VEDITHA REDDY, IAS)

Dr. RITA SHARMA
Additional Director of Education
(School/Exam)

सत्यमेव जयते

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D.O. No. DE.5/228/Exam/Message/SM/2018/570
Dated: .. *02/07/2025* ....

MESSAGE

"Children are not things to be molded, but are people to be unfolded." - Jess Lair

In line with this insightful quote, the Directorate of Education, Delhi, has always made persistent efforts to nurture and unfold the inherent potential within each student. This support material is a testimony to this commitment.

The support material serves as a comprehensive tool to facilitate a deeper understanding of the curriculum. It is crafted to help students not only grasp essential concepts but also apply them effectively in their examinations. We believe that the thoughtful and intelligent utilization of these resources will significantly enhance the learning experience and academic performance of our students.

Our expert faculty members have dedicated themselves to the support material to reflect the latest CBSE guidelines and changes. This continuous effort aims to empower students with innovative approaches, fostering their problem-solving skills and critical thinking abilities.

I extend my heartfelt congratulations to the entire team for their invaluable contribution to creating a highly beneficial and practical support material. Their commitment to excellence ensures that our students are well-prepared to meet the challenges of the CBSE examinations and beyond.

Wishing you all success and fulfilment in your educational journey.

(Dr. Rita Sharma)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
2025-2026

MATHEMATICS

Class : XI

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1][**संपूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य**] बनाने के लिए, तथा उसके समस्त नागरिकों को :

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की **समता**

प्राप्त कराने के लिए,

तथा उन **सब में**

व्यक्ति की गरिमा और [2][**राष्ट्र की एकता**

और अखंडता] सुनिश्चित करने वाली **बंधुता**

बढ़ाने के लिए

दृढ़संकल्प होकर **अपनी इस संविधान सभा में** आज तारीख 26 नवंबर, 1949 ई. को **एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।**

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य'' के स्थान पर प्रतिस्थापित।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''राष्ट्र की एकता'' के स्थान पर प्रतिस्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

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1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51 क

मूल कर्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की संप्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण बनाए रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभावों से परे हो, ऐसी प्रथाओं का त्याग करे जो महिलाओं के सम्मान के विरुद्ध हों;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की, जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणिमात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे, जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई ऊँचाइयों को छू सके; और

(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य को शिक्षा के अवसर प्रदान करे।

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

2025-2026

SUPPORT MATERIAL

Class : XI

MATHEMATICS

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Mathematics (XI-XII)
(Code No. 041)
Session - 2025-2026

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. Senior Secondary stage is a launching stage from where the students go either for higher academic education in mathematics or for professional course like Engineering, Physical and Biological Science, Commerce or Computer Applications. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in Focus Group on Teaching of Mathematics 2005 which is to meet the emerging needs of all categories of students. Motivating the topics from real life situations and other subject areas, greater emphisis has been laid on application of various concepts.

Objectives

The board objectives of teaching Mathematics at senior school stage intend to help the students :

- to acquire knowledge and critical understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles, symbols and mastery of underlying processes and skills.
- to feel the flow of reasons while proving a result or solving a problem.
- to apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- to develop positive attitude to think, analyze and articulate logically.
- to develop interest in the subject by participating in related competitions.
- to acquaint students with different aspects of Mathematics used in daily life.
- to develop an interest in students to study Mathematics as a discipline.
- to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases.
- to develop reverence and respect towards great Mathematicians for their contirbutions to the field of Mathematics.

COURSE STRUCTURE
CLASS XI (2025-2026)

One Paper — Total Period–240 [35 Minutes each]

Three Hours — **Max Marks: 80**

No.	Units	Marks
I.	Sets and Functions	23
II.	Algebra	25
III.	Coordinate Geometry	12
IV.	Calculus	08
V.	Statistics and Probability	12
	Total	80
	Internal Assessment	20

*No chapter/unit-wise weightage. Care to be taken to cover all the chapters.

Unit-I: Sets and Functions

1. **Sets** **(20) Periods**

Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers especially intervals (with notations). Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.

2. **Relations & Functions** **(20) Periods**

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R).Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions** **(20) Periods**

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing *sin (x±y)* and *cos (x±y)* in terms of sin*x*, sin*y*, cos*x* & cos*y* and their simple applications. Deducing identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$cos\alpha - cos\beta = -2sin\frac{1}{2}(\alpha + \beta)sin\frac{1}{2}(\alpha - \beta)$$

Identities related to sin2*x*, cos2*x*, tan2 *x*, sin3*x*, cos3*x* and tan3*x*.

Unit-II: Algebra

1. **Complex Numbers and Quadratic Equations** **(10) Periods**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane

2. **Linear Inequalities** **(10) Periods**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line.

3. **Permutations and Combinations** **(10) Periods**

Fundamental principle of counting. Factorial *n.* (n!) Permutations and combinations, derivation of Formulae for ^nP_r and ^nC_r and their connections, simple applications.

4. **Binomial Theorem** **(10) Periods**

Historical perspective, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, simple applications.

5. **Sequence and Series** **(10) Periods**

Sequence and Series. Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of *n* terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M.

Unit-III: Coordinate Geometry

1. Straight Lines (15) Periods

Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point -slope form, slope-intercept form, two-point form, intercept form, Distance of a point from a line.

2. Conic Sections (25) Periods

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-dimensional Geometry (10) Periods

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points.

Unit-IV: Calculus

1. Limits and Derivatives (40) Periods

Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V Statistics and Probability

1. Statistics (20) Periods

Measures of Dispersion: Range, Mean deviation, variance and standard deviation of ungrouped/grouped data.

2. Probability (20) Periods

Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

MATHEMATICS

QUESTION PAPER DESIGN

CLASS – XI (2025-2026)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weight age
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper.

However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: Please refer the guidelines given under XII Mathematics Syllabus:

CLASS–XI (2025-26)

The following topics are included in the syllabus but will be assessed only formatively to reinforce understanding without adding to summative assessments. This reduces academic stress while ensuring meaningful learning. Schools can integrate these with existing chapters as they align well. Relevant NCERT textual material is enclosed for reference.

S. No. Content

Unit–I : Sets and Functions

1. **Sets**
   Practical problems on Union and Intersection of two sets.
2. **Relations and Functions**
   Composition of Functions
3. **Trigonometric Functions**
   General solution of trigonometric equations of the type $\sin y = \sin a$, $\cos y = \cos a$ and $\tan y = \tan a$.

Unit – II : Algebra

1. **Principle of Mathematical Induction**
   Process of the proof by induction, motivating the application of the metghod by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.
2. **(Complex Numbers) and Quadratic Equations**
   Polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients) in the complex number system.
3. **Linear Inequalities**
   Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.
4. **Binomial Theorem**
   General and middle term in binomial expansion.
5. **Sequence and Series**
   Formulae for the following special sums $\sum_{k=1}^{n} k, \sum_{k=1}^{n} k^2, \sum_{k=1}^{n} k^3$

Unit–III : Coordinate Geometry

1. **Straight Lines**
   Normal form. General equation of a line.
2. **Introduction to Three-dimensional Geometry**
   Section formula.

Unit–IV : Calculus

1. **Limits and Derivatives**
   Derivatives of composite functions (Chain rule).

Unit–V : Statistics and Probability

1. **Probability**
   Random experiments; outcomes, sample space (set representation).

CONTENTS

CHAPTER - 1

SETS AND FUNCTIONS

KEY POINTS

- Definition of Set : Set is well defined collection of objects.
- Objects in Set are called elements of Set.
- Elements are said to be 'belong to' set.

  Example: A = {a, b, c, d} is a Set and a, b, c, d are element of Set A

  Here a, b, c, d belongs to A or a, b, c, d $\in$ A

- Representation of Sets:
  - (a) Roster or Tabular form

    e.g.: Set of all Natural Numbers less than 5 = {1, 2, 3, 4}

  - (b) Set-builder form

    e.g.: Set of all Natural Numbers less than 5 = $\{x : x \in N, x < 5\}$

- **Types of sets:**
  - (a) Empty /Null/Void Set: Set which does not contain any element. It is denoted by ϕ or { }
  - (b) Finite set : Set having finite number of elements
  - (c) Infinite set: Set having infinite number of elements
  - (d) Singleton set : Set having only one element
- Cardinal number of finite set: Number of distinct elements of set. It is denoted by n(A).
- Equivalent sets: Two or more finite sets having same number of elements or same cardinal number.

- Subset: A set A is said to be subset of a set B iff a $\in$A $\Rightarrow$ a $\in$B. $\forall a \in A$

  We write it as A $\subseteq$ B.

  Note: ϕ and A itself are always subsets of set A.
- Super set: If A $\subseteq$ B then B is superset of A.
- Proper subset : If A $\subseteq$ B, but A ≠ B then A is proper subset of B.

  We write it as A $\subset$ B.
- Number of subsets of a set A= $2^{n(A)}$
- Number of proper subsets of a Set A = $2^{n(A)} - 1$
- **Equal sets:** Two or more sets having exactly same elements.

  A = B $\Leftrightarrow$A $\subseteq$B and B $\subseteq$ A.

- **Power set:** The collection of all subsets of a set A.It is denoted by P(A)

  P(A) = {X: X$\subseteq$ A}

  n[P(A)] = $2^{n(A)}$

- **Types of Intervals**

  (a) Open Interval (a, b) = { x $\in$ R : a < x < b }

  (b) Closed Interval [a, b] = { x $\in$ R : a $\leq$ x $\leq$ b }

  (c) Semi open or Semi closed Interval,

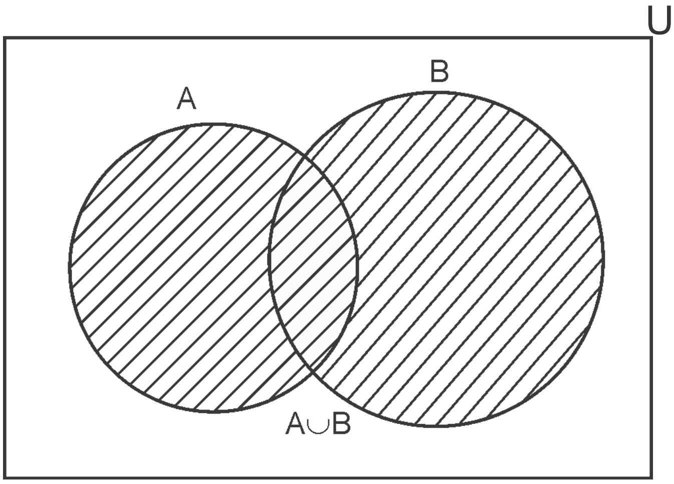
  (a,b] = { x $\in$ R : a < x ≤b}

  [a,b) = { x $\in$ R : a ≤ x < b}

- Venn diagram and operations on sets

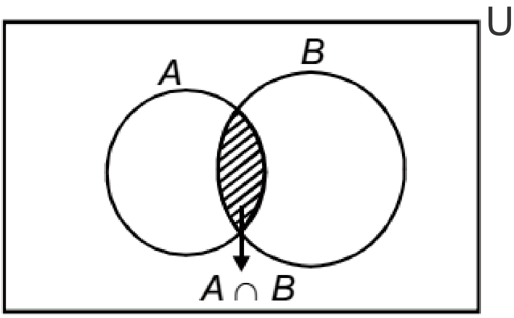
  (a) Union of two sets A and B :

  A $\cup$ B = { x : x $\in$ A or x $\in$ B }

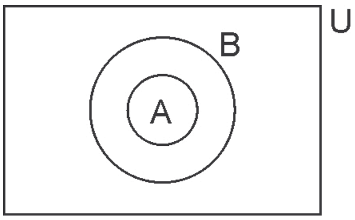


(b) Intersection of two sets A and B :

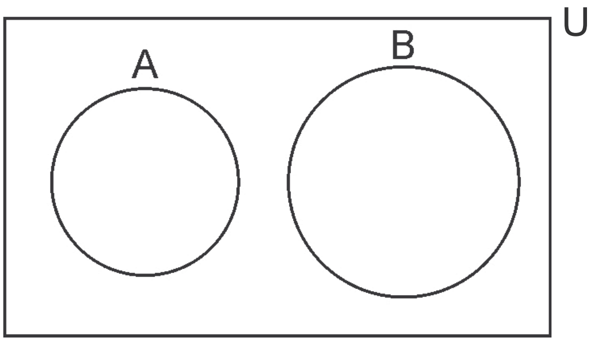
$A \cap B = \{ x : x \in A \text{ and } x \in B\}$



- Subset and superset: $A \subset B$

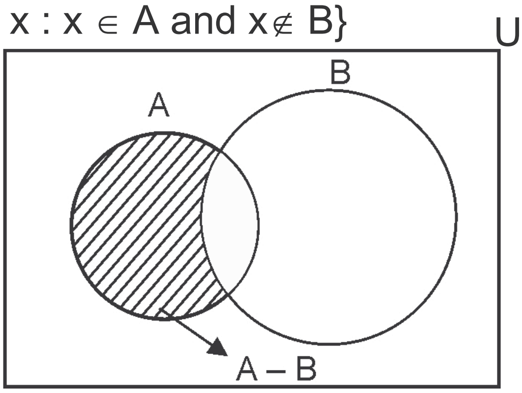


- Disjoint sets: Two sets A and B are said to be disjoint if $A \cap B = \phi$



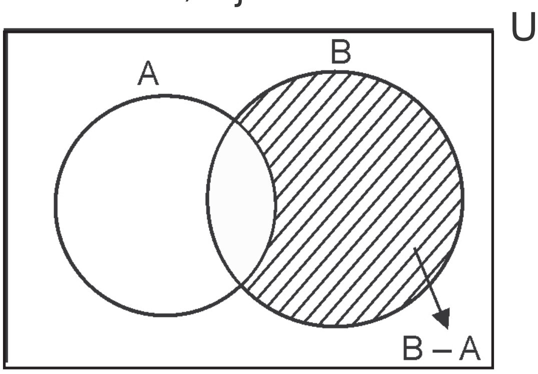
(c) Difference of sets A and B is,

$A - B = \{ x : x \in A \text{ and } x \notin B\}$



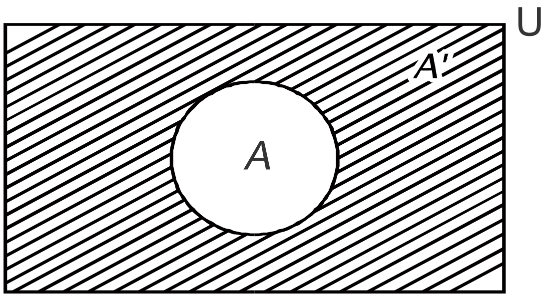
(d) Difference of sets B and A is,

$B - A = \{ x : x \in B \text{ and } x \notin A\}$



(e) Complement of a set A, denoted by A' or A^c

$A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



- Properties of complement of sets :

1. Complement laws

   (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Note **:** This law can be extended to any number of sets.

3. $\phi' = \bigcup$ and $\bigcup' = \phi$

4. If $A \subset B$ then $B' \subset A'$

- Laws of Algebra of sets

(i) $A \cup \phi = A$

(ii) $A \cap \phi = \phi$

- $A - B = A \cap B' = A - (A \cap B)$
- Commutative Laws :–

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

- Associative Laws :–

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive Laws :–

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$
- $n(A \cup B) + n(A \cap B) = n(A) + n(B)$
- If A and B are disjoint, then $n(A \cup B) = n(A) + n(B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

MIND MAP

SETS

Introduction

A set is collection of well-defined distinguised objects. The sets are usually denoted by capital letters A, B, C etc., and the members or elements of the set are denoted by lower case letters a, b, c etc., if x is a member of the set A, we write $x \in A$ (read as 'x' belongs to A) and if x is not a member of set A, we write $x \in A$(read as 'x' doesn't belongs to A). if x and y both belong to A, we write $x,y \in A$.
Some examples of sets are: A: odd number less then 10
N: the set of all rational numbers
B: the vowels in the English alphabates
Q: the set of all rational numbers.

Cardinal Number

The number of elements in a finite set is represented by **n (A)**, known as cardinal number.
Eg.: A = {a, b, c, d, e} Then, n(A) = 5

Subset

Let A and B be two sets. if every element of A is an element of B,then A is called a subset of B and written as $A \subset B$ or $B \supset$ (read as 'A' is contained in 'B' or 'B contains A'). is called superset of A.
Note:
1. Every set is a subset and superset of itself.
2. If A is not a subset of B, we write $A \not\subset B$.
3. The empty set is the subset of every set.
4. If A is a set with n(A) = m, then no. of element A are 2^n and the number of proper subsets of A are $2^n - 1$

Eg. Let A = {3,4}, then subsets of A are ϕ, {3}, {4}, {3, 4}. Here, n(A) = 2 and number of subsets of A = $2^2 = 4$.

Rerpresentation of sets

Set Builder Form Or Rule Method

In this form, we write a variable (say x) representing any member of the set followed by property satisfied by each member of the set.
e.g.: The set A of all prime number less than 10 in set builder from is written as
A = {x | x is a prime number less than 10}
the symbol "|" stands for the word "such that", sometimes, we use symbol ":" in place of symbol "|"

Roster Or Tambular Form

In this form, we first list all the members of the set within braces (curly brackets) and seprate these by commas.
E.g. The set of all natural number less than 10 in this form is written as: A = {1,3,5,7,9}
In roaster from, every element of sets is listed only once.
The order in which the elements are listed is immaterial.
E.g. Each of the following sets denotes the same set
{1, 2, 3}, {3, 2, 1},{1, 3, 2}

Types Of Sets

Empty Set Or Null Set

A set which has no element is called null set. It is denoted by symbol ϕ or {}.
E.g: Set of all real number whose squre is –1.
In set-builder formL: {x:x is a real number whose squre is –1}
in roaster from: {} or ϕ

Finite And Infinite Set

A set which has finite number of element is called a finite set. Otherwise, it is called an infinite set.
E.g.: The set of all days in a week is a finite set where as the set of all integers, denoted by
{......–2, –1, 0, 1, 2,......} or {x | x is an integer} is an infinite set.
An empty set ϕ which has no element is a finite set A is called empty or void or null set.

Equal Set

Two sets A and B are set to be equal, written as A = B, if every element of A is in B and every element of B is in A.
e.g.: (i) A = {1, 2, 3, 4} and B {3, 1, 4, 2} then A= B
(ii) A = {x: x –5 =0} and B = {x: x is an intergral positive root
Then A = B.

Singleton Set

A set having one element is called singleton set.
e.g.: (i) {0} is a singleton set, whose only member is 0.
(ii) A = {x: 1<x <3, x is a natural number} is a singleton set which has only one member which is 2.

Equivalent Set

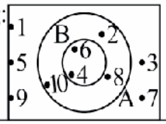
Two finite sets A and B are said to be equivalent, if = n(A) = n(B), Clearly, equal set are equivalent but equivalent set need not to be equal.
e.x. The sets A = {4, 5, 3, 2} and B = {1, 6, 8, 9} are equivalent, but are not equal.

SETS

Venn Diagram

A Vann diagram is an illustration of the relationships between and among sets, groups of objects that share something common these diagrams consist of rectangle and closed curves usually circles

Eg:



In the given venn diagram $U = \{1,2,3,.....10\}$ universe set of which $A = \{2,4,6,8,10\}$ and $B = \{4,6\}$ are subsets and also $B \subset A$

Algebra of sets

1. For any set A, we have
   (a) $A \cup A = A$, (b) $A \cap A = A$, (c) $A \cup \phi = A$, (d) $A \cap \phi = \phi$, (e) $A \cup U = U$, (f) $A \cap \underline{U} = A$, (g) $A - \phi = A$, (h) $A - A = \phi$,
2. For any two sets A and B we have
   (a) $A \cup B = B \cup A$, (b) $A \cap B = B \cap A$, (c) $A - B \subseteq A$, (d) $B - A \subseteq B = U$
3. For any two sets A and B we have
   (a) $A \cup (B \cup C) = (A \cup B) \cup C$, (b) $A \cap (B \cap C) = (A \cap B) \cap C$
   (c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, (d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (e) $A - (B \cup C) = (A - B) \cap (A - C)$, (f) $A - (B \cap C) = (A - B) \cup (A - C)$

Operations On Sets

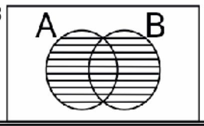
Union

The union of two sets A and B, written as $A \cup B$ (read as A union B) is the set of all elements which are either in A or in B in both.

Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

clearly, $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ and $x \in A \cup B \Rightarrow x \in A$ and $x \in B$

$A \cup B$



eg: If $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$ then $A \cup B = \{a, b, c, d, e, f\}$

Intersection

The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B.

Thus, $A \cap B \Rightarrow \{x : x \in A \text{nd } x \in B\}$

Clearly, $x \in A \cap B \Rightarrow \{x \in A \text{nd } x \in B\}$ and $x \in A \cap B \Rightarrow \{x \in A$ or $x \in B\}$.

$A \cap B$

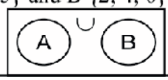
Eg: If $A = \{a,b,c.d\}$ and $B = \{c,d,e,f\}$

Then $A \cap B = \{c,d\}$

Disjoint Sets

Two sets A and B are said to be disjoint, if $A \cap B = \phi$ i.e, A and B have no common element. e.g: it $A = \{1, 3, 5\}$ and B $\{2, 4, 6\}$

Then, $A \cap B = \phi$, so A and B are disjoint.



Universat Set

The set containing all objects of element and of which all other sets are subsets is known as universal sets and denoted by U.

E.g: For the set of all intergers, the universal set can be the set of rational numbers or the set R of real numbers

Power Set

The set of all subset of a given set A is called power set of A and denoted by P(A).

Eg: If A=(1,2,3,), then P (A)} {ϕ}, {1}, {2}, {3}, {1,2},{1,3},{2,3},{1,2,3}

Clearly, if A has n elements, then its power set P (A) contains exactly 2^n elements.

Symmeteric Defference

The symmetric diference of two sets A and B, denoted by A Δ B, in defined as $(A \Delta B) = (A-B) \cup (B-A)$

Eg. If A= {1,2,3,4,5}

and B = {1,3,5,7,9} then

$(A \Delta B) = (A-B) \cup (B-A)$

$= \{2, 4,\} \cup \{7,9\}$

$= \{2,4,7,9\}$

Difference Of Sets

If A and B are two sets, then their difference A-B is defined as:

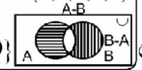
$A-B = \{x : x \in A \text{ and } x \in B\}$

Similarly, $B - A = \{x : x \in B \text{ and } x \in A\}$

Eg. If A = (1,2,3,4,5,) and B={1,3,5,7,9)

then

A-B={2,4} and B-A= {7,9}



Compliment Of Set

If U is a universal set and A is a subset of U, then complement of A is the set which contains those elements of U, which are not present in A and is denoted by A' or A^c. Thus,

$A^c = \{x : x \text{EU and } x \text{eA}\}$

e.g.: If U {1, 2, 3, 4,...} and A {2, 4, 6, 8,...} then AC {1,3,5,7,...}

Properties of complement

Complement law:

(1) $A \cup A' = U$ (ii) $A \cap A' = \phi$,

- De morgan's Law:

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

- Double Complement law:

$(A')' = A$

- Law of empty set and universal set

$\phi' = U$ and $L = \phi$

Subsets Of A Sets Of Real Number 'R'

- The set of natural numbers N={1, 2, 3, 4, 5,---}
- The set of integers Z=(...–3, –2, –1, 0, 1, 2, 3,– – –}
- The set of irrational numbers, T= {x: x ∈ R and r ∈ Q}
- The set of rational number Q= {x: x = p, q ∈ Z and q ≠ 0} P
- Relation among these subsets are $N \subset Z \subset Q$, $Q \subset R$, $T \subset R$, $N \not\subset T$

Interval Notation

Let a and b be real numbers with a < b

Set of Real Numbers	Interval Notation	Region on the real number line
• {x|a < x <b}	(a, b)	○———○
• {x|a ≤ x <b}	[a, b)	●———○
• {x|a < x ≤b}	(a, b]	○———●
• {x|a ≤ x ≤b}	[a, b]	●———●
• {x|x < b}	(–∞ , b)	◄———○
• {x|x ≤ b}	[–∞ , b]	◄———●
• {x|x > a}	(a, ∞)	○———►
• {x|x≥a}		●———►
• R	(–∞, ∞)	◄———►

VERY SHORT ANSWER TYPE QUESTIONS

1. Write set $\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},...\frac{19}{20}\right\}$ in set builder form.

2. Write the set {x : x $\in Z^+$, $x^2 < 4$} in Roster form.

   Let A = {1,3,5,7,9}. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces: – (Question- 3,4)

3. (i) 2 _____ A (ii) {3} ____ A (iii) {3, 5} ____ A

4. Write the set A = { x : x is an integer, $-1 \leq x < 4$} in roster form

5. Write the set B = {3,9,27,81} in set-builder form.

   Which of the following are empty sets? Justify. (Question- 6,7)

6. A = { x : x $\in$ N and 3 <x <4}

7. B = { x : x $\in$N and $x^2 = x$}

   Which of the following sets are finite or infinite? Justify. (Question-8, 9)

8. The set of all the points on the circumference of a circle.

9. B = { x : x $\in$ N and x is an even prime number}

10. Are sets A = { –2,2}, B = { x : x $\in$ Z, $x^2 - 4 = 0$} equal? Why?

11. Write (–5,9] in set-builder form

12. Write {x : x $\in$ R, $-3 \leq x < 7$} as interval.

13. If A = { 1,3,5}, how many elements has P(A)?

14. Write all the possible subsets of A = {5,6}.

    If A = Set of letters of the word 'DELHI' and B= the set of letters the words 'DOLL' find (Question- 15, 16, 17)

15. A $\cup$ B

16. $A \cap B$

17. $A - B$

18. Describe the following sets in Roster form

(i) The set of all letters in the word 'ARITHMETIC'.

(ii) The set of all vowels in the word 'EQUATION'.

19. Write the set $A = \{x : x \in Z, x^2 < 25\}$ in roster form.

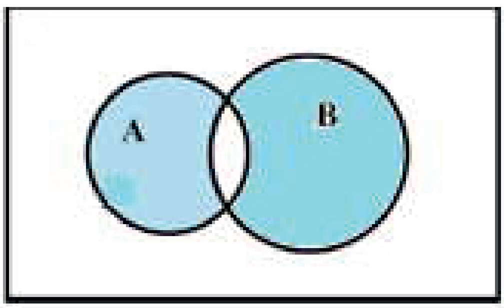
20. Write the set B = {x : x is a two digit number, such that the sum of its digits is 7}

SHORT ANSWER TYPE QUESTIONS

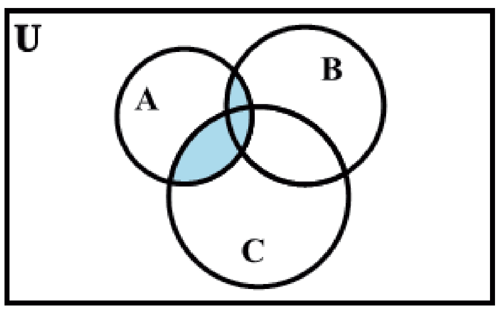
21. Are sets $A = \{1,2,3,4\}$, $B = \{ x : x \in N \text{ and } 5 \leq x \leq 7\}$ disjoint? Justify?

What is represented by the shaded regions in each of the following Venn-diagrams? (Question 22, 23)

22.



23.



24. If A = { 1, 3, 5, 7, 11, 13, 15, 17}

$B = \{2, 4, 6, 8 \ldots 18\}$, $U = \{1, 2, 3, \ldots\ldots\ldots\ldots.. 20\}$

Where U is universal set then find $A' \cup [(A \cup B) \cap B']$

25. Two sets A and B are such that

    $n(A \cup B) = 21$, $n(A) = 10$, $n(B) = 15$, find $n(A \cap B)$ and $n(A - B)$

26. Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$ Verify the following identity

    $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

27. If $U = \{x : x \in N \text{ and } x \leq 10\}$

    $A = \{x : x \text{ is prime and } x \leq 10\}$
    $B = \{x : x \text{ is a factor of } 24\}$

    Verify the following result

    (i) $A - B = A \cap B'$ (ii) $(A \cup B)' = A' \cap B'$ (iii) $(A \cap B)' = A' \cup B'$

28. For any sets A and B show that

    (i) $(A \cap B) \cup (A - B) = A$ (ii) $A \cup (B - A) = A \cup B$

29. On the Real axis, if $A = [0,3]$ and $B = [2,6]$, then find the following

    (i) A' (ii) $A \cup B$ (iii) $A \cap B$ (iv) $A - B$

30. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

31. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

32. Two sets A and B are such that $n(A \cup B) = 21$, $n(A' \cap B') = 9$, $n(A \cap B) = 7$ find $n(A \cap B)'$.

LONG ANSWER TYPE QUESTIONS

33. In a group of 84 persons, each plays at least one game out of three viz. tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis.

34. Using properties of sets and their complements prove that

(i) $(A \cup B) \cap (A \cup B') = A$

(ii) $A - (A \cap B) = A - B$

(iii) $(A \cup B) - C = (A - C) \cup (B - C)$

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$

(v) $A \cap (B - C) = (A \cap B) - (A \cap C)$.

35. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.

36. A survey shows that 63% people watch news channel A whereas 76% people watch news channel B. If x% of people watch both news channels, then prove that $39 \leq x \leq 63$.

37. From 50 students taking examination in Mathematics, Physics and chemistry, each of the students has passed in at least one of the subject, 37 passes Mathematics, 24 Physics and 43Chemistry. At most 19 passed Mathematics and Physics, almost 29 Mathematics and chemistry and at most 20 Physics and chemistry. What is the largest possible number that could have passes in all the three subjects?

CASE STUDY TYPE QUESTIONS

38. In a survey of of 600 students of class XI, 150 are using YouTube videos and 225 are consulting books (other than text book) as a learning resource. 100 were using both YouTube videos and books as a learning resource.

i. How many students are using either books or YouTube videos as the learning resource?

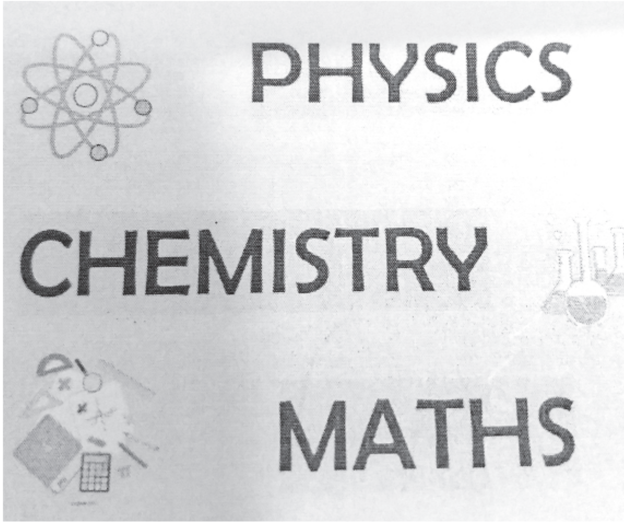
ii. How many students are neither using YouTube videos nor books as the learning resource?

iii. How many students are using YouTube videos only as the learning resource?

iv. How many students are using books only as the learning resource?

v. What can be the maximum number of students who will use YouTube video or books as learning resources?

39. In a class 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics.Of these 13 took both Chemistry and Mathematics, 12 took both physics and chemistry and 11 took both Physics and mathematics. If 6 students were offered all the three subjects, find:



i. The total number of students are

(a) 47 (b) 37 (c) 35 (d) 49

ii. How many took Mathematics but not Chemistry?

(a) 11 (b) 1 (c) 6 (d) 12

iii. How many took exactly one of the three subjects?

(a)12 (b) 11 (c) 13 (d) 1

iv. How many took exactly two of these subjects?

(a) 11 (b) 13 (c) 12 (d) 18

v. Number of students who took Physics or Mathematics but not Chemistry:

(a)12 (b) 13 (c) 11 (d) 18

40. In a town of 10,000 families, it was found that 40% families go to shop A for their home needs groceries, 20% families go to the shop B and 10% families go to shop C. 5% families go to shops A and B, 3% go to B and C and 4% families go to A and C. 2% families go to all the three shops A, Band C. Find:

i. The number of families which go to shop A only;

(a) 4000 (b) 3300 (c) 3700 (d) 4200

ii. The number of families which don't visit/purchase from any of A, B and C.

(a) 4000 (b) 7000 (c) 3300 (d) 6000

iii. The number of families which don't visit/purchase from any of A, B and C.

(a) 300 (b) 200 (c) 100 (d) 600

iv. The number of families that purchase from exactly one shop.

(a) 4700 (b) 4000 (c) 5200 (d) 3800

v. The number of families that buy from at least one of the shops A, B or C.

(a) 4000 (b) 6000 (c) 7000 (d) 1000

Multiple Choice Questions

41. In set builder method the null set is represented by

(a) { } (b) ϕ (c) $\{ x : x \neq x\}$ (d) $\{ x : x = x\}$.

42. If A and B are two given sets, then $A \cap (A \cap B)'$ is equal to

(a) A (b) B′ (c) ϕ (d) A – B.

43. If A and B are two sets such that $A \subset B$ then $A \cap B'$ is

(a) A (b) B′ (c) ϕ (d) $A \cap B$.

44. If $n(A \cup B) = 18$, $n(A - B) = 5$, $n(B - A) = 3$ then $n(A \cap B)$ is

(a) 18 (b) 10 (c) 15 (d) 12

45. For any two sets A and B, $A \cap (A \cup B)'$ is equal to

(a) A (b) B (c) ϕ (d) $A \cap B$

46. If n(A) = 5 and n(B) = 7, then maximum number of elements in $A \cup B$ is

(a) 7 (b) 5 (c) 12 (d) None of these

47. $n[P\{P(\phi)\}] =$

(a) 2 (b) 4 (c) 8 (d) 0

48. If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is

(a) 120 (b) 30 (c) 31 (d) 32

49. For any two sets A and B, $(A - B) \cup (B - A) =$

(a) $(A - B) \cup A$ (b) $(B - A) \cup B$

(c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) \cap (A \cap B)$

50. If X = $\{8^n - 7n - 1 : n \in N)$ and $y = \{49n - 49 : n \in N\}$, then

(a) $X \subset Y$ (b) $Y \subset X$

(c) X = Y (d) $X \cap Y = \phi$

51. Let n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$, then $n(A^c \cap B^c) =$

(a) 400 (b) 600

(c) 300 (d) 200

52. If a set A has n elements, then the total number of subsets of A is

(a) n (b) n^2

(c) 2^n (d) 2n

53. The number of non-empty subsets of the set {1, 2, 3, 4} is

(a) 15 (b) 14

(c) 16 (d) 17

54. If A and B are two sets, then $A \cup B = A \cap B$ iff

(a) $A \subseteq B$ (b) $B \subseteq A$

(c) A = B (d) None of these

55. Let A and B be two sets. Then

(a) $A \cup B \subseteq A \cap B$ (b) $A \cap B \subseteq A \cup B$

(c) $A \cap B = A \cup B$ (d) None of these

56. If Q = $\left\{x : x = \frac{1}{y}, \text{where } y \in N\right\}$, then

(a) $0 \in Q$ (b) $1 \in Q$

(c) $2 \in Q$ (d) $\frac{2}{3} \in Q$

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes. (a), (b), (c) and (d) given below:

(a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

57. **Assertion:** The number of non-empty subsets of the set {a, b, c, d} are 15.

Reason: Number of non-empty subsets of a set having n elements are $2^n - 1$.

58. Suppose A, B and C are three arbitrary sets and U is a universal set.

Assertion: If B = U – A, then n(B) = n(U) – n(A).

Reason: If C = A – B, then n(C) = n(A) – n(B).

59. **Assertion:** Let A = {1, {2, 3}}, then

P(A) = {{1}, {2, 3}, ϕ, {1, {2, 3}}}

Reason: Power set is set of all subsets of A.

60. **Assertion:** The subsets of the set {1, {2}} are { }, {1}, {{2}} and {1, {2}}.

Reason: The total number of proper subsets of a set containing n elements is $2^n - 1$.

ANSWERS

1. $\left\{x : x = \dfrac{n}{n+1}, n \in N, n \leq 19\right\}$

2. {1}

3. (i) $\notin$ (ii) $\notin$ (iii) $\notin$

4. A = {–1, 0, 1, 2, 3}

5. B = { x : x = 3^n, n $\in$ N and 1 $\leq$ n $\leq$ 4}

6. Empty set because no natural number is lying between 3 and 4

7. Non-empty set because B = {1}

8. Infinite set because circle is a collection of infinite points whose distances from the centre is constant called radius.

9. Finite set because B = {2}

10. Yes, because $x^2 - 4 = 0 \Rightarrow x = 2$ or – 2 both are integers

11. {x : x $\in$ R, –5 < x ≤ 9}

12. [–3,7)

13. $2^3 = 8$

14. ϕ, {5}, {6}, {5,6}

15. A $\cup$ B = {D, E, L, H, I, O}

16. A $\cap$ B = {D, L}

17. A – B = {E, H, I}

18. (i) {A, R, I, T, H, M, E, C} (ii) {E, U, A, I, O}

19. { –4, –3, –2, –1, 0, 1, 2, 3, 4}

20. {16, 25, 34, 43, 52, 61, 70}

21. Yes, because A $\cap$ B = ϕ

22. (A – B) $\cup$(B – A) or A Δ B

23. A $\cap$(B $\cup$ C)

24. $\cup$ = {1, 2, 3, ... 20}

25. $n(A \cap B) = 4$, $n(A - B) = 6$

29. (i) $(-\infty, 0) \cup (3, \infty)$(ii) $[0, 6]$ (iii) $[2, 3]$ (iv) $[0, 2)$

30. **Hint:** U = set of people surveyed
A = set of people who play cricket
B = set of people who play tennis

Number of people who play neither cricket nor tennis

$= n[(A \cup B)'] = n(U) - n(A \cup B)$

$= 450 - 200 = 250$

31. There are 280 students in the group.

32. 23 [Hint: Find n(U)]

33. 6

35. n = 3 m = 6

36. [Hint: Take $n(\cup) = 100$]

37. 14

38. i. 275 ii. 325 iii. 50 iv. 125

39. i. (c) ii. (a) iii. (b) iv. (d) v. (a)

40. i. (b) ii. (a) iii. (d) iv. (c) v. (b)

41.(c)	42.(d)	43.(c)
44.(b)	45.(c)	46.(c)
47.(a)	48.(c)	49.(c)
50.(a)	51.(c)	52.(c)
53.(a)	54.(c)	55.(b)
56.(b)	57.(a)	58.(c)
59.(d)	60.(b)	

CHAPTER – 2

RELATIONS AND FUNCTIONS

KEY POINTS

- **Ordered Pair:** An ordered pair consists of two objects or elements in a given fixed order.

  Remarks: An ordered pair is not a set consisting of two elements. The ordering of two elements in an ordered pair is important and the two elements need not be distinct.

- **Equality of Ordered Pair:** Two ordered pairs (x_1, y_1) & (x_2, y_2) are equal if $x_1 = x_2$ and $y_1 = y_2$.

  i.e. $(x_1, y_1) = (x_2, y_2) \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$

- **Cartesian product of two sets:** Cartesian product of two non-empty sets A and B is given by A × B and $A \times B = \{(x, y) : x \in A$ and $y \in B\}$.

- **Cartesian product of three sets:** Let A, B and C be three sets, then A × B × C is the set of all ordered triplet having first element from set A, 2nd element from set B and 3rd element from set C.

  i.e., $A \times B \times C = \{(x, y, z) : x \in A, y \in B \text{ and } z \in C\}$.

- **Number of elements in the Cartesian product of two sets:** If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

- **Relation:** Let A and B be two non-empty sets. Then a relation from set A to set B is a subset of A × B.

- **No. of relations:** If n(A) = p, n(B) = q then no. of relations from set A to set B is given by 2^{pq}.
- **Domain of a relation:** Domain of R = {a : (a,b) $\in$ R}
- **Range of a relation:** Range of R = { b : (a,b) $\in$ R}
- Co-domain of R from set A to set B = set B.
- Range $\subseteq$ Co-domain
- **Relation on a set:** Let A be non-empty set. Then a relation from A to A itself. i.e., a subset of A × A, is called a relation on a set.
- **Inverse of a relation:** Let A, B be two sets and Let R be a relations from set A to set B.

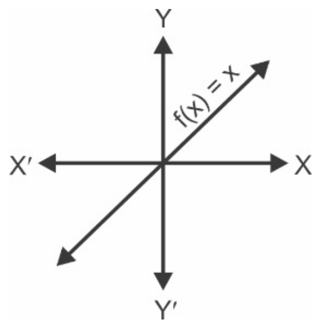
  Then the inverse of R denoted R^{-1} is a relation from set B to A and is defined by R^{-1} = {(b, a) : (a, b) $\in$ R}
- **Function:** Let A and B be two non-empty sets. A relation from set A to set B is called a function (or a mapping or a map) if each element of set A has a unique image in set B.

  Remark: If (a, b) $\in$ f then 'b' is called the image of 'a' under f and 'a' is called pre-image of 'b'.
- **Domain and range of a function:** If a function 'f' is expressed as the set of ordered pairs, the domain of 'f' is the set of all the first components of members of f and range of 'f' is the set of second components of member of 'f'.

  i.e., D_f = {a : (a, b) $\in$ f} and R_f = {b : (a, b) $\in$ f}
- **No. of functions:** Let A and B be two non-empty finite sets such that n(A) = p and n(B) = q then number of functions from A to B = q^p.
- **Real valued function:** A function f : A $\to$ B is called a real valued function if B is a subset of R (real numbers).

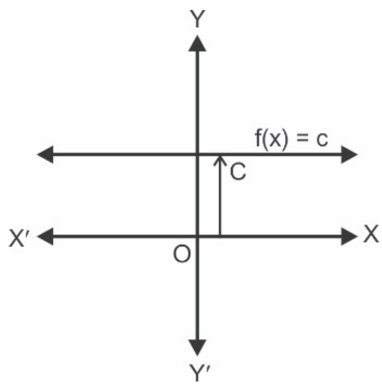
- **Identity function:** f : R→R given by f(x) = x ∀ x ∈ R (real number)

  Here, D_f = R and R_f = R



- **Constant function:** f : R → R given by f(x) = c for all x ∈ R where c is any constant

  Here, D_f = R and R_f = {c}

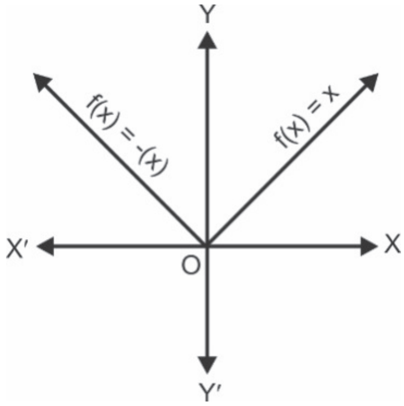


- **Modulus function:** f : R → R given by f(x) = |x| ∀ x ∈ R

  Here, D_f = R and R_f = [0, ∞)

  Remarks : $\sqrt{x^2} = |x|$

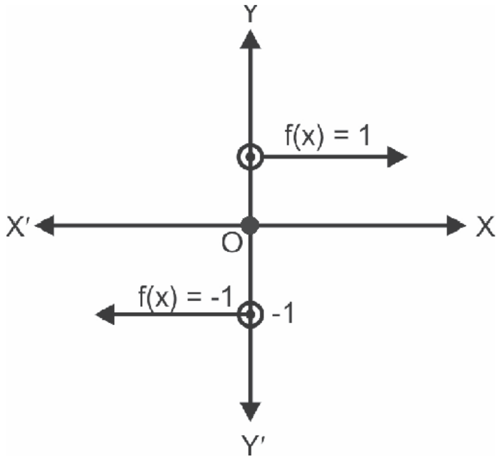
  or $f(x) = |x| = \begin{bmatrix} x : x \geq 0 \\ -x : x < 0 \end{bmatrix}$



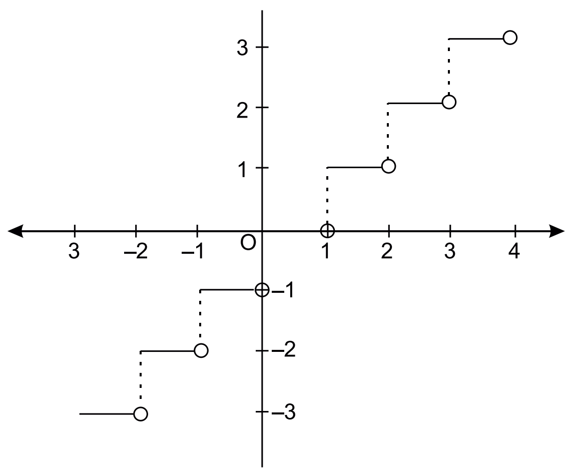
- **Signum function:** f : R → R defined by $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$

Or

$$f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases}$$

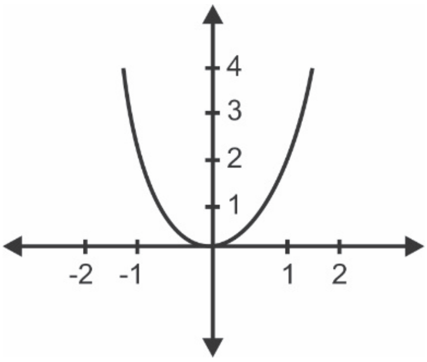


- **Greatest Integer function:** f : R → R defined by f(x) = [x], x ∈ R assumes the value of the greatest integer, less than or equal to x. Here, D_f = R and R_f = Z



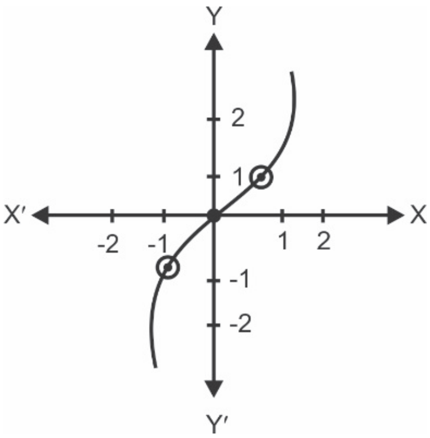
- Graph for f : R → R, defined by $f(x) = x^2$

  Here, $D_f = R$ and $R_f = [0, \infty)$

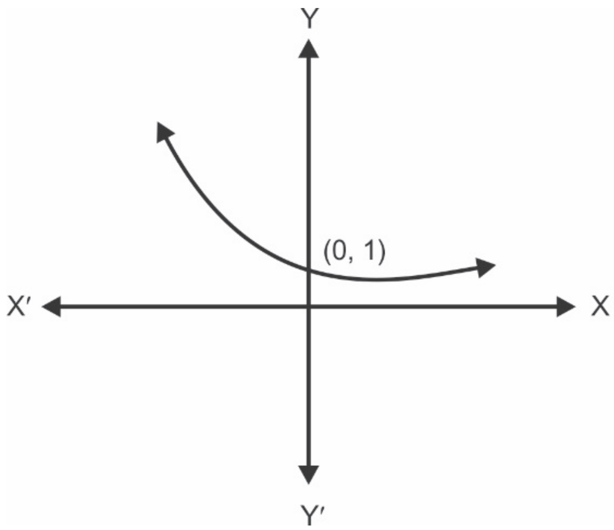


- Graph for f : R → R, defined by $f(x) = x^3$

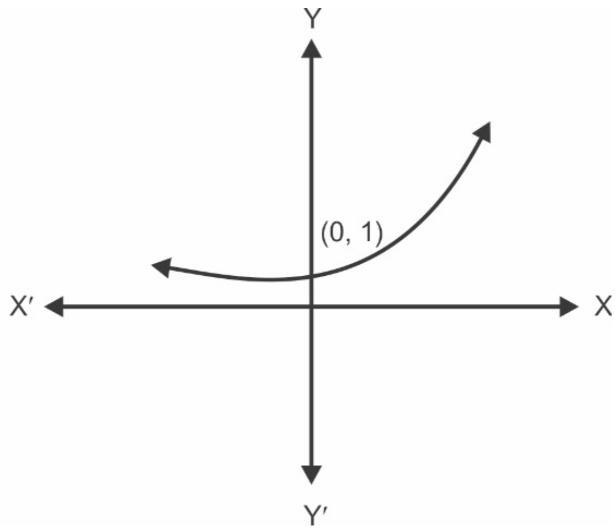
  Here $D_f = R$ and $R_f = R$



- **Exponential function:** f : R → R, defined by $f(x) = a^x$, $a > 0$, $a \neq 1$



(0 < a < 1)



(a > 1)

When $0 < a < 1$ When $a > 1$

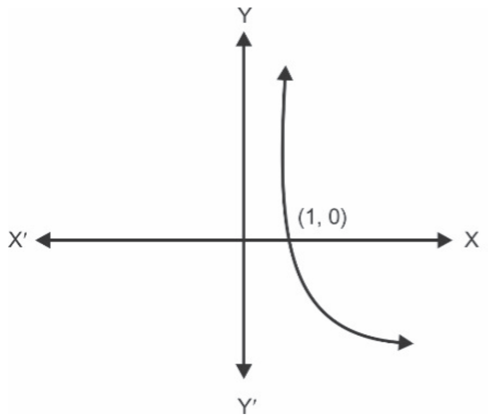
$$f(x) = a^x = \begin{cases} >1 & \text{for } x < 0 \\ =1 & \text{for } x = 0 \\ <1 & \text{for } x > 0 \end{cases}$$

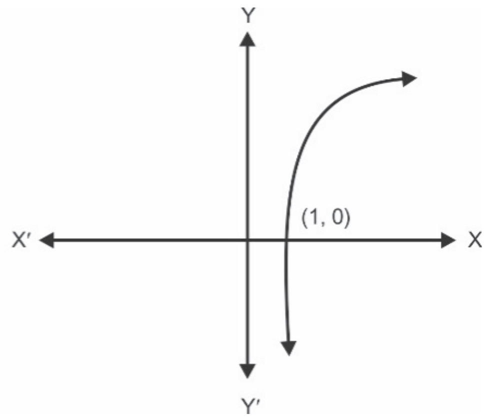
$$f(x) = a^x = \begin{cases} <1 & \text{for } x < 0 \\ =1 & \text{for } x = 0 \\ >1 & \text{for } x > 0 \end{cases}$$

- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots \infty, \quad 2 < e < 3$$

- Logarithmic functions, $f : (0, \infty) \to R$; $f(x) = \log_a x$, $a > 0$, $a \neq 1$





When, $0 < a < 1$
$D_f = (0, \infty)$
$R_f = R$

When, $a > 1$
$D_f = (0, \infty)$
$R_f = R$

- **Natural logarithm function:** $f(x) = \log_e x$ or $\ln(x)$.

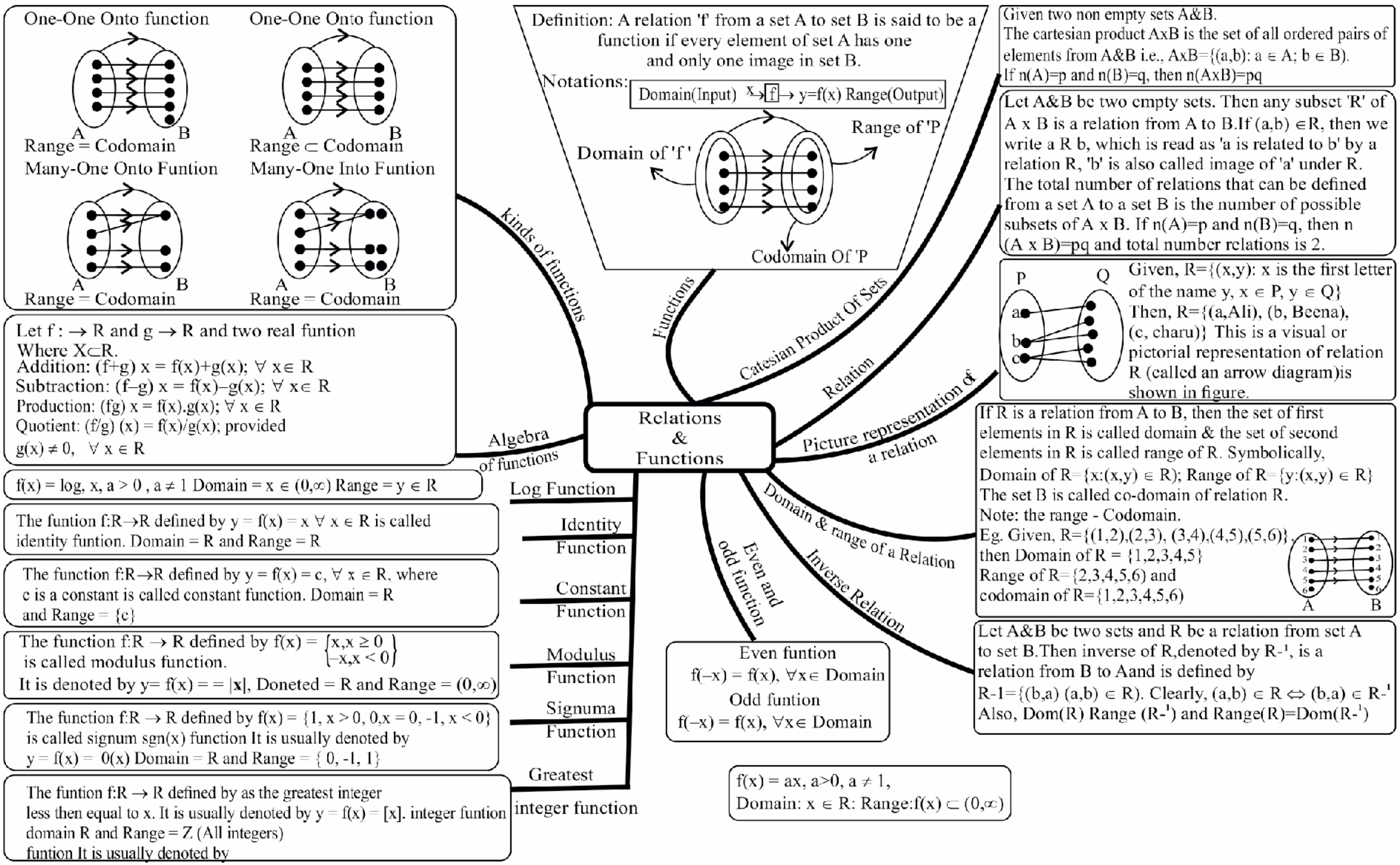
- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $X \subset R$ then

$(f \pm g)(x) = f(x) \pm g(x) \ \forall\ x \in X$

$(fg)(x) = f(x)\, g(x) \ \forall\ x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \forall\ x \in X \text{ provided } g(x) \neq 0$$

MIND MAP



Relations & Functions
Functions
Definition: A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.
Notations: Domain(Input) x → f → y=f(x) Range(Output)
Domain of 'f'
Range of 'P
Codomain Of 'P
kinds of functions
One-One Onto function
A B
Range = Codomain
One-One Onto function
A B
Range ⊂ Codomain
Many-One Onto Funtion
A B
Range = Codomain
Many-One Into Funtion
A B
Range = Codomain
Algebra of functions
Let f : → R and g → R and two real funtion
Where X⊂R.
Addition: (f+g) x = f(x)+g(x); ∀ x∈ R
Subtraction: (f–g) x = f(x)–g(x); ∀ x∈ R
Production: (fg) x = f(x).g(x); ∀ x ∈ R
Quotient: (f/g) (x) = f(x)/g(x); provided
g(x) ≠ 0, ∀ x ∈ R
Log Function
f(x) = log, x, a > 0 , a ≠ 1 Domain = x ∈ (0,∞) Range = y ∈ R
Identity Function
The funtion f:R→R defined by y = f(x) = x ∀ x ∈ R is called identity funtion. Domain = R and Range = R
Constant Function
The function f:R→R defined by y = f(x) = c, ∀ x ∈ R, where c is a constant is called constant function. Domain = R and Range = {c}
Modulus Function
The function f:R → R defined by f(x) = {x,x ≥ 0; –x,x < 0} is called modulus function.
It is denoted by y= f(x) = = |x|, Doneted = R and Range = (0,∞)
Signuma Function
The function f:R → R defined by f(x) = {1, x > 0, 0,x = 0, -1, x < 0} is called signum sgn(x) function It is usually denoted by y = f(x) = 0(x) Domain = R and Range = { 0, -1, 1}
Greatest integer function
The funtion f:R → R defined by as the greatest integer less then equal to x. It is usually denoted by y = f(x) = [x]. integer funtion domain R and Range = Z (All integers)
funtion It is usually denoted by
Catesian Product Of Sets
Given two non empty sets A&B.
The cartesian product AxB is the set of all ordered pairs of elements from A&B i.e., AxB={(a,b): a ∈ A; b ∈ B).
If n(A)=p and n(B)=q, then n(AxB)=pq
Relation
Let A&B be two empty sets. Then any subset 'R' of A x B is a relation from A to B.If (a,b) ∈R, then we write a R b, which is read as 'a is related to b' by a relation R, 'b' is also called image of 'a' under R.
The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A x B. If n(A)=p and n(B)=q, then n (A x B)=pq and total number relations is 2.
Picture representation of a relation
P Q
a b c
Given, R={(x,y): x is the first letter of the name y, x ∈ P, y ∈ Q}
Then, R={(a,Ali), (b, Beena), (c, charu)} This is a visual or pictorial representation of relation R (called an arrow diagram)is shown in figure.
Domain & range of a Relation
If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. Symbolically,
Domain of R={x:(x,y) ∈ R); Range of R={y:(x,y) ∈ R}
The set B is called co-domain of relation R.
Note: the range - Codomain.
Eg. Given, R={(1,2),(2,3), (3,4),(4,5),(5,6)},
then Domain of R = {1,2,3,4,5}
Range of R={2,3,4,5,6) and
codomain of R={1,2,3,4,5,6)
A B
Inverse Relation
Let A&B be two sets and R be a relation from set A to set B.Then inverse of R,denoted by R-¹, is a relation from B to Aand is defined by
R-1={(b,a) (a,b) ∈ R). Clearly, (a,b) ∈ R ⇔ (b,a) ∈ R-¹
Also, Dom(R) Range (R-¹) and Range(R)=Dom(R-¹)
Even and odd function
Even funtion
f(–x) = f(x), ∀x∈ Domain
Odd funtion
f(–x) = f(x), ∀x∈ Domain
f(x) = ax, a>0, a ≠ 1,
Domain: x ∈ R: Range:f(x) ⊂ (0,∞)

Very Short Answer Type Question

1. Find a and b if (a – 1, b + 5) = (2, 3)

   If A = {1,3,5}, B = {2,3}, find : (Question- 2, 3)

2. A × B

3. B × A

   Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4, 5)

4. $A \times (B \cap C)$

5. $A \times (B \cup C)$

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from P to Q

7. If R = {(x,y): x,y $\in$ Z, $x^2 + y^2 = 64$}, then,

   Write R in roster form

   Which of the following relations are functions? Give reason. (Questions 18 to 20)

8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = { (2,1), (2,2), (2,3), (2,4)}

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

SHORT ANSWER TYPE QUESTIONS

11. If A and B are finite sets such that n(A) = 5 and n(B) = 7, then find the number of functions from A to B.

12. If $f(x) = x^2 - 3x + 1$ find $x \in R$ such that f (2x) = f (x)

Let f and g be two real valued functions, defined by, f(x) = x, g(x) = |x|.Find: (Question 13 to 16)

13. f + g

14. f – g

15. fg

16. $\frac{f}{g}$

17. If $f(x) = x^3$, find the value of, $\frac{f(5)-f(1)}{5-1}$

18. Find the domain of the real function, $f(x)=\sqrt{x^2-4}$

19. Find the domain of the function, $f(x)=\frac{x^2+2x+3}{x^2-5x+6}$

Find the range of the following functions. (Question- 20, 21)

20. $f(x)=\frac{1}{4-x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,
$R = \{(x, y): x, y \in Z, xy = 4\}$

Find the range of the following relations: (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R=\left\{\left(x,\frac{1}{x}\right): x\in Z, 0<x<6\right\}$

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

R = {(x, y): x $\in$ A, y $\in$B and y = x^2}

(a) Depict this relation using arrow diagram.
(b) Find domain of R.
(c) Find range of R.
(d) Write co-domain of R.

26. If A = {2,4,6,9}, B = {4,6,18,27,54} and a relation R from A to B is defined by R = {(a,b): a$\in$ A, b$\in$ B, a is a factor of b and a < b}, then write R in Roster form. Also find its domain and range.

27. Find the domain and range of,

f(x) = |2x – 3| – 3

28. Draw the graph of the Constant function f : R $\rightarrow$ R; f(x) = 2 $\forall$ x $\in$ R. Also find its domain and range.

29. Draw the graph of the function |*x* – 2|

Find the domain and range of the following real functions (Question 30-35)

30. $f(x) = \sqrt{x^2 + 4}$

31. $f(x) = \dfrac{x+1}{x-2}$

32. $f(x) = \dfrac{|x+1|}{x+1}$

33. $f(x) = \dfrac{x^2-9}{x-3}$

34. $f(x) = 1 - |x-3|$

35. $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

36. Determine a quadratic function f defined by $f(x) = ax^2 + bx + c$. If $f(0) = 6$; $f(2) = 11$, $f(-3) = 6$

37. Draw the graph of the function $f(x) = \begin{cases} 1+2x & x<0 \\ 3+5x & x \geq 0 \end{cases}$ also find its range.

38. Draw the graph of following function

$$f(x) = \begin{cases} \dfrac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Also find its range.

Find the domain of the following function.

39. $f(x) = \dfrac{1}{\sqrt{x+|x|}}$

40. $f(x) = \dfrac{1}{\sqrt{x-|x|}}$

41. $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

42. $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$

43. Find the domain for which the following functions:
$f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

44. If $f(x) = x - \dfrac{1}{x}$ prove that $\left[f(x)\right]^3 = f(x^3) + 3f\left(\dfrac{1}{x}\right)$.

45. If [x] denotes the greatest integer function. Find the solution set of equation, $[x]^2 + 5[x] + 6 = 0$.

46. If $f(x) = \frac{ax - b}{bx - a} = y$. Find the value of f(y).

Long Answer Type Questions

47. Draw the graph of following function and find range (R_f) of $f(x) = |x - 2| + |2 + x| \quad \forall \quad -3 \leq x \leq 3$.

48. Find domain and range $f(x) = \frac{1}{2\sin 3x}$

CASE STUDY TYPE QUESTIONS

49. To make himself self-dependent and to earn his living, a person decided to setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture.

i. If x units of hand sanitizers are manufactured per month. What is the cost function?

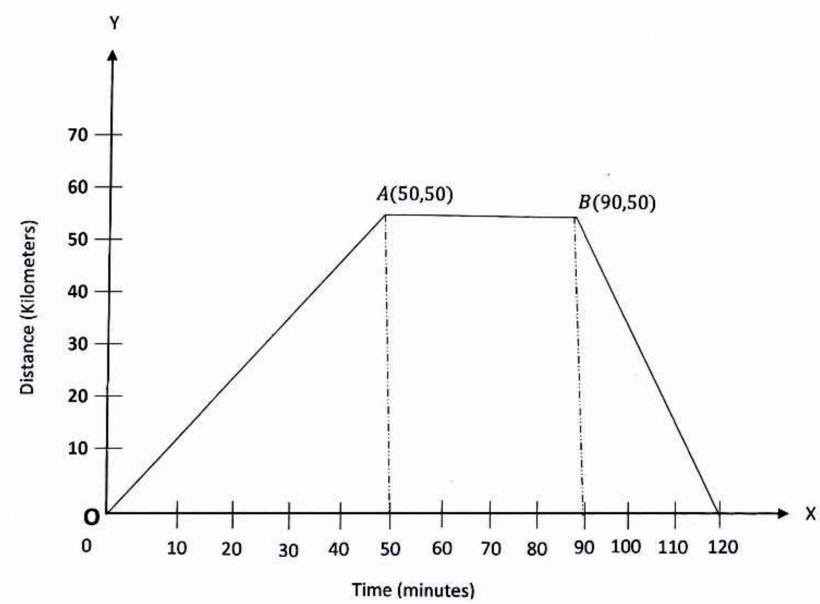
ii. If each unit is sold for Rs. 45. What is the selling (revenue) function?

iii. What is the profit function?

iv. For Break even (No Profit, no loss situation) in a month, how many units should be manufactured and sold?

50. This is a graph showing how far the distances have been travelled by Sunita (in her car) in a given time.

She drove, stopped, does her work and returned back.



i. The line OA of the graph represents the function:

(a) $x=10y$ (b) $y=10x$

(c) $y=x$ (d) $y=\frac{x}{2}$

ii. At what distance from the starting point Sunita stopped to do her work?

(a) 30m | (b) 40m

(c) 50m | (d) 60m

iii. How much time Sunita took to complete her work?

(a) 30 min | (b) 40 min

(c) 50 min | (d) 60 min

iv. Line AB represents the constant function:

(a) y = 50 | (b) x = 50

(c) y = 10 | (d) x = 9

v. How much time Sunita took to reach at a distance of 40 km. from the initial point?

(a) 30 min | (b) 40 min

(c) 50 min | (d) 1 hour

Multiple Choice Questions

51. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5} then (A – B) × (B – C)

(a) {(1, 2), (1, 5), (2, 5)} | (b) {1, 4}

(c) {1, 4} | (d) None of these.

52. If R is a relation on set A = {1, 2, 3, 4, 5, 6, 7, 8} given by xRy $\Leftrightarrow$ y = 3x, then R = ?

(a) {(3, 1), (6, 2), (8, 2), (9, 3)} | (b) {(3, 1), (6, 2), (9, 3)}

(c) {(3, 1), (2, 6), (3, 9)} | (d) None of these.

53. Let A = {1, 2, 3}, B = {4, 6, 9} is relation R from A to B defined by R = {(x, y) x is greater than y}, the range of R is -

(a) {1, 4, 6, 9} | (b) {4, 6, 9}

(c) {1} | (d) None of these.

54. If R be a relation from a set A to a set B then -

(a) R = A $\cup$ B | (b) R = A $\cap$ B

(c) R $\subseteq$ A × B | (d) R $\subseteq$B × A.

55. If $2f(x)-3f\left(\frac{1}{x}\right)=x^2$ $(x \neq 0)$, then f(2) is equal to -

(a) $\frac{-7}{4}$ (b) $\frac{5}{2}$

(c) –1 (d) None of these.

56. Range of the function f(x) = cos[x] for $\frac{-\pi}{2} < x < \frac{\pi}{2}$ is -

(a) {–1, 1, 0} (b) {cos1, cos2, 1}

(c) {cos1, –cos1, 1} (d) {–1, 1}.

57. If f(x) = $\log\left(\frac{1+x}{1-x}\right)$ and g(x) = $\frac{3x+x^3}{1+3x^2}$ then f{g(x)} is equal to -

(a) f(3x) (b) $\{f(x)\}^3$

(c) 3f(x) (d) –f(x).

58. If f(x) = cos(logx) then value of $f(x).f(y)-\frac{1}{2}\left\{f\left(\frac{x}{y}\right)+f(xy)\right\}$ is -

(a) 1 (b) –1

(c) 0 (d) ±1.

59. Doman of f(x) = $\sqrt{4x-x^2}$ is -

(a) R – [0, 4] (b) R – (0, 4)

(c) (0, 4) (d) [0, 4].

60. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function then -

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$

(c) $x \in [2, 3]$ (d) $x \in [2, 4)$.

61. If A = {2, 3, 5}, B = {2, 5, 6}, then (A – B) × (A $\cap$ B) is

(a) {(3, 2), (3, 3), (3, 5)}
(b) {(3, 2), (3, 5), (3, 6)}
(c) {(3, 2), (3, 5)}
(d) None of these

62. The relation R defined on the set of natural numbers as {(a, b), a – b = 3}, is given by

(a) {(1, 4), (2, 5), (3, 6), ...}
(b) {(4, 1), (5, 2), (6, 3), ...}
(c) {(1, 3), (2, 6), (3, 9), ...}
(d) None of these

63. If R = {(x, y)| x, y $\in$ Z, $x^2 + y^2 < 4$} is a relation is Z, then domain of R is

(a) {0, 1, 2}
(b) {0, – 1, – 2}
(c) {–2, –1, 0, 1, 2}
(d) None of these

64. Let n(A) = n. Then the number of all relations on A is

(a) 2^n
(b) $2^{n!}$
(c) 2^{n^2}
(d) None of these

65. If n(A) = 4, n(B) = 3, n(A × B × C) = 24, then n(C) =

(a) 288
(b) 1
(c) 12
(d) 2

66. If $f(x) = \frac{x}{x-1}$, then $\frac{f(a)}{f(a+1)} =$

(a) f(–a)
(b) $f\left(\frac{1}{a}\right)$
(c) $f(a^2)$
(d) $f\left(\frac{-a}{a-1}\right)$

Direction: Each of these questions contains two statements Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes. (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reasoni is correct.

67. Let A = {a, b, c, d, e, f, g, h} and R = {(a, b), (b, b), (a, g), (b, a), (b, g), (g, a), (g, b), (g, g), (b, b)}

Consider the following statements:

Assertion: $R \subset A \times A$.

Reason: R is not a relation on A.

68. Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by {(a, b) : a, b $\in$ A. b is exactly divisible by a}.

Assertion: The relation R in Roster form is {(6, 3), (6, 2), (4, 2)}.

Reason: The Domain and Range of R is {1, 2, 3, 4, 6}

69. **Assertion:** If (x + 1, y – 2) = (3, 1), then x = 2 and y = 3.

Reason: Two ordered pairs are equal, if their corresponding elements are equal.

70. **Assertion:** If $f(x) = \frac{1}{x-2}$, $x \neq 2$ and $g(x) = (x - 2)^2$, then

$(f+g)(x) = \frac{1+(x-2)^3}{x-2}$, $x \neq 2$.

Reason: If f and g are two functions, then their sum is defined by $(f + g)(x) = f(x) + g(x)\ \forall\ x \in D_1 \cap D_2$, where D_1 and D_2 are domains of f and g, respectively.

ANSWERS

1. a = 3, b = –2

2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}

3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}

4. {(1,4), (2,4)}

5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)}

6. $2^6 = 64$

7. R = { (0,8), (0,–8), (8,0), (–8,0)}

8. Not a function because 4 has two images.

9. Not a function because 2 does not have a unique image.

10. Function because every element in the domain has its unique image.

11. 7^5

12. 0,1

13. $f + g = \begin{cases} 2x & x \geq 0 \\ 0 & x < 0 \end{cases}$

14. $f - g = \begin{cases} 0 & x \geq 0 \\ 2x & x < 0 \end{cases}$

15. $fg = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

16. $\frac{f}{g} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$ and Note:- $\frac{f}{g}$ is not defined at x = 0

17. 31

18. $(-\infty, -2] \cup [2, \infty)$

[Hint: Put $x^2 - 4 \geq 0$]

19. R – {2,3}

20. $(-\infty, 0) \cup [1/4, \infty)$

21. $[2,\infty)$

22. {–4, –2, –1,1,2,4}

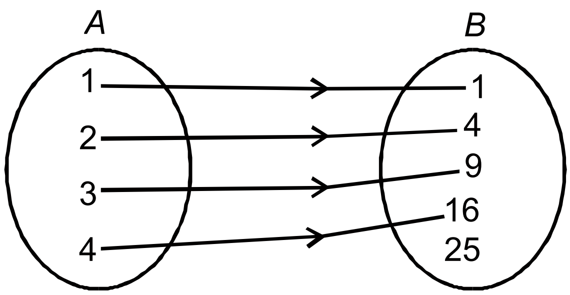
23. {2,4,6,8} [Hint: Use roster Form]

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

23. {2,4,6,8} [Hint: Use roster Form]

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

25. (a)



(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

26. R = { (2,4) (2,6) (2,18) (2,54) (6,18) (6,54) (9,18) (9,27) (9,54) }

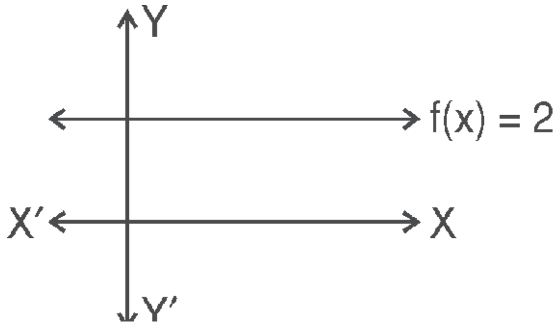
Domain is R = {2,6,9}

Range of R = { 4, 6, 18, 27, 54}

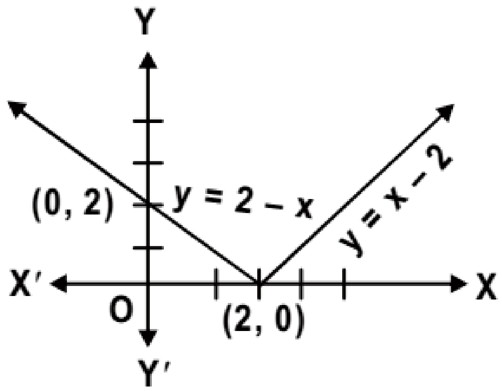
27. Domain is R

Range is $[-3, \infty)$

28. Domain = R,Range = {2}



29. Hint: $|x - 2| = \begin{cases} x-2 : x \geq 2 \\ 2-x : x < 2 \end{cases}$



30. Domain = R,

    Range =[2, ∞)

31. Domain = R – {2}

    Range = R – {1}

32. Domain = R – {–1}

    Range = {1, –1}

33. Domain = R – {3}

    Range = R – {6}

34. Domain = R

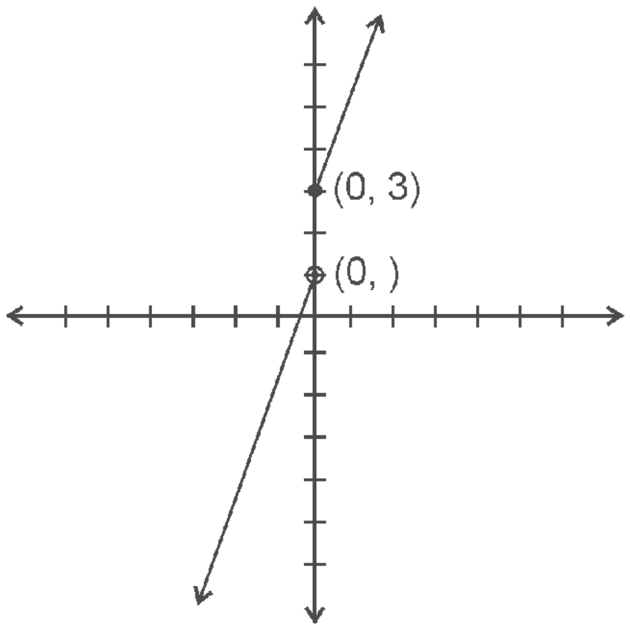
    Range =(–∞, 1]

35. Domain = (–3, 3)

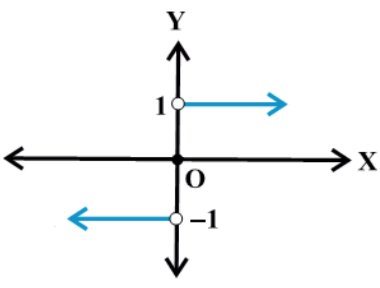
Range = $\left[\frac{1}{3}, \infty\right)$

36. $\frac{1}{2}x^2 + \frac{3}{2}x + 6$

37. $(-\infty, 1) \cup [3, \infty)$



38. Range of f = {–1,0,1}



39. $(0, \infty)$

40. ϕ(given function is not defined)

41. $(-\infty, -2) \cup (4, \infty)$

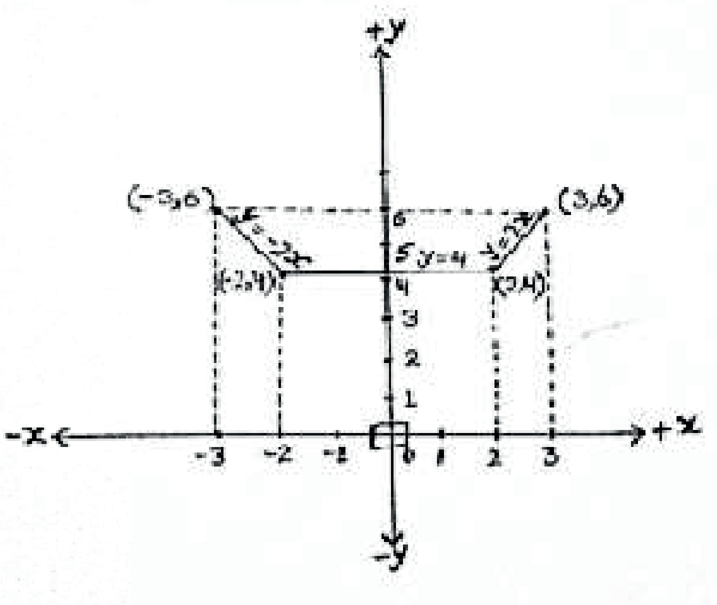
42. $(-\infty, -1) \cup (1, 4]$

43. $\left\{-2, \frac{1}{2}\right\}$

45. [–3,–1)

46. x

47. R_f = [4, 6] and graph is



48. Domain = R
Range = [1/3, 1]

49. i. 15000 + 30x ii. 45x iii. 15(x – 1000) iv. 1000

50. i. (c) ii. (c) iii. (b) iv. (a) v. (b)

51. (b) 52. (d) 53. (c)

54. (c) 55. (a) 56. (b)

57. (c) 58. (c) 59. (d)

60. (d) 61. (c) 62. (b)

63. (c) 64. (c) 65. (d)

66. (c) 67. (c) 68. (d)

69. (a) 70. (a)

CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- 1 radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.
- π radian=180 degree

  $$1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ} = 57^{\circ}\ 16'\ 22'' \text{ (Appr.)}$$

- If an arc of length 'ℓ' makes an angle 'θ' radian at the centre of a circle of radius 'r', we have $\theta = \frac{\ell}{r}$.
- I degree is $\left(\frac{1}{360}\right)^{th}$ part of a circle. One degree is further divided into 60 parts called minutes and one minute is further divided into 60 parts called seconds.
- $360°$ = one complete revolution
- $1° = 60'$ (minutes)
- $1' = 60''$(second)

Quadrant →	I	II	III	IV
t-functions which are postive	All	sinx cosecx	tanx cotx	cosx secx

-

Function	Domain	Range
Sinx	R	[–1, 1]
Cosx	R	[–1, 1]
Tanx	$R - \left\{(2n+1)\frac{\pi}{2}\right\}; \quad n \in Z$	R
cosecx	$R - \{n\pi\}; n \in Z$	R – (–1, 1)
Secx	$R - \left\{(2n+1)\frac{\pi}{2}\right\}; \quad n \in Z$	R – (–1, 1)
Cotx	$R - \{n\pi\}; n \in Z$	R

- **Trigonometric Identities:**

(i) sin (x + y) = sinx cosy+cosxsiny

(ii) sin (x – y) = sinx cosy–cosxsiny

(iii) cos (x + y) = cosxcosy–sinxsiny

(iv) cos (x – y) = cosx cosy + sinxsiny

(v) $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x . \tan y}$

(vi) $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x . \tan y}$

(vii) $\cot(x + y) = \dfrac{\cot x . \cot y - 1}{\cot y + \cot x}$

(viii) $\cot(x - y) = \dfrac{\cot x . \cot y + 1}{\cot y - \cot x}$

(ix) $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$

(x) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

(xi) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

(xii) $\sin 3x = 3\sin x - 4\sin^3 x$

(xiii) $\cos 3x = 4\cos^3 x - 3\cos x$

(xiv) $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

(xv) $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

(xvi) $\cos x - \cos y = 2\sin\dfrac{x+y}{2}\sin\dfrac{y-x}{2} = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

(xvii) $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

(xviii) $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

(xix) $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

(xx) $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

(xxi) $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

(xxii) $2\sin x \sin y = \cos(x-y) - \cos(x+y)$

(xxiii) $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

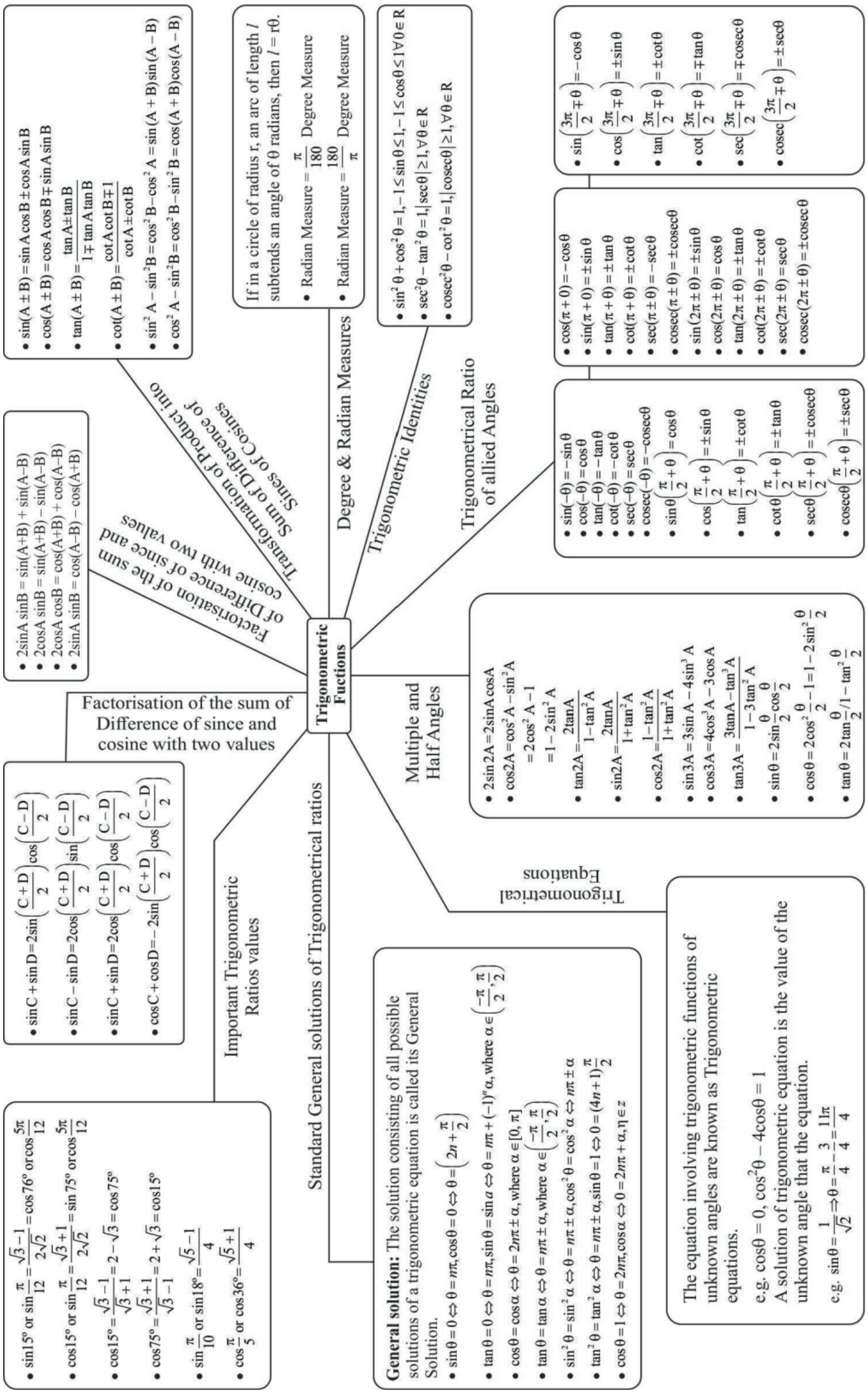
(xxiv) $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

(xxv) $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

sign '+' or '–' will be decided according to the quadrant in which angle lies.

- Maximum and minimum values of the expression Acosθ + Bsinθ are $\sqrt{A^2+B^2}$ and $-\sqrt{A^2+B^2}$ respectively, where A and B are constants.

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. Write the radian measure of 5° 37' 30".
2. Write the degree measure of $\frac{11}{16}$ radian.
3. Write the value of $\tan\left(\frac{19\pi}{3}\right)$.
4. What is the value of sin (–1125°).
5. Write the value of 2sin75° sin15°.
6. What is the maximum value of 3 – 7cos5x.
7. Express sin12θ + sin4θ as the product of sines and cosines.
8. Express 2cos4x sin2x as an algebraic sum of sines and cosines.
9. Write the maximum value of cos(cosx) and also write its minimum value.
10. Write is the value of $\tan\frac{\pi}{12}$.

SHORT ANSWER TYPE QUESTIONS

11. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°.
12. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$ Find cos A, sin $2A$.
13. What is the sign of cos x/2 – sin x/2 when

    (i) $0 < x < \pi/4$ (ii) $\frac{\pi}{2} < x < \pi$
14. Prove that cos 510° cos 330° + sin 390° cos 120° = –1.
15. Find the maximum and minimum value of 7 cos x + 24 sin x.
16. Evaluate $\sin(\pi + x)\sin(\pi - x)\,\text{cosec}^2 x$.
17. Find the angle in radians between the hands of a clock at 7 : 20 PM.

18. If $\cot\alpha = \frac{1}{2}$, $\sec\beta = \frac{-5}{3}$ where $\pi<\alpha< 3\pi/2$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of $\tan(\alpha + \beta)$.

19. If $\cos x = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$. Find the value of $\cos x/2$, $\tan x/2$

20. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$

21. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

22. Find the minimum and maximum value of $\sin^4 x + \cos^2 x$; $x \in R$

23. Find x if $\tan(x - 15°) = \tan(x + 15°)$

24. If $\sec x = \sqrt{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find the value of

$$\frac{1 - \tan x - \operatorname{cosec} x}{1 - \cot x - \operatorname{cosec} x}$$

25. If $f(x) = \frac{\cot x}{1 + \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$ then find $f(\alpha).f(\beta)$.

26. Prove that $\tan 70° = \tan 20° + 2\tan 50°$

27. Prove that $\tan 13x = \tan 4x + \tan 9x + \tan 4x \tan 9x \tan 13x$.

[Hint: $13x = 9x + 4x$]

Prove the following Identities

28. $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta.\cos 4\theta$.[Hint: Break into sin and cos]

29. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$.

30. $\dfrac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x . \sin x + \cos 6x . \cos x} = \tan 2x$.

[Hint: Transformation formula form product to sum or different]

31. $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$.[Hint: Use half angle formula]

32. $\tan\alpha.\ \tan(60° - \alpha).\ \tan(60° + \alpha) = \tan 3\alpha.$

33. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$.[Hint: Use half angle formula]

34. $\dfrac{\cos x}{1 - \sin x} = \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$.

35. $\cos 10° + \cos 110° + \cos 130° = 0.$

36. $\dfrac{\sin(x + y) - 2\sin x + \sin(x - y)}{\cos(x + y) - 2\cos x + \cos(x - y)} = \tan x$

37. $\sin x + \sin 2x + \sin 4x + \sin 5x = 4\cos\dfrac{x}{2}.\cos\dfrac{3x}{2}.\sin 3x$

[Hint: Use transformation formula from sum of product.]

38. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

39. Find the value of $\sqrt{3}$ cosec 20° – sec 20°

40. Draw the graph of cos$x$, sin$x$ and tan$x$ in $[0, 2\pi]$.

41. Draw sin$x$, sin$2x$ and sin$3x$ on same graph and with same scale.

42. Evaluate: $\tan\left(\dfrac{13\pi}{12}\right)$

43. If $\tan A - \tan B = x$, $\cot B - \cot A = y$ prove that $\cot(A - B) = \frac{1}{x} + \frac{1}{y}$

44. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$ then prove that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

45. Find the range of $5 \sin x - 12 \cos x + 7$.

46. Show that $\cos^2 + \cos^2\left(x + \frac{2\pi}{3}\right) + \cos^2\left(x - \frac{2\pi}{3}\right) = \frac{3}{2}$

[Hint: Use $\cos 2\theta = 2\cos^2\theta - 1$]

47. Show that $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$

[Hint: Use transformation formula sum to product]

Long Answer Type Questions

48. Find $\cos \pi/8$

49. Prove that $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$.

50. $\cos\frac{\pi}{5}.\cos\frac{2\pi}{5}.\cos\frac{4\pi}{5}.\cos\frac{8\pi}{5} = \frac{1}{16}$

51. $\cos 20°.\cos 40°.\cos 80° = \frac{1}{8}$

52. Evaluate: $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$

[Hint: Use $\cos 2\theta = 2\cos^2\theta - 1$]

53. If $\cos x = \cos\alpha.\cos\beta$ then prove that $\tan\left(\frac{x+\alpha}{2}\right).\tan\left(\frac{x-\alpha}{2}\right) = \tan^2\frac{\beta}{2}$

54. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ then prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$.

55. If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$ then prove that

$$\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) = 1 - 2a^2 - 2b^2$$

56. If α and β are the solution of the equation, $a\tan\theta + b\sec\theta = c$, then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2 - c^2}$.

57. Prove that

$$\cos^2 x + \cos^2 y - 2\cos x.\cos y.\cos(x+y) = \sin^2(x+y)$$

58. Prove that :

$$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$$

59. Prove that : $\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16.\sin A}$.

[Hint: Use transformation formula]

60. Evaluate: $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

61. Prove that : $4\sin\alpha.\sin\left(\alpha+\frac{\pi}{3}\right).\sin\left(\alpha+\frac{2\pi}{3}\right) = \sin 3\alpha$.

[Hint: Use transformation formula of product to sum or diff.]

62. If sin A + sin B = p, cosA + cosB = q show that

(i) $\sin(A+B) = \frac{2pq}{p^2+q^2}$

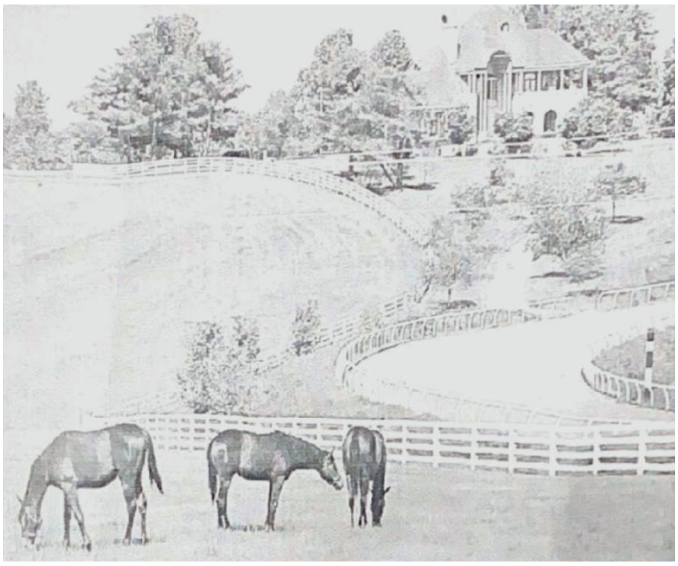
(ii) $\cos(A+B) = \frac{p^2-q^2}{p^2+q^2}$

(iii) $\tan(A+B) = \frac{p^2+q^2}{p^2-q^2}$

63. Show that $\sin^3 x+\sin^3\left(\frac{2\pi}{3}+x\right)+\sin^3\left(\frac{4\pi}{3}+x\right)=\frac{-3}{4}\sin 3x$

CASE STUDY TYPE QUESTIONS

64. After retirement, Mr. D. N. Sharma purchased a farm house in shape of quadrilateral ABCD with ∠A = 90°, ∠B = 72°, ∠C = 108° and ∠D = 90°. He also purchased a horse and cow. One day, he tied the horse with a rope at vertex B and oserved that it describes an arc of length 88 m when it moves along a circular path keeping the rope tight.



Based on above information answer the following :-

i. What is radian measure of ∠B ?

ii. What is length of rope?

iii. What will be the length of arc described by horse if he doubles the rope length?

iv. What will be the length of arc described by cow if it is tied at vertex c with the rope of same length as horse?

65. While playing with this nephew Shashank, Mr. V.S. Malik observes a vertical pole in park. A wire is tied from top of pole to a point on ground level. Mr. Malik asks Shashank some

mathematics related questions. Mr. Shashank is Class-XI student and very intelligent in Maths. Using some tools he measure the distance of point at ground where wire is tied as 10 m. and angle between wire and ground level as 75°.

Based on above information answer the following :-

i. What is the value of tan75° ?

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(c) $\frac{\sqrt{3}}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}}{\sqrt{3}-1}$

ii. What is the height of pole?

(a) $10\left(\sqrt{3}+1\right)$ (b) $10\left(\sqrt{3}-1\right)$

(c) $10\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $10\frac{\sqrt{3}-1}{\sqrt{3}+1}$

iii. What is the value of sin75° ?

(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

iv. What is the length of wire?

(a) $10\sqrt{2}\left(\sqrt{3}+1\right)$ (b) $10\left(\sqrt{3}+1\right)$

(c) $10\sqrt{2}\left(\sqrt{3}-1\right)$ (d) $10\left(\sqrt{3}-1\right)$

iii. What is the value of sin105° ?

(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

66. The greatest value of sinx cosx is -

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) 1/2

67. The value of tan1° × tan2° × tan3° ……..… tan89° is -

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) Not defined.

68. The value of cos1° × cos2° × cos3° ……..… cos179° is -

(a) $\frac{1}{\sqrt{2}}$ (b) 0 (c)1 (d) –1.

69. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is -

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2.

70. The value of sin50° – sin70° + sin10° is equal to -

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) 2.

71. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to -

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) 2.

72. If $\alpha + \beta = \frac{\pi}{4}$, then value of $(1 + \tan\alpha) . (1 + \tan\beta)$ is -

(a) 1 (b) 2

(c) –2 (d) Not defined.

73. If $\cos x = \frac{1}{2}\left(a + \frac{1}{a}\right)$,then $\cos 3x$ is -

(a) $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ (b) $\frac{3}{2}\left(a^3 + \frac{1}{a^3}\right)$

(c) $\frac{1}{2}\left(a^3 - \frac{1}{a^3}\right)$ (d) $\frac{3}{2}\left(a^3 - \frac{1}{a^3}\right)$.

74. If $P = 2\sin^2 x - \cos^2 x$, then P lies in the interval -

(a) [1, 3] (b) [1, 2]

(c) [–1, 2] (d) None of these.

75. If $\frac{\pi}{4} < x < \frac{\pi}{2}$, then write the value of $\sqrt{1 - \sin 2x}$ is -

(a) cosx – sinx (b) cosx + sinx

(c) sinx – cosx (d) 2.

76. If $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$, then θ lies in which quadrant

(a) First (b) Second

(c) Third (d) Fourth

77. If sin $(\alpha - \beta) = \frac{1}{2}$ and $\cos(\alpha + \beta) = \frac{1}{2}$, where, α and β are positive acute angle, then

(a) α = 45°, β = 15° (b) α = 15°, β = 45°

(c) α = 60°, β = 15° (d) None of these

78. If tan $\theta = -\frac{1}{\sqrt{10}}$ and θ lies in the fourth quadrant, then cos θ =

(a) $1/\sqrt{11}$ (b) $-1/\sqrt{11}$

(c) $\sqrt{\frac{10}{11}}$ (d) $-\sqrt{\frac{10}{11}}$

79. tan 15° =

(a) $\frac{1}{3}$ (b) $\sqrt{3} - 2$

(c) $2 - \sqrt{3}$ (d) None of these

80. If $\sin \alpha = \frac{-3}{5}$, where, $\pi < a < \frac{3\pi}{2}$, then $\cos \frac{1}{2} \alpha =$

(a) $\frac{-1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{10}}$

(c) $\frac{3}{\sqrt{10}}$ (d) $\frac{-3}{\sqrt{10}}$

81. Which of the following number(s) is/are rational

(a) sin 15° (b) cos 15°

(c) sin 15° cos 15° (d) sin 15° cos 75°

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

82. **Assertion:** $\dfrac{\cos(\pi + x).\cos(-x)}{\sin(\pi - x).\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$

Reason: cos (π + θ) = – cos θ and cos (–θ) = cos θ. Also sin (π – θ) = sin θ and sin (– θ) = – sin θ.

83. **Assertion:** coses *x* is negative in third and fourth quadrants.

Reason: cot *x* decreases from 0 to – ∞ in first quadrant and increases from 0 to ∞ in third quadrant.

84. **Assertion:**The degree measure corresponding to (–2) radian is – 114° 19 min.

Reason: The degree measure of a given radian measure $= \dfrac{180}{\pi}$ × Radian measure.

85. **Assertion:** The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of 30° and 70° is 21 : 10.

Reason: Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

ANSWERS

1. $\frac{\pi}{32}$

2. 39° 22' 30"

3. $\sqrt{3}$

4. $-\frac{1}{\sqrt{2}}$

5. $\frac{1}{2}$

6. 10

7. $2 \sin 8\theta \cos 4\theta$

8. $\sin 6x - \sin 2x$

9. 1 and –1

10. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

11. 70m

12. $\frac{-4}{5}, \frac{-24}{25}$ [Hint: For f(x) = a sin θ + b cos θ Max value = $\sqrt{a^2+b^2}$ Min value = – $\sqrt{a^2+b^2}$]

13. (i) +ve (ii) –ve

15. Max value = 25

Min value = –25

[Hint: For f(x) = a sin q + b cos q Max value = $\sqrt{a^2+b^2}$ Min value = $\sqrt{a^2+b^2}$]

16. –1

17. $\frac{5\pi}{9}$

18. $\frac{2}{11}$

19. $-1/\sqrt{3}, -2$

20. $\pi/4$

21. 70 m

22. $\min = \frac{3}{4}, \max = 1$

23.30° [Hint: Break into sin and cos and use sin(A – B)]

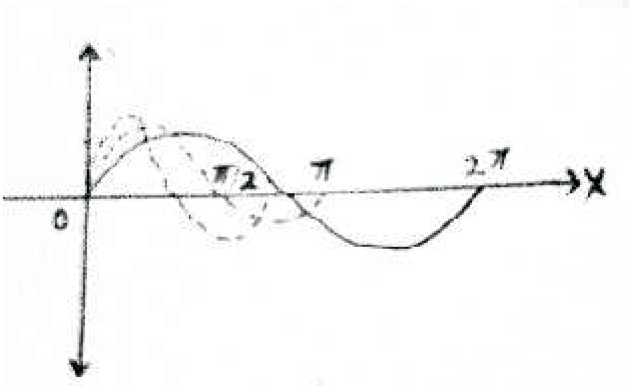
24.1 [Hint: Break into sum and cos and rationalise]

25. $\frac{1}{2}$

39. 4

40.

41.



42.(i) $2-\sqrt{3}$

52. $\frac{3}{2}$

45.[–6, 20]

48. $=\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

60. $\frac{1}{8}$

64. i. 2π/5 ii. 70 m iii. 176 m iv. 144 m

65. i. (b) ii. (c) iii. (c) iv. (a) v. (c)

66. (d) 67. (c) 68. (c)

69. (c)	70. (b)	72. (c)
73. (a)	74. (c)	75. (c)
76. (c)	77. (a)	78. (c)
79. (c)	80. (a)	81. (c)
82. (a)	83. (c)	84. (d)
85. (d)		

CHAPTER - 4

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $i^2 = -1$; $i^4 = 1$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, if atleast one number is positive.
- A number of the form z = a + ib, where a, b $\in$ R is called a complex number.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- a + ib = c + id $\Leftrightarrow$ a = c, and b = d
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$ but if b, d = 0 and a > c then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- 0 + *i* 0 is additive identity of a complex number.
- –z = –a –ib is called the Additive Inverse or negative of z = a + ib

- 1 + i 0 is multiplicative identity of complex number.

- $z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\overline{z}}{|z|^2}$ is called the multiplicative Inverse of

  z = a + ib (a ≠ 0, b ≠ 0)

- $|z| = \sqrt{a^2 + b^2}$ is called modulus of *z* = *a* + *ib*

- $\overline{z} = a - ib$ is called conjugate of *z* = *a* + *ib*

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane

- $|z_1 + z_2| \leq |z_1| + |z_2|$; $|z_1 - z_2| \geq |z_1| - |z_2|$

- $|z_1 z_2| = |z_1|.\ |z_2|$; $\left|\frac{z_1}{z_2}\right| = \frac{|Z_1|}{|Z_2|}$

- $|z^n| = |z|^n$; $|z| = |\overline{z}| = |-z| = |-\overline{z}|$

- $(\overline{z_1 \pm z_2}) = \overline{z}_1 \pm \overline{z}_2$; $\overline{\left(\frac{Z_1}{Z_2}\right)} = \frac{\overline{Z}_1}{\overline{Z}_2}$

- $(\overline{z^n}) = (\overline{z})^n$

- $z\ .\ \overline{z} = |z|^2$

- For the quadratic equation $ax^2 + bx + c = 0$, a, b, c ∈ R, a ≠ 0,if $b^2 - 4ac < 0$ then it will have complex roots given by,

  $$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

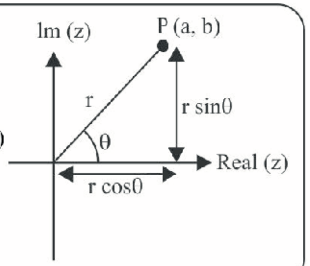
W. R. Hamilton
(1805-1865)

MIND MAP

Trigonometric Fuctions

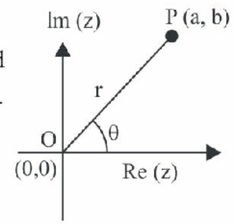
Polar Representation

Let $a = r\cos\theta$
$b = r\sin\theta$
where,
$and\ \theta = \arg(z)$
$\therefore z = a + ib = r(\cos\theta + i\sin\theta)$
The arument 'θ' of complex number $z_1 = a + ib$ is called principal argument of z if $-\pi < \theta \le \pi$.



Modulus & Argument of Complex Number

If $z = a + ib$ is a complex number
(i) Distance of z from origin is called as modulus of complex number z.
It is denoted by $r = |z| = \sqrt{a^2 + b^2}$
(*ii*) Angle θ made by OP with +ve direction of X-axis is called argument of z.

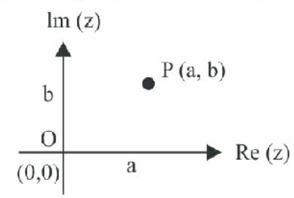


Powers of 'i'

$i = \sqrt{-1},\ i^2 = -1$
In general, $i^{4l+r} = \begin{cases} 1; r = 0 \\ i; r = 1 \\ -1; r = 2 \\ -i; r = 3 \end{cases}$

Argand Plane

A complex number $z = a + ib$ can be represented by a unique point $P(a, b)$ in the argand plane



$z = a + ib$ is represented by a poin $P(a, b)$

Square root of Complex Number

Let $x + iy = \sqrt{a + ib}$, squaring both side, we get $(x + iy)^2 = a + ib$ *i.e.* $x^2 - y^2 = a, 2xy = b$ solving these equations, we get square root of z.

Multiplicative Inverse of complex number

For a non-zero complex number $z = a + ib (a \ne 0, b \ne 0)$, there exists a complex number $\frac{a}{a^2 + b^2} + \frac{-b}{a^2 + b^2} i$ denoted by $\frac{1}{z}$ or z^{-1}, called multiplicative inverse of Z
Such that: $(a + ib)\left(\frac{a}{a^2 + b^2} + i\frac{-b}{a^2 + b^2} = 1 + 0i = 1\right)$

Solution of Quadratic Equation

General form of quadratic equation in x is $ax^2 + bx + c = 0$,
whered $a, b, c \in R$ and $a \ne 0$
The solutions of given quadratic equation are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\because b^2 - 4ac < 0$
Note: • A polynomial equation has atleast one root.
• A polynomial equation of degree n has n roots.

Definition of complex numbers

A number of the form $a + ib$, where $a, b \in R$ and $i = \sqrt{-1}$ is called a comples number and denoted by 'i'
$z = a + ib \rightarrow$ Imaginary part
↓
Real part
Conjugate of a complex number : For a given complex number $r = a + ib$, its conjugate is defined as $\bar{z} = a - ib$

Algebra of complex Numbers

Let: $z_1 = a + ib$ and $z_1 = c + id$ be two complex numbers, where $a, b, c, d \in R$ and $i = \sqrt{-1}$
1. Addition: $z_1 + z_2 = (a + ib) + (c + id) = (a + c) + i(b + d)$
2. Subtraction: $z_1 - z_2 = (a + ib) - (c + id) = (a - c) + (b - d)i$
3. Multiplication: $z_1 . z_2 = (a + ib)(c + id)$
$= a(c + id) + ib(c + id)$
$= (ac - bd) + (ad + bc)i \ (\because i^2 = -1)$
4. Division: $\frac{z_1}{z_1} = \frac{a + ib}{c + id} \cdot \frac{c - id}{c - id}$
$= \left(\frac{ac + bd}{c^2 + d^2}\right) + \left(\frac{bc - ad}{c^2 + d^2}\right)i$
Note : If $a + ib = c + id$
$\Rightarrow d = c \ \& \ d = d$

VERY SHORT ANSWER TYPE QUESTIONS

1. Write the value of $i + i^{10} + i^{20} + i^{30}$

2. Write the additive Inverse of $6i - i\sqrt{-49}$

3. Write the multiplicative Inverse of $1 + 4\sqrt{3}\, i$

4. Write the conjugate of $\dfrac{2-i}{(1-2i)^2}$

5. Write in the form of $a + ib$: $\dfrac{1}{-2+\sqrt{-3}}$

6. Multiply $2 - 3\,i$ by its conjugate.

7. What is the least integral value of k which makes the roots of the equation $x^2 + 5x + k = 0$ imaginary?

8. Find the real value of 'a' for which $3i^3 - 2ai^2 + (1-a)i$ is real.

9. Find the value of $\left(-\sqrt{-1}\right)^{4n-3}$, when n $\in$ N.

10. If a complex number lies in the third quadrant, then find the quadrant of it's conjugate.

11. Find the value of $\sqrt{-25} \times \sqrt{-9}$

12. Evaluate :

    (i) $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

    (ii) $i\sqrt{-16} + i\sqrt{-25} + \sqrt{49} - i\sqrt{-49} + 14$

    (iii) $\left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3$

    (iv) $\dfrac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

13. Find x and y if (x + iy) (2 – 3i) = 4 + i.

14. If n is any positive integer, write value of $\frac{i^{4n+1} - i^{4n-1}}{2}$

15. If z_1 = √2(cos 30° + i sin 30°), z_2= √3(cos 60 + i sin 30°)

Find Re ($z_1 z_2$)

16. If $|z+4| \leq 3$ then find the greatest and least values of $|z+1|$.

17. Find the real value of a for which $3i^3 - 2ai^2 + (1-a)i + 5$ is real.

SHORT ANSWER TYPE QUESTIONS

18. If x + iy = $\sqrt{\frac{1+i}{1-i}}$ prove that $x^2 + y^2 = 1$

19. Find real value of θ such that, $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number.

20. If $\left|\frac{z-5i}{z+5i}\right| = 1$ show that z is a real number.

21. Find real value of x and y if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$.

22. If $(1+i)(1+2i)(1+3i).......(1+ni) = x+iy$.

Show, $2.5.10..........(1+n^2) = x^2 + y^2$

23. If z = 2 – 3i show that $z^2 - 4z + 13 = 0$, hence find the value of $4z^3 - 3z^2 + 169$.

24. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = a + ib$, find a and b.

25. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$.

26. If $\left(\frac{2+2i}{2-2i}\right)^n = 1$, find the least positive integral value of n

27. If $(x+iy)^{\frac{1}{3}} = a+ib$ prove $\left(\frac{x}{a}+\frac{y}{b}\right) = 4\left(a^2-b^2\right)$.

28. Solve

(i) $x^2 - \left(3\sqrt{2}-2i\right)x - 6\sqrt{2}i = 0$ (ii) $ix^2 - 4x - 4i = 0$

29. Solve $|z + 1| = z + 2(1 + i)$

30. If $|z^2 - 1| = |z|^2 + 1$, then show that z lies on imaginary axis.

[Hint: Take $z = x + iy$]

31. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represent a circle find its centre and radius.

32. Find all non-zero complex number z satisfying $\overline{z} = iz^2$.

33. If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$.

34. If z_1, z_2 are complex numbers such that, $\frac{2z_1}{3z_2}$ is purely imaginary number then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$.

35. If z_1 and z_2 are complex numbers such that,

$\left|1-\overline{z_1}z_2\right|^2 - \left|z_1 - z_{2^2}\right|^2 = k\left(1-|z_1|^2\right)\left(1-|z_2|^2\right)$. Find value of k.

LONG ANSWER TYPE QUESTIONS

36. Find number of solutions of $z^2 + |z|^2 = 0$.

37. If z_1, z_2 are complex numbers such that $\left|\dfrac{z_1 - 3z_2}{3 - z_1.\bar{z}_2}\right| = 1$ *and* $|z_2| \neq 1$ then find $|z_1|$.

38. Evaluate $x^4 - 4x^3 + 4x^2 + 8x + 44$, When $x = 3 + 2i$

39. If $z = x + iy$ and $w = \dfrac{1-iz}{z-i}$ show that if $|w| = 1$ then z is purely real.

40. If $\left(\dfrac{1+i}{1+2^2 i}\right) \times \left(\dfrac{1+3^2 i}{1+4^2 i}\right) \times .... \times \left(\dfrac{1+(2n-1)^2 i}{1+(2n)^2 i}\right) = \dfrac{a+ib}{c+id}$ then show that $\dfrac{2}{17} \times \dfrac{82}{257} \times .... \times \dfrac{1+(2n-1)^4}{1+(2n)^4} = \dfrac{a^2+b^2}{c^2+d^2}$.

41. Find the values of x and y for which complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate to each other.

42. Show that the complex number z_1, z_2 and z_3 satisfying $\dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{1 - i\sqrt{3}}{2}$ are the vertices of a equilateral triangle.

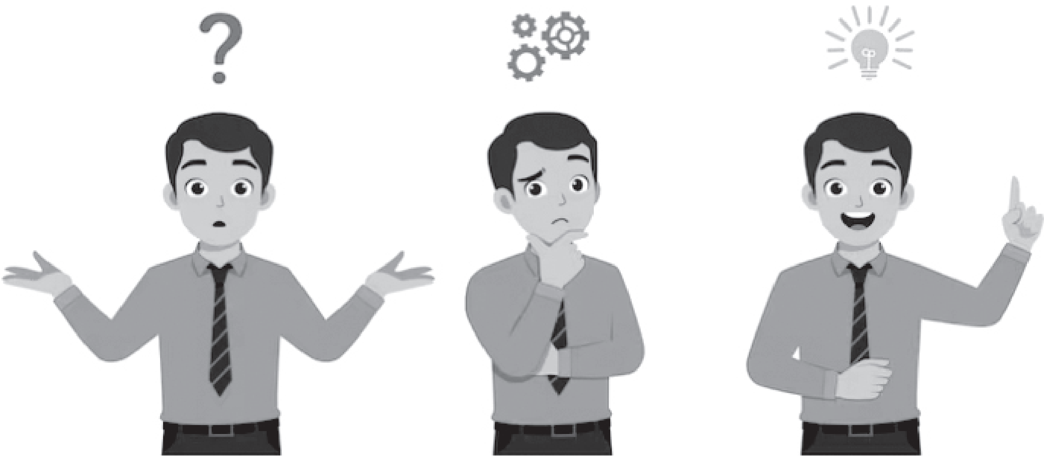
43. If $f(z) = \dfrac{7-z}{1-z^2}$ where $z = 1 + 2i$ then show that $|f(z)| = \dfrac{|z|}{2}$.

44. If z_1, z_2, z_3 are complex numbers such that

$$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$ then find the value of $|z_1 + z_2 + z_3|$

CASE STUDY TYPE QUESTIONS

45. While solving a typical equation a person finds that one of the root of the equation is a complex number $z = \frac{1+2i}{1-3i}$, help him to find



i. The standard form of z

(a) $-\frac{1}{2}+\frac{i}{2}$ (b) $\frac{1}{2}-\frac{i}{2}$ (c) $-\frac{1}{2}-\frac{i}{2}$ (d) $\frac{1}{2}+\frac{i}{2}$

ii. If $z = 2x+(4-y)i$, then

(a) $x=\frac{1}{4},\ y=\frac{7}{2}$ (b) $x=-\frac{1}{4},\ y=\frac{7}{2}$

(c) $x=\frac{1}{4},\ y=-\frac{7}{2}$ (d) $x=-\frac{1}{4},\ y=-\frac{7}{2}$

iii. Conjugate of Z is

(a) $\frac{1-2i}{1-3i}$ (b) $\frac{1+2i}{1+3i}$ (c) $\frac{1+2i}{1-3i}$ (d) $\frac{1-2i}{1+3i}$

iv. The modulus of z is

(a) 1/3 (b) 1/2 (c) $1/\sqrt{2}$ (d) $1/\sqrt{3}$

v. z lies in

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

Multiple Choice Questions

46. $(\sqrt{-2})(\sqrt{3})$ is equal to

(a) $\sqrt{6}$ (b) $-\sqrt{6}$

(c) $i\sqrt{6}$ (d) None of these

47. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, $x^2 + y^2$ is equal to

(a) $\frac{(a^2+1)^4}{4a^2+1}$ (b) $\frac{(a+1)^2}{4a^2+1}$

(c) $\frac{(a^2-1)^2}{(4a^2-1)^2}$ (d) None of these

48. If z = $\frac{1}{1-\cos\theta - i\sin\theta}$, then Re(z) =

(a) 0 (b) ½

(c) cot θ/2 (d) ½ cot θ /2

49. If f(z) = $\frac{7-z}{1-z^2}$, where z = 1 + 2i, then |f(z)| is

(a) $\frac{|z|}{2}$ (b) |z|

(c) 2|z| (d) None of these

50. The value of $(1+i)^4 + (1-i)^4$ is

(a) 8 (b) 4

(c) – 8 (d) – 4

51. The equation $|z+1-i|=|z-1+i|$ represent a

(a) Straight line (b) Circle

(c) Parabola (d) Hyperbola

52. The value of $\frac{i^{4n+1}-i^{4n-1}}{2}$ is

(a) 2i (b) –2i

(c) i (d) –i

53. If three complex number z_1, z_2 and z_3 are in A.P, then points representing them lies on

(a) Circle (b) Parabola

(c) Hyperbola (d) Straight line

54. The sum of series $i+i^2+i^3+...$ up to 1000 terms is

(a) 0 (b) *i*

(c) –*i* (d) None of these

55. If $z_1=\sqrt{3}+i\sqrt{3}, z_2=\sqrt{3}+i$, then the point $\frac{z_1}{z_2}$ lies in

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

56. If $i=\sqrt{-1}$ then $1 + i^2 + i^3 - i^6 + i^8$ is equal to

(a) 2 – *i* (b) 1

(c) 3 (d) – 1

57. The complex number $\frac{1+2i}{1-i}$ lies in which of the complex plane

(a) First (b) Second

(c) Third (d) Fourth

58. If $\frac{c+i}{c-i} = a + ib$ where a, b, c are real, than $a^2 + b^2 =$

(a) 1 (b) – 1

(c) c^2 (d) $-c^2$

59. If the conjugate of (x + iy) (1 – 2i) be 1 + i, then

(a) $x = \frac{1}{5}$ (b) $y = \frac{3}{5}$

(c) $x + iy = \frac{1-i}{1-2i}$ (d) $x - iy = \frac{1-i}{1+2i}$

60. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) An ellipse (b) The imaginary axis

(c) A circle (d) The rea axis

61. If z = 1 + i, then the multiplicative inverse of z^2 is (where $i = \sqrt{-1}$)

(a) 2 i (b) 1 – i

(c) – i/2 (d) i/2

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct, reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct, reason is not a correct explanation for assertion.

(c) Assertin is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

62. **Assertion:** If$|z_1 + z_2|^2 = |z_1|^2$ |+ $|z_2|^2$, then $\frac{z_1}{z_2}$ is purely imaginary.

Reason: If z is purely imaginary, then $z + \overline{z} = 0$

63. **Assertion:** Consider z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then $\text{Im}\left(\frac{z_1}{z_2}\right) = 0.$

Reason: arg(z) = 0 ⇒ z is purely real.

64. **Assertion:** If P and Q are the points in the plant XOY representing the complex numbers z_1 and z_2 respectively then distance $|PQ| = |Z_2 - Z_1|$

Reason: Locus of the point P(z) satisfying $|z - (2 + 3i)| = 4$ is a straight line.

65. **Assertion:** The equation $ix^2 - 3ix + 2i = 0$ has non-real roots.

Reason: If a, b, c are real and $b^2 - 4ac \geq 0$, then the roots of the equation $ax^2 + bx + c = 0$ are real and if $b^2 - 4ac < 0$, then roots of $ax^2 + bx + c = 0$ are non-real.

ANSWERS

1. $-1 + i$
2. $-7 - 6i$
3. $\frac{1}{49} - \frac{4\sqrt{3}i}{49}$
4. $\frac{-2}{25} + \frac{11i}{25}$
5. $-\frac{2}{7} - \frac{i\sqrt{3}}{7}$
6. 13
7. 7
8. –2
9. $-i$
10. First
11. –15
12. (i) 0 (ii) 19 (iii) –8 (iv) $\frac{-7}{\sqrt{2}}i$

13. $x = \frac{5}{13}, \quad y = \frac{14}{13}$

14. i

15. 0 (zero)

16. 6 and zero

17. a = –2

19. $\theta = (2n+1)\frac{\pi}{2}$

21. x = 3, y = –1

23. zero

24. a = 0, b = –2

25. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

26. n = 4

28. (i) $3\sqrt{2}$ and $-2i$ (ii) –2i, –2i

29. ½, –2*i*

31. radius= $\frac{2}{3}$

32. $z = 0, \ i, \ \frac{\sqrt{3}}{2} - \frac{1}{2}i, \ -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

34. 1

35. K=1

36. Infinitely many solutions of the form $z = 0 \pm iy; \ y \in R$

37. $|z_1| = \sqrt{x^2 + y^2}$ [Hint: $|z|^2 = z \times \bar{z}$]

38. 5

40. When x = 1, y = –4 or x = –1, y = –4

41. 1 (one)

44. 1 [Hint: $|z|^2 = z \times \bar{z}$, $|\bar{z}| = |z|$]

45. i. (a)	ii. (b)	iii. (d)	iv. (c)	v. (b)
46. (b)	47. (a)		48. (b)	49. (a)
50. (c)	51. (a)		52. (c)	53. (d)
54. (a)	55. (d)		56. (a)	57. (b)
58. (a)	59. (c)		60. (b)	61. (c)
62. (b)	63. (a)		64. (c)	65. (d)

CHAPTER - 5

LINEAR INEQUALITIES

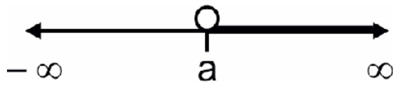
KEY POINTS

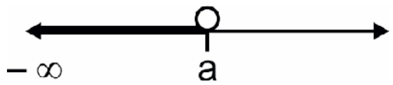
- **Inequalities:** A statement involving '<', '>','≥'or '≤'is called inequality.
  - Inequalities which do not involve variables are called numerical inequalities.
  - Inequalities which involve variables are called literal inequalities.
    Eg., $3x - 4 \leq 15$ and $4x - 3y \geq 5$
  - Inequalities involving the symbols '>' or '<' are called strict inequalities.
  - Inequalities involving the symbols '≥' or '≤' are called slack inequalities.
- **Linear inequalities in one variable:** The inequalities of form $ax + b > 0$, $ax + b < 0$, $ax + b \geq 0$ or $ax + b \leq 0$; $a \neq 0$ are called linear inequalities in one variable.

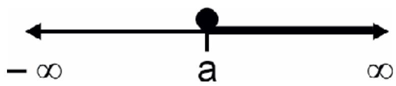
  The set of real numbers which satisfy a given linear in equality is called the solution set of the inequality.
- **Algebraic solutions of linear inequalities in one variables:**
  - **Rule-1**
    Equal numbers may be added (or subtracted from) to both sides without affecting sign of inequalities.

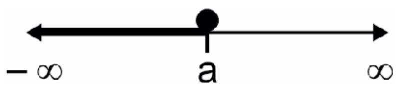
- **Rule-2**
  (i) If both sides of inequality are multiplied (or divided) by same positive number, then sign of inequality remains unchanged.
  (ii) If both sides are multiplied (or divided) by any negative number, then sign of inequality is reversed.

▶ **Graphical representation of solutions on number line:**

(i) $x > a \Leftrightarrow a < x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$ 

(ii) $x < a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$ 

(iii) $x \geq a \Leftrightarrow a \leq x < \infty \Leftrightarrow x \in [a, \infty) \Leftrightarrow$ 

(iv) $x \leq a \Leftrightarrow -\infty < x \leq a \Leftrightarrow x \in (-\infty, a] \Leftrightarrow$ 

(v) $a < x < b \Leftrightarrow x \in (a, b) \Leftrightarrow$ 

(vi) $a \leq x \leq b \Leftrightarrow x \in [a, b] \Leftrightarrow$ 

▶ **Linear inequalities in two variables:** The inequalities of form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$ or $ax + by + c \leq 0$ are linear inequalities in two variables. $(a, b \neq 0)$

Eg., $4x - 3y < 15$ and $-4x + 15y + 3 \geq 4$

▶ **Graphical solution of linear inequalities in two variables**

- A line divides the Cartesian plane into two parts. Each part is known as a half plane.

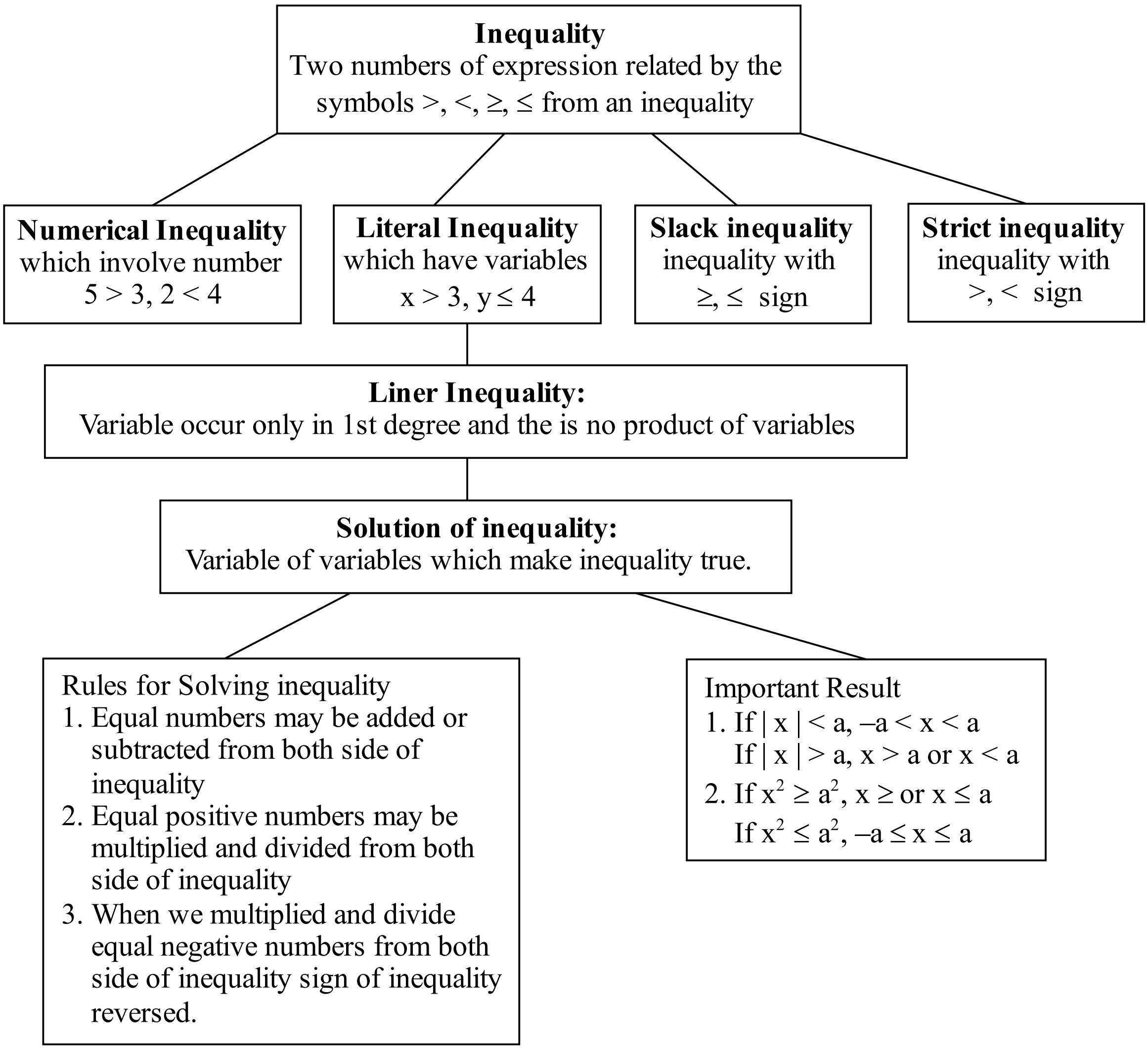
- The region containing all the solutions of the inequality is called solution region.
- In order to identify the half plane represented by an inequality (solution region), it is just sufficient to take any point (a, b) not on the line and check whether it satisfy the inequality or not.

  (i) If it satisfies, then the regions containing that point (a, b) is solution region.

  (ii) If it does not satisfy, then the other region is solution region.
- If inequality contains '$\geq$' or '$\leq$', then points on line ax + by = c are also included in solution region. In this case we draw dark line while sketching graph of ax + by = c.
- If inequality contains '>' or '<', then points on line ax + by = c are not included in solution region. In this case we draw dotted line while sketching graph of ax + by = c.

Note: While solving system of linear inequalities in two variables, the common of solution regions of each inequality is solution region of system.

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. Solve $5x < 24$ when $x \in N$

2. Solve $3 - 2x < 9$ when $x \in R$. Express the solution in the form of interval.

3. Show the graph of the solution of $2x - 3 > x - 5$ on number line.

4. Solve $0 < \frac{-x}{3} < 1, \quad x \in R$

5. Solve $-3 \leq -3x + 2 < 4, \quad x \in R$.

6. Draw the graph of the solution set of $x + y \geq 4$.

7. Draw the graph of the solution set of $x < y$.

8. Solve the inequality for real *x*: $\frac{x^2}{x-2} > 0$.

SHORT ANSWER TYPE QUESTIONS

9. Solve $\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0, \quad x \in R$.

10. Solve $\frac{x+3}{x-1} > 0, \quad x \in R$.

Solve the inequalities for real x and represent solution on number line

11. $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}, \quad x \in R$.

12. $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2, \quad x \in R$.

13. $-5 \leq \frac{2-3x}{4} \leq 9, \quad x \in R$.

14. $\frac{x+3}{x-2} > 0, \quad x \in R$

15. $\frac{x-3}{x-5} > 2$

16. $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$

17. $\frac{2x+3}{x-3} \le 4$

18. Find the pair of consecutive even positive integers which are greater than 5 and are such that their sum is less than 20.

19. A company manufactures cassettes and its cost and revenue functions are C(x) = 26000 + 30x and R(x) = 43x respectively, where x is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realise some profit. [Profit = R(x) – C(x)]

20. While drilling a hole in the earth, it was found that the temperature (T°C) at x km below the surface of the earth was given by T = 30 + 25(x – 3), when $3 \le x \le 15$.

Between which depths will the temperature be between 200°C and 300°C?

21. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3rd reading that will result in acidity level being normal.

Solve the following systems of inequalities for all $x \in R$

22. $2(2x+3)-10 < 6(x-2), \quad \frac{2x-3}{4}+6 \ge 4+\frac{4x}{3}$

23. $|2x-3| \le 11, \quad |x-2| \ge 3$

24. $\frac{4x}{3}-\frac{9}{4} < x+\frac{3}{4}, \quad \frac{7x-1}{3}-\frac{7x+2}{6} > x$

25. Solve $\frac{|x|-1}{|x|-2} \ge 0 \quad x \in R, \quad x \ne \pm 2$

26. In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks be should score in fifth paper.

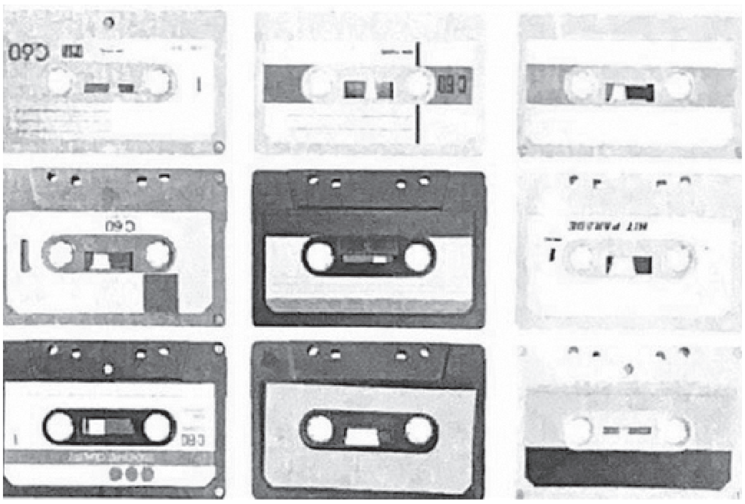
27. A milkman has 80% milk in this stock of 800 litres of adultered milk. How much 100% pure milk is to be added to it so that purity is between 90% and 95%?

28. $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}, \quad \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

29. $\frac{x}{2x+1} \geq \frac{1}{4}, \quad \frac{6x}{4x-1} < \frac{1}{2}$

30. $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 45$.

31. A company produced cassettes, one cassette Cost Company Rs. 30 and also an additional fixed cost 26000 per week. The company sold each Cassette at Rs. 43. If x is number of cassettes produced and sold by the company in a week. From the following information find



i. The cost function of the company

(a) 26000 + 30x (b) 26000+43x

(c) 30 +26000x (d) 43 + 26000x

ii. The revenue function of the company

(a) 30x (b) 26000x

(c) 43x (d) 13x

iii. The profit function of the company

(a) –26000 + 73x (b) –26000+ 13x

(c) 26000+43x (d) 26000+ 30x

iv. How many cassettes must be produced by the company in a week to realize some profit?

(a) more than 2000 (b) less than 2000

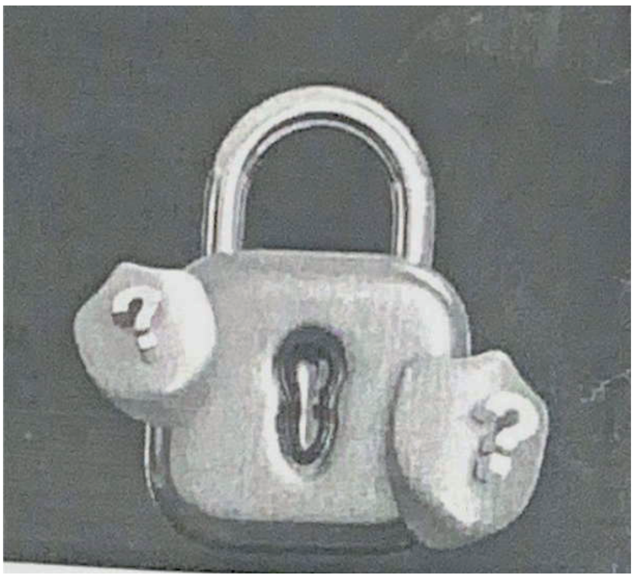
(c) more than 5000 (d) less than 5000

v. If company incurred an additional cost of Rs. 3 on each cassette per week. How many cassettes must be produced by the company in a week so that there is no profit no loss?

(a) 2000 (b) 5000

(c) 2600 (d) 1000

32. A and B tried to find the solution of the inequality $|x - 1| + |x - 2| \geq 4$. Help them to find the solution of the inequality



i. When $x < 1$

(a) $(-\infty, -1/2)$ (b) $(-\infty, -1)$

(c) $(-\infty, -1/2]$ (d) $(-\infty, 1/2)$

ii. When $1 \leq x < 2$

(a) $(-\infty, \infty)$ (b) $(-\infty, -1)$

(c) Infinite solution (d) no solution

iii. When $2 \leq x < \infty$

(a) $(-\infty, \infty)$ (b) $(7/2, \infty)$

(c) $[7/2, \infty)$ (d) no solution

iv. When $x \in R$

(a) $(-\infty, -1/2] \cup [7/2, \infty)$ (b) $(-\infty, -1/2) \cup [7/2, \infty)$

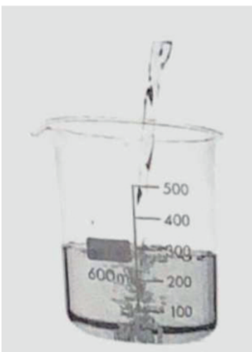
(c) $(-\infty, -1/2] \cup (7/2, \infty)$ (d) $(-\infty, -7/2] \cup [1/2, \infty)$

v. When $x > 4$

(a) $(-\infty, 4)$ (b) $(-\infty, 4]$

(c) $(4, \infty)$ (d) $[4, \infty)$

33. A student have solution of 640 litres of 8% boric acid. He wants to dilute it by using 2% solution of boric acid.



(a) How many minimum litres of 2% boric acid he must add so that resulting solution have more than 4% boric acid?

(b) How many minimum litres 2% boric acid he must add so that resulting soltution have less than 6% boric acid?

(c) How many litres of water he must add so that resulting solution have more 4% but less than 6% of boric acid?

34. If $|x + 3| \geq 10$, then

(a) $x \in (-13, 7]$ (b) $x \in (-13, 7]$

(c) $x \in (-\infty, -13] \cup [7, \infty)$ (d) $x \in [-\infty, -13] \cup [7, \infty)$

35. If $\frac{|x-7|}{(x-7)} \geq 0$, then

(a) $x \in [7, \infty)$ (b) $x \in (7, \infty)$

(c) $x \in (-\infty, 7)$ (d) $x \in (-\infty, 7]$

Multiple Choice Questions

36. If $-4x > 20$ and $x \in z^+$ then x belongs to -

(a) {–6, –7, –8, ……..} (b) ϕ

(c) {–4, –3, –2, –1} (d) {1, 2, 3, 4, ……..}.

37. If $\frac{x-3}{x-2} > 0$ then x belongs to -

(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, -3) \cup (-5, \infty)$

(c) $(-\infty, 3] \cup [5, \infty)$ (d) $(3, 5)$

38. Solution set for inequality $|x - 1| \leq 5$ is -

(a) [–6, 4] (b) [–4, 0]

(c) [–4, 6] (d) [0, 6].

39. Solution set for inequality $\frac{1}{x-2} < 0$ is -

(a) $(2, \infty)$
(b) ϕ
(c) $(0, 2)$
(d) $(-\infty, 2)$.

40. Solution set for inequality $5x - 3 < 3x + 1, x \in N$ is -

(a) $(-\infty, 2)$
(b) $\{0, 1, 2\}$
(c) $\{1\}$
(d) ϕ.

41. Which of the following point lies in solution region of inequality $3x - y \leq 5$?

(a) $(5, 1)$
(b) $(1, 5)$
(c) $(2, 0)$
(d) $(2, -1)$.

42. If $x > 0$ and $y < 0$ then (x, y) lies in -

(a) I quadrant
(b) II quadrant
(c) III quadrant
(d) IV quadrant.

43. If $x^2 > 9$ then x belongs to -

(a) $(-3, 3)$
(b) $(0, 3)$
(c) $(3, \infty)$
(d) $(-\infty, -3) \cup (3, \infty)$.

44. Solution set for inequality $-8x \leq 5x - 3 < 7$ is -

(a) $(-1, 2)$
(b) $(2, 3)$
(c) $[-1, 2)$
(d) $[2, 3]$.

45. The graph of the inequalities

$x \leq 0, y \geq 0, 2x + y + 6 \leq 0$ is

(a) a triangle
(b) a square
(c) unbounded
(d) none of these

Directions: Each of these questions contains two statements Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is correct explanation for assertion.

(b) Assertion is correct, reason is correct, reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

45. **Assertion:** The inequality ax + by < 0 is strict inequality.

    Reason: The inequality ax + b $\geq$ 0 is slack inequality.

46. **Assertion:** The inequality ax + by < 0 is strict inequality.

    Reason: The inequality ax + b $\geq$ 0 is slack inequality.

47. **Assertion:** If a < b, c < 0, then $\frac{a}{c} < \frac{b}{c}$.

    Reason: If both sides are divided by the same negative quantity, then the inequality is reversed.

48. **Assertion:** |3x – 5| > 9 $\Rightarrow$ x $\in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$

    Reason: The region containing all the solutions of an inequality is called the solution region.

49. **Assertion:** A line divides the Cartesian plane in two part(s).

    Reason: If a point P(α, β) on the line ax + by = c, then aα + bβ = c.

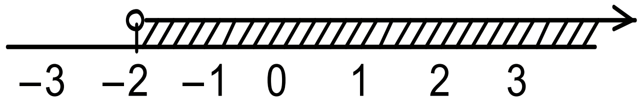
50. **Assertion:** Each part in which a line divides the Cartesian plane, is known as half plane.

    Reason: A point in the Cartesian palne will either lie on a line or will lie in either of half plane I or II.

ANSWERS

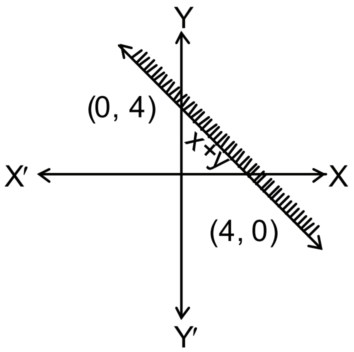
1. {1,2,3,4}

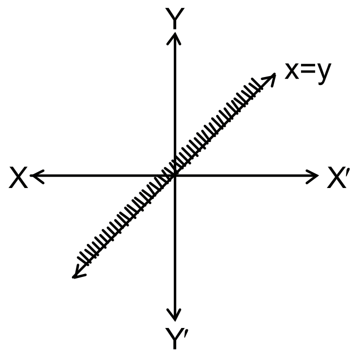
2. (–3, ∞)

3. 

4. –3 < x < 0

5. $\left(\dfrac{-2}{3}, \dfrac{5}{3}\right]$

6. 

7. 

8. (2, ∞)

9. (–∞, –1] ∪ [2, 3) ∪ (4, ∞)

10. (–∞, –3) ∪ (1, ∞)

11. $\left(-\infty, \dfrac{63}{10}\right]$

12. $\left(-\infty, \dfrac{-13}{2}\right)$

13. $\left[\dfrac{-34}{3}, \dfrac{22}{3}\right]$

14. (–∞, –3) ∪ (2, ∞)

15. (5, 7)

16. (–∞, 2]

17. $(-\infty, -3) \cup \left[\dfrac{15}{2}, \infty\right)$

18. (6, 8) and (8, 10)

19. More than 2000 cassettes

20. Between 9.8 m and 13.8 m

21. Between 6.27 and 8.07.

22. Solution set $=\phi$

23. $[-4, -1] \cup [5, 7]$

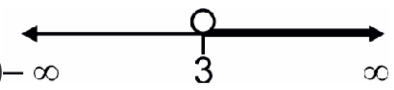
24. $(4, 9)$

25. $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

26. He must score greater than or equal to 52 and less than 77.

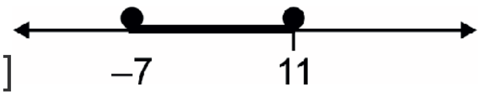
27. Between 800 litre and 2400 litre

28. $(3, \infty)$



29. No solution

30. $[-7, 11]$



31. i. (a) ii. (c) iii. (b) iv. (a) v. (c)

32. i. (c) ii. (d) iii. (c) iv. (a) v. (c)

33. i. 1280 L ii. 320 L iii. $\frac{640}{3} < x < 640$

34. (c)

35. (b)

36. (b)	37. (a)	38. (c)
39. (d)	40. (c)	41. (b)
42. (d)	43. (d)	44. (c)
45. (c)	46. (b)	47. (d)
48. (b)	49. (b)	50. (b)

CHAPTER - 6

PERMUTATIONS AND COMBINATIONS

KEY POINTS

- **Fundamental principle of counting**

- **Multiplication Principle:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of simultaneous occurrence of the two events in order is m × n.

- **Fundamental Principle of Addition:** If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in (m + n) ways.

- **Factorial:** Factorial of a natural number n, denoted by n! or n is the continued product of first n natural numbers.

  $n! = n \times (n-1) \times (n-2) \times \ldots\ldots\ldots \times 3 \times 2 \times 1$

  $= n \times ((n-1)!)$

- **Permutation:**A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.

- The number of permutation of n different objects taken r at a time where$0 \leq r \leq n$ and the objects do not repeat is denoted by ^nP_r or P(n, r) where,

$$^nP_r = \frac{n!}{(n-r)!}$$

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, …….. p_k are of k^{th} kind and the rest if any, are of different kinds, is $\dfrac{n!}{(p_1!)(p_2!)\,.......(p_k!)}$

▶ **Combination:**Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n distinct objects taken r at a time where,

$0 \le r \le n$, is denoted by ${}^nC_r$ or C(n, r) where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

▶ **Some important result:**

(i) 0! = 1

(ii) ${}^nC_0 = {}^nC_n = 1$

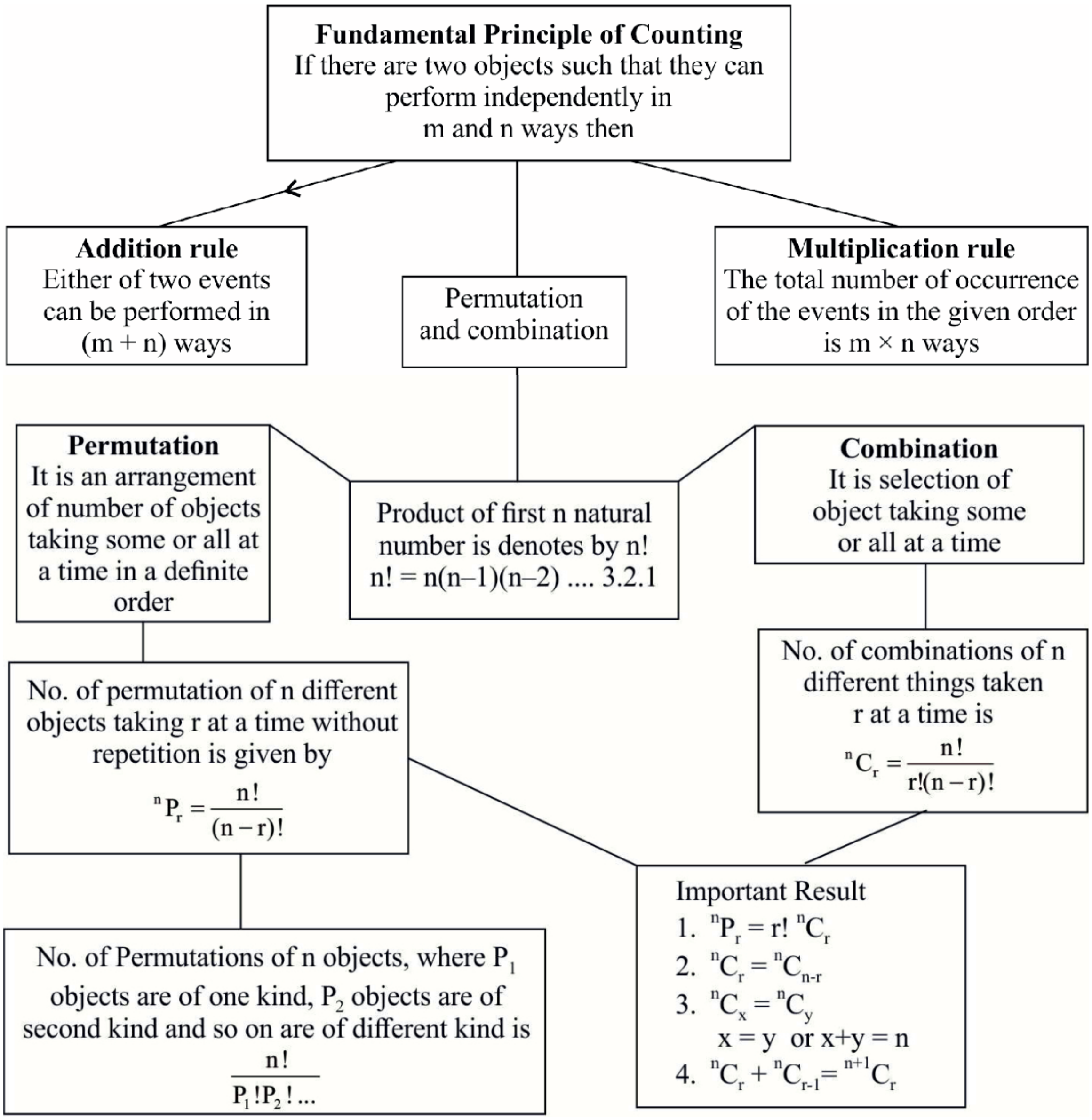
(iii) ${}^nC_r = {}^nC_{n-r}$ where $0 \le r \le n$, and r are positive integers

(iv) ${}^nP_r = \underline{|n}\,{}^nC_r$ where $0 \le r \le n$, r and n are positive integers.

(v) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

(vi) If ${}^nC_a = {}^nC_b$ if either a = b or a + b = n

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If ${}^{n}P_4 : {}^{n}P_2 = 12$, find n.

7. How many different words (with or without meaning) can be made using all the vowels at a time?

8. In how many ways 4 boys can be chosen from 7 boys to make a committee?

9. How many different words can be formed by using all the letters of word "SCHOOL"?

10. In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

11. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

12. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

13. If ${}^{20}C_r = {}^{20}C_{r-10}$ then find the value of ${}^{18}C_r$

SHORT ANSWER TYPE QUESTIONS

14. Find n, ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$.

15. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$, find r.

16. If ${}^{n}P_{r} = 336$, ${}^{n}C_{r} = 56$, find n and r. Hence find ${}^{n-1}C_{r-1}$.

17. A convex polygon has 65 diagonals. Find number of sides of polygon. (Hint: No. Of diagonals = ${}^{n}C_{2} - n$)

18. In how many ways can a cricket team of 11 players be selected out of 16 players, if two particular players are always to be selected?

19. From a class of 40 students, in how many ways can five students be chosen for an excursion party.

20. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together?

21. If ${}^{n}C_{12} = {}^{n}C_{13}$ then find the value of the ${}^{25}C_{n}$.

22. In how many ways can the letters of the word "PENCIL" be arranged so that I is always next to L.

23. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice.

24. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

25. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

26. A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?

27. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

28. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

29. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

30. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

31. A polygon has 35 diagonal. Find the number of its sides.

32. Determine the number of 5 cards combinations out of a pack of 52 cards if at least 3 out of 5 cards are ace cards?

33. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

34. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.

35. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has:

    (i) no girl

    (ii) at least 3 girls

    (iii) at least one girl and one boy?

36. In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter vote for at least one candidate, then find the number of ways in which he can vote.

37. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in

how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

LONG ANSWER TYPE QUESTIONS

38. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

39. There are 15 points in a plane out of which only 6 are in a straight line, then
    (i) How many different straight lines can be made?
    (ii) How many triangles can be made?

40. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that
    (i) No two girls sit together?
    (ii) All the girls never sit together?

41. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

42. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,
    (i) 3 are red and 1 is black.
    (ii) All 4 cards are from different suits.
    (iii) Atleast 3 are face cards.
    (iv) All 4 cards are of the same colour.

43. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

44. How many different four letter words can be formed (with or without meaning) using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R.

45. If all letters of word 'MOTHER' are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word 'MOTHER'?

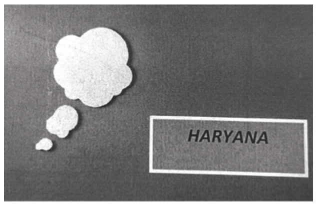
46. From 6 different novels and 3 different dictionaries, 4 novels and a dictionary is to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then find the number of such arrangements.

47. The set S= {1,2,3,.......12} is to be partitioned into three sets A, B, and C of equal sizes. $A \cup B \cup C = S$, $A \cap B = B \cap C = C \cap A = \phi$. Find the number of ways to partition S.

48. If ${}^{15}C_{3r} : {}^{15}C_{r+1} = 11 : 3$, find r

CASE STUDY TYPE QUESTIONS

49. Anita is doing an experiment in which she has to arrange the alphabets of the word "HARYANA" in all possible orders and notes the observations. Help her to find the answers of the following:-



i. Number of words starting with A

(a) 360 (b) 720

(c) 1440 (d) 2880

ii. Number of words having H at end

(a) 72 (b) 120

(c) 240 (d) 480

iii. Number of words having H and N together

(a) 120 (b) 60

(c) 280 (d) 240

iv. Number of words having begin with H and end with N

(a) 20 (b) 24

(c) 60 d) 48

v. Number of words having vowels together

(a) 240 (b) 120

(c) 240 (d) 720

50. A Company wants to appoint 5 persons, 3 for post A and 2 for post B for its upcoming office in Delhi. They have invited the applications for the same. 14 candidates have applied for the post A and 13 have applied for the post B

i. Find the total number of ways in which the company can make a selection for all the posts.

(a) 5! (b) C(14,3).C(13,2)

(c) P(13,2)P(14,3) (d) none of these

ii. Find the number of ways of selecting one woman for each post, if 3 women have applied for post A and 7 women have applied for post B

(a) 6 (b) 21

(c) 6930 (d) 182

iii. On the day of interview, the candidates were seated in a hall having two chambers. The chairs in both the chambers are placed in line. If the candidates for the two posts are to be seated in two different chambers. Find the total number of ways in which all the candidate can be seated.

(a) 3!2! (b) 11!11!

(c) 14!13! (d) 14! × 13! × 2

iv. During appointment procedure they came to know about a candidate whose resume is excellent and should be selected for the post B. In how many ways can the total selections now be made?

(a) 12 × C(14,3) (b) 4

(c) 168 (d) 13 × C(13,3)

v. While checking the applications the management observed that one candidate each who have applied for post A and B are not fit for the job. So they cannot be appointed. In how many ways can now the post is filled?

(a) 2184 (b) 24024

(c) 18876 (d) 1716

Multiple Answer Type Questions

51. What is the number of ways of arrangement of letters of word 'BANANA' so that no two N's are together -

(a) 40 (b) 60

(c) 80 (d) 100.

52. What is the value of n, if P(15, n – 1) : P(16, n – 2) = 3 : 4 ?

(a) 10 (b) 12

(c) 14 (d) 15.

53. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants can't occur together is -

(a) 4! (b) 3! × 4!

(c) 7! (d) None of these.

54. If 7 points out of 12 are in the same straight line, then what is the number of triangles formed?

(a) 84 (b) 175

(c) 185 (d) 201

55. In how many ways can be bowler take four wickets in a single 6 balls over?

(a) 6 (b) 15

(c) 20 (d) 30.

56. What is the number of signals that can be sent by 6 flags of different colours taking one or more at a time?

(a) 45 (b) 63

(c) 720 (d) 1956.

57. There are 6 letters and 3 post boxes. The number of ways in which these letters can be posted is –

(a) 6^3 (b) 3^6

(c) ${}^6P_3$ (d) ${}^6C_3$.

58. If ${}^mC_1 = {}^nC_2$, then –

(a) 2m = n (b) 2m = n(n + 1)

(c) $2m = n(n - 1)$ (d) $2n = m(m - 1)$.

59. ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_x$, then x = ?

(a) r (b) r – 1

(c) n (d) r + 1.

60. ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then value of r is –

(a) 12 (b) 8

(c) 6 (d) 10.

61. If ${}^nP_s = 60\ {}^{n-1}P_3$ the value of *n* is

(a) 6 (b) 10

(c) 1 (d) 16

62. The number of ways 10 digit numbers can be written using the digits 1 and 2 is

(a) 2^{10} (b) ${}^{10}C_2$

(c) 10! (d) ${}^{10}C_1 + {}^9C_2$

63. The number of ways in which 8 students can be seated in a line is

(a) 5040 (b) 50400

(c) 40230 (d) 40320

64. There are 10 true-false questions in an examination. These questions can be answered in

(a) 20 ways (b) 100 ways

(c) 512 ways (d) 1024 ways

65. In how many ways can we paint the six faces of a cube with six different colours?

(a) 30 (b) 6

(c) 6! (d) None of these

Directions: Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) givne below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion in incorrect, reason is correct.

66. **Assertion:** If the letters W.I.F.E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24^{th} position.

Reason: The number of ways of arranging four distinct objects taken all at a time is C(4, 4).

67. **Assertion:** A number of four different digits is formed with the help of the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. Then, number of numbers which are exactly divisible by 4 is 200.

Reason: A number divisible by 4, if unit place digit is divisible by 4.

68. **Assertion:** Product of five consecutive natural numbers is divisible by 4!

Reason: Product of n consecutive natural number is divisible by (n + 1)!

69. **Assertion:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Reason: The number of ways of choosing any 3 places, from 9 different places is ${}^9C_3$.

70. **Assertion:** A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number formed are 216.

Reason: If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

ANSWERS

1. $\frac{6!}{2} = 360$ 2. $\frac{9!}{2!}$

3. 100 4. 45

5. 120 6. n = 6

7. 120 8. 35

9. 360 10. 63

11. $3^6=729$ 12. 66

13. 816 14. n = 9

15. r = 7 16. n = 8, r = 3 and 21

17. 13 18. 2002

19. 40_{C_5} 20. $\frac{3!}{2!} \times 4!$

21. 1 22. 120

23. $90 \times {}^{10}P_8$ 24. 56

25. 350 26. 266

27. 36 28. PAANVR

29. ${}^{13}C_{10} + {}^{13}C_8$ 30. 5040

31. 10 32. 4560

33. 576 34. 2454

35. (i) 21; (ii) 91; (iii) 441

36. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

37. 48,144

38. 420

39. (i)91

(ii)435

40. (i)$7! \times {}^{8}P_5$

(ii)$12! - 8! \times 5!$

41. 24480

42. ${}^{52}C_4$

(i) ${}^{26}C_1 \times {}^{26}C_3$

(ii) $(13)^4$

(iii) 9295 (Hint : Face cards : 4J + 4K + 4Q)

(iv) $2 \times {}^{26}C_4$

43. 265 (*Hint* : make 3 cases i.e.

(i) All 3 letters are different

(ii) 2 are identical 1 different

(iii) All are identical, then form the words.)

44. 59

45. 309

46. $4!\,{}^{6}C_4\,{}^{3}C_1$

47. ${}^{12}C_4\,{}^{8}C_4\,{}^{4}C_4$

48. r = 3

49. i. (a) ii. (b) iii. (d) iv. (a) v. (b)

50. i. (b) ii. (c) iii. (d) iv. (a) v. (c)

51. (a)	52. (c)	53. (a)
54. (c)	55. (b)	56. (b)
57. (b)	58. (c)	58. (d)
60. (a)	61. (b)	62. (a)
63. (d)	64. (d)	65. (a)
66. (c)	67. (c)	68. (c)
69. (a)	70. (d)	

CHAPTER - 7

BINOMIAL THEOREM

KEY POINTS

- **Binomial Theorem for Positive Integers :**
  - $(x + y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots\ldots\ldots$

    $\ldots\ldots\ldots\ldots + {}^nC_r x^{n-r} y^r + \ldots\ldots\ldots\ldots {}^nC_n x^0 y^n,$

    Where n is any positive integer.
  - It is written as $(x+y)^n = \sum_{r=0}^{n} {}^nc_r x^{n-r} y^r$
  - Total number of terms in expansion $(x + y)^n$ is $(n + 1)$
  - General Term = $T_{r+1} = {}^nC_r x^{n-r} y^r$, where $0 \le r \le n$.

- **Middle Term :**
  - If n is even, then there is only one middle term

    M.T. = $\left(\frac{n}{2}+1\right)^{th}$ term
  - If n is odd, then there are two middle terms

    First M.T. = $\left(\frac{n+1}{2}\right)^{th}$ term

    Second M.T. = $\left(\frac{n+1}{2}+1\right)^{th}$ term

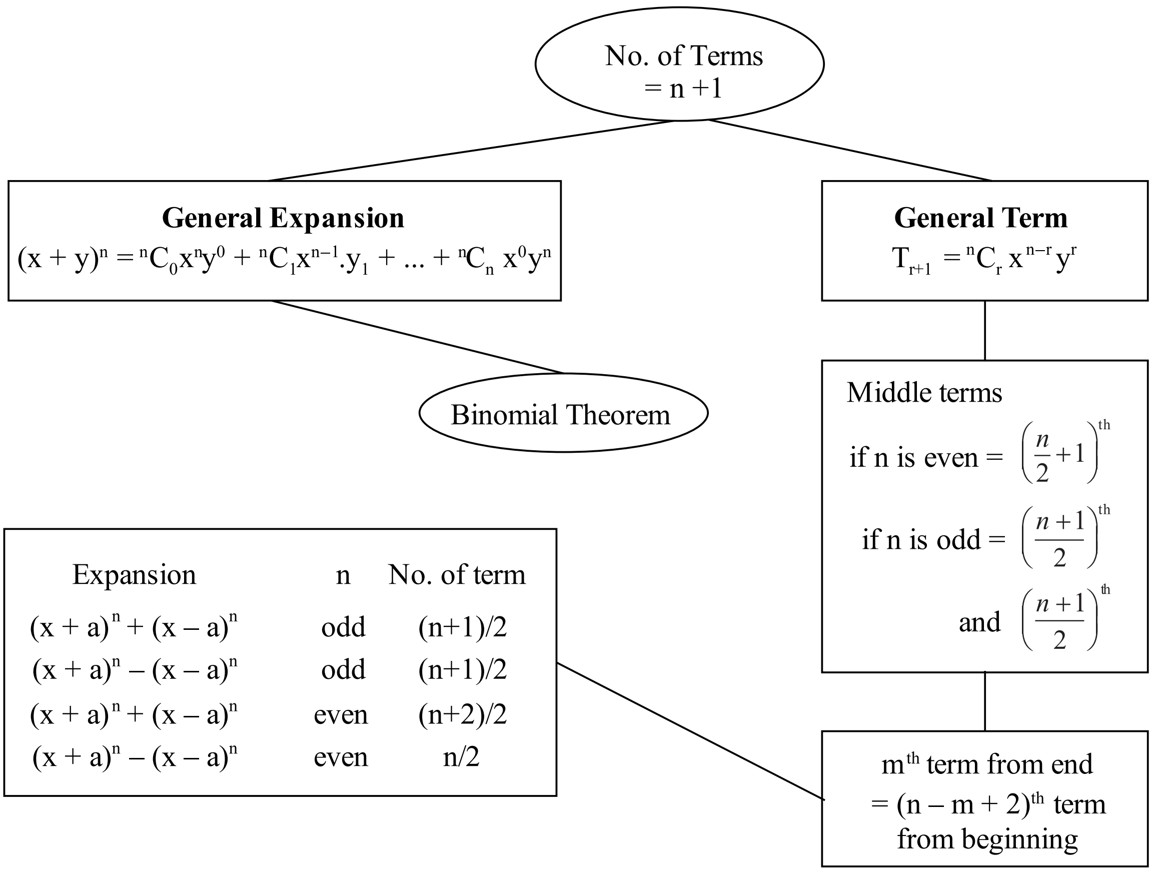
Some important observations :

- In expansion $(x + y)^n$

  ${}^nC_r, {}^nC_{r-1}, \ldots {}^nC_0$ are called **binomial coefficients**

- Sum of indices of x and y is n in each of the expansion.
- T_{r+1} = [(r + 1)th term from beginning] = ${}^nC_r x^{n-r} y^r$
- T'_{r+1} = [(r + 1)th term from end] = ${}^nC_{n-r} x^r y^{n-r}$
- $(x - y)^n = \Sigma(-1)^r \; {}^nC_r \; x^{n-r} y^r$
- $(1 + x)^n = \sum_{r=0}^{n} {}^nc_r x^r$
- $(1 - x)^n = \sum_{r=0}^{n} (-1)^r \; {}^nc_r x^r$

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. Write number of terms in the expansion of $\left\{\left(2x+y^{3}\right)^{4}\right\}^{7}$.

2. Expand $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^{6}$ using binomial theorem.

3. Write value of

   ${}^{2n-1}C_5 + {}^{2n-1}C_6 + {}^{2n}C_7$ use $\left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

4. Which term is greater $(1.2)^{4000}$ or 800?

5. Find the coefficient of x^{-17},in the expansion of $\left(x^{4}-\frac{1}{x^{3}}\right)^{15}$.

6. Find the sum of the coefficients in $(x + y)^8$

   [**Hint :** Put x = 1, y = 1]

7. If ${}^{n}C_{n-3} = 120$, find n.

8. Find number of terms in expansion of $(-2x + 3y)^{17}$.

9. Find term independent of x in expansion of $\left(x-\frac{1}{3x^{2}}\right)^{9}$

10. Find the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$

11. If the coefficient of x in $\left(x^{2}+\frac{\lambda}{x}\right)^{5}$ is 270, then find λ.

12. Find the coefficient of x^5 in $(x + 3)^8$

13. Find 4^{th} term from the end in expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$

14. Find number of terms in $(x + y)^5 + (x - y)^5$

15. Find coefficient f x^5 in $(1 + x)^{10}$

16. Find number of terms in

(i) $(3x - y)^{10} + (3x + y)^{10}$

(ii) $(2x + y)^8 - (2x - y)^8$

(iii) $(x + 5y)^{15} - (x - 5y)^{15}$

(iv) $(5x - y)^9 + (5x - y)^9$

17. Using Binomial Theorem, Evaluate.

(i) 96^3 [Hint $(100 - 4)^3$] (ii) $(101)^4$ (iii) $(10.1)^5$

SHORT ANSWER TYPE QUESTIONS

18. How many term are free from radical signs in the expansion of $\left(x^{\frac{1}{5}} + y^{\frac{1}{10}}\right)^{55}$.

19. Find the constant term in expansion of $\left(x - \frac{1}{x}\right)^{10}$.

20. Find 4th term from end in the expansion of $\left(\frac{x^3}{2} + \frac{2}{x^2}\right)^9$.

21. Find middle term in the expansion of $(x - 2y)^8$.

22. Which term is independent of x in the expansion of $\left(3x^3 - \frac{1}{2x^3}\right)^{10}$.

23. Find the 11th term from end in expansion of $\left(2x - \frac{1}{x^2}\right)^{25}$.

24. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

25. In $\left(3x^2 - \frac{1}{x}\right)^{18}$ which term contains x^{12}.

26. In $\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}x^2}\right)^{10}$ find the term independent of x.

27. Evaluate $\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5$ using binomial theorem.

28. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

29. Find the coefficients of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

30. Show that $3^{2n+2} - 8n - 9$ is divisible by 8. [$3^{2n+2} = 9.9^n = 9(1 + 8)^n$]

31. If the term free from x in the expansion of $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405. Find the value of k.

32. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1024}$.

33. If a, b, c and d in any binomial expansion be the 6^{th}, 7^{th}, 8^{th} and 9^{th} terms respectively, then prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$.

34. If in the expansion of $(1 + x)^n$, the coefficients of three consecutive of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.

35. Show that $2^{4n+4} - 15n - 16$ where $n \in N$ is divisible by 225. $[2^{4n+4} = 2^4.2^{4x} = 16(1 + 15)^n]$

36. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

37. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$.

38. Find the value of r, if the coefficient of $(2r + 4)^{th}$ term and $(r - 2)^{th}$ term in the expansion of $(1 + x)^{18}$ are equal.

39. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

40. The coefficient of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 6 : 30. Find n.

LONG ANSWER TYPE QUESTIONS

41. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.

42. If the 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n.

43. If the coefficients of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ are equal, then show that ab = 1.

44. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is 1:6 , find n.

45. If p is a real number and if middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, find p.

46. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms inthe expansion of $(1 + x)^n$

Prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

47. Find the remainder left out when $8^{2n} – (62)^{2n+1}$ is divided by 9.

48. Find the value of $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$ and show that $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

49. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2-\frac{1}{3}x\right)^9$.

50. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^4$are in A.P find the value of r.

51. If the expansion of $(1 – x)^{2n–1}$, the coefficients of x^r is denoted by a_r, then prove $a_{(r–1)} + a_{(2n–r)} = 0$.

52. If the coefficient of 5^{th}, 6^{th} and 7^{th} terms in the expansion of $(1 + x)^n$ are in A.P., then find the value of n.

53. The coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in A.P. Prove that $2n^2 – 9n + 7 = 0$.

54. Show that the middle term in the expansion of $\left[x-\frac{1}{x}\right]^{2n}$ is $\frac{1\cdot3\cdot5.....(2n-1)}{n!}(-2n)^n$.

55. If n is a positive integer, find the coefficient of x^{-1} in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$

Multiple Choice Questions

56. The middle term of $\left[2x - \frac{1}{3x}\right]^{10}$ is -

(a) ${}^{10}C_4 \frac{2^4}{3^4}$

(b) $-{}^{10}C_5 \frac{2^5}{3^5}$

(c) $-{}^{10}C_4 \frac{2^4}{3^5}$

(d) ${}^{10}C_5 \frac{2^5}{3^5}$.

57. For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by -

(a) 125

(b) 225

(c) 450

(d) 625.

58. What is the coefficient of x^n in $(x^2 + 2x)^{n-1}$?

(a) $(n - 1)\, 2^{(n-2)}$

(b) $(n - 1) \times 2^{(n-1)}$

(c) $(n - 1)\, 2^n$

(d) $n \,.\, 2^{(n-1)}$.

59. The coefficient of x^{-3} in the expansion of $\left[x - \frac{m}{x}\right]^{11}$ is –

(a) $-924\, m^7$

(b) $-792\, m^5$

(c) $-792\, m^6$

(d) $-330\, m^7$.

60. In the expansion of $\left[x^2 - \frac{1}{3x}\right]^9$, the term without x is equal to -

(a) $\frac{28}{81}$ (b) $\frac{-28}{243}$

(c) $\frac{28}{243}$ (d) None of these.

61. If in the expansion of $(1 + x)^{20}$, the coefficients of r^{th} and $(r + 4)^{th}$ term are equal, then r is equal to -

(a) 7 (b) 8

(c) 9 (d) 10.

62. If in the expansion of $(1 + x)^5$, the coefficients of $(r - 1)^{th}$ and $(2r + 3)^{th}$ terms are equal, then the value of r -

(a) 5 (b) 6

(c) 4 (d) 3.

63. The total number of terms in expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is -

(a) 202 (b) 51

(c) 50 (d) None of these.

64. The middle term in the expansion of $\left[\frac{2x}{3} - \frac{3}{2x^2}\right]^{2n}$ is -

(a) ${}^{2n}C_n$ (b) $(-1)^{n\,2n}C_n\, x^{-n}$

(c) ${}^{2n}C_n\, x^{-n}$ (d) None of these.

65. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the some, then the value of a is -

(a) $\frac{-7}{9}$ (b) $\frac{-9}{7}$

(c) $\frac{7}{9}$ (d) $\frac{9}{7}$.

66. The total number of term in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is

(a) 102 (b) 25

(c) 26 (d) 28

67. If z = $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then

(a) Re(z) = 0 (b) Im(z) = 0

(c) Re(z) > 0, In (z) > 0 (d) Re(z) > 0, In (z) < 0

68. The two successine terms in the expansion of $(1 + x)^{24}$ whose coefficients are in the ratio 1 : 4 are

(a) 3rd and 4th (b) 4th and 5th

(c) 5th and 6th (d) 6th and 7th

69. Constant term in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ is

(a) 152 (b) – 152

(c) – 252 (d) 252

70. The coelficent of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

(a) $\frac{405}{256}$ (b) $\frac{504}{259}$

(c) $\frac{450}{263}$ (d) none of these

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices. Only one of which is the correct answer. You have to select one of the codes. (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct. Reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct: reason in not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

71. **Assertion:** The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$.

Reason: The coefficient of x^6 in the expansion of $(1 + x)^n$ is ${}^nC_6$.

72. **Assertion:** The r^{th} term from the end in the expansion of $(x + a)^n$ is ${}^nC_{n-r+1}\, x^{r-1}\, a^{n-r+1}$.

Reason: The r^{th} term from the end in the expansion of $(x + a)^n$ is $(n - r + 2)^{th}$ term.

73. **Assertion:** In the expansion of $(x + 2y)^8$, the middle term is 4^{th} term.

Reason: If n is even in the expansion of $(a + b)^n$, then $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

74. **Assertion:** General term of the expansion $(x + 2y)^9$ is ${}^9C_r2^r.\, x^{9-r}y^r$.

Reason: General term of the expansion $(x + a)^n$ is given by $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

75. **Assertion:** In the binomial expansion $(a + b)^n$, r^{th} term is ${}^nC_ra^{n-r}.\, b^r$.

Reason: If n is odd, then there are two middle terms.

ANSWERS

1. 29

2. $\frac{x^{32}}{a^{32}a} - \frac{6x}{} + 15\frac{x}{a}\ 20 + 15 - \frac{a}{xx}\frac{6a}{x}{}^{23}_{23} + \frac{a}{}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. –1365

6. 256

7. n = 10

8. 18

9. ${}^{9}C_{3} \times \left(\frac{-1}{3}\right)^{3}$

1 ${}^{10}C_5$

11. 3

1 1512

13. $\frac{672}{3}$

14. 3

15. C_5^{10}

16. (i) 6 (ii) 4 (iii) 8 (iv) 5

17. (i) 884736 (ii) 104060401 (iii) 105101.00501

18. 6 terms (0, 10, 20, 30, 40, 40, 50)

19. $-252 = -{}^{10}C_5$

20. $\frac{672}{3}$

21. $1120\ x^4\ y^4$

22. $\frac{-15309}{8}$

23. $C_{15}^{25} \times \ \frac{10}{\ \ \ 20}$

24. a = 3, b = 2, n = 3

25. 9^{th} term

26. $T_3 = \frac{5}{6}$

27. 82

28. 28

29. 10

31. k = ±3

32. 129 integral terms

34. n = 8, 4^{th}, 5^{th} and 6^{th}

38. r = 6

40. n = 41

42. a = 2, x = 1, n = 7

44. 9

45. p = ± 2

47. 2

48. 198

49. $\frac{17}{54}$

50. 5

52. n = 7 or 14

55. ${}^{2n}C_{n-1}$

56. (b)

57. (b)

58. (a)

59. (d)

60. (c)

61. (c)

62. (a)

63. (b)

64. (b)

65. (d)

66. (c)

67. (b)

68. (c)

69. (c)

70. (a)

71. (d)

72. (a)

73. (d)

74. (a)

75. (d)

CHAPTER - 8

SEQUENCES AND SERIES

KEY POINTS

- In general, listing of any collection of objects in certain order is **sequence**.
- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- Let a_1, a_2, a_3, ……….. be a sequence, then the expression $a_1 + a_2 + a_3 +$ ……….. is called series associated with given sequence.
- A sequence containing finite number of terms is called finite sequence.
- A sequence is infinite, if it is not finite sequence.
- A sequence is said to be a progression if all the terms of the sequence can be expressed by same formula
- **Arithmetic Progression:** A sequence is called an arithmetic progression if the difference between of a term and its previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=d) for all $n \in N$.
- General A.P. isa, a + d and a + 2d, ..., where a = first term and d = common difference.
- $a_n = a + (n - 1)d = n^{th}$ term of A.P. $= l$
- S_n = Sum of first n terms of A.P. $= \frac{n}{2}[a + l]$, where l = last term N.

$$= \frac{n}{2}[2a + (n-1)d]$$

- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P.

  ak, bk, ck also in A.P., k ≠ 0

  $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where k ≠ 0.

- If a, A, b are in A.P., then A is called **arithmetic** mean of a and b.

- Arithmetic mean between a and b is = $\frac{a+b}{2}$.

- If A_1, A_2, A_3, ...A_n are n numbers inserted between a and b, such that the resulting sequence is A.P.

  then, $A_n = a + nd$ where $d = \frac{b-a}{n+1}$

- $S_k - S_{k-1} = a_k$

- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term.

- If a, b, c are in A.P. then 2b = a + c.

- Three terms of A.P. can be chosen as a – d, a, a + d

- Four terms of A.P. can be chosen as a – 3d, a – d, a + d, a + 3d.

- **G.P. (Geometrical Progression)**

  (i) a, ar, ar^2, ...(General G.P.)

  Where a = First term

  And r = common ratio

(ii) $a_n = ar^{n-1}$

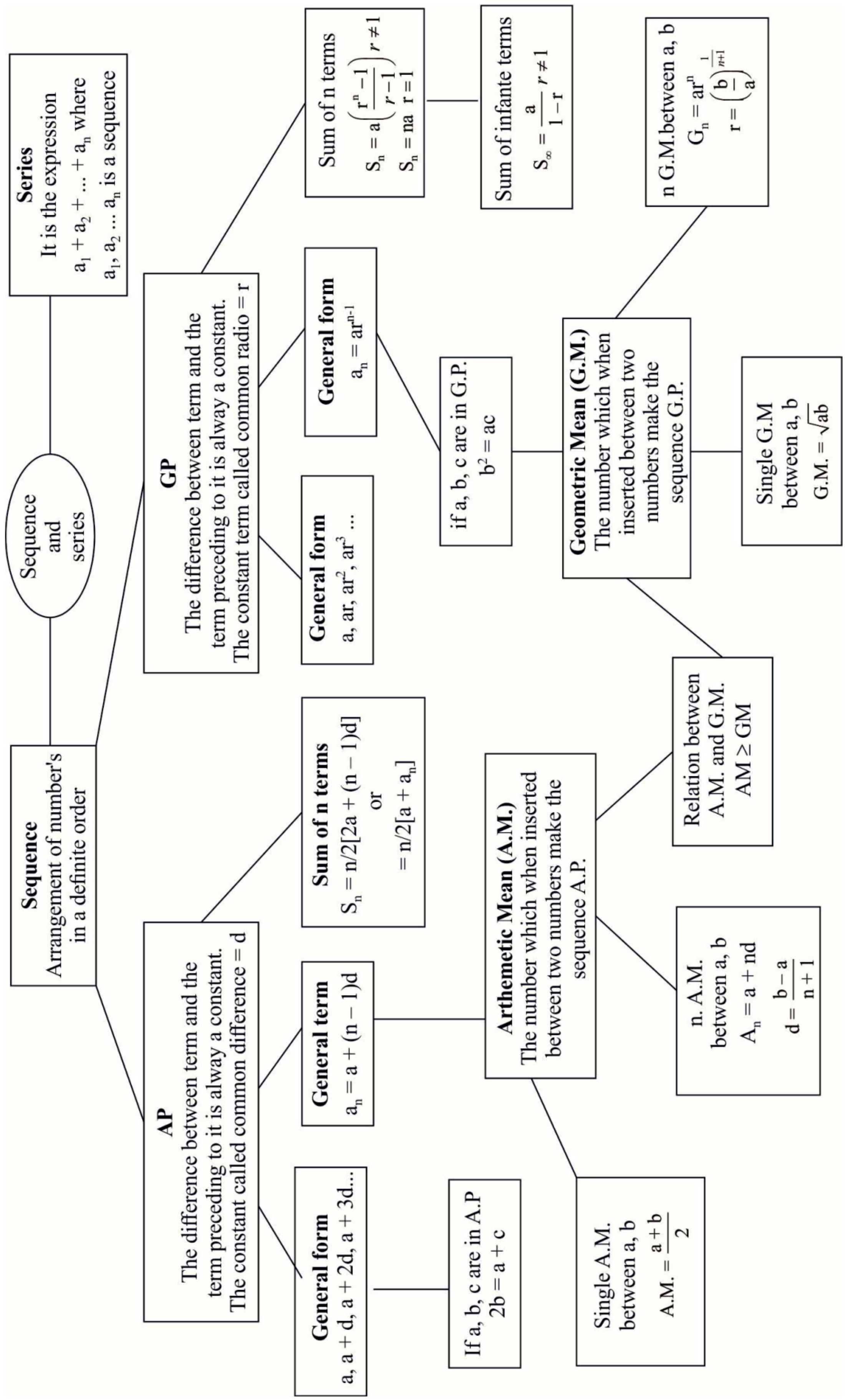
(iii) $S_n = \frac{a(r^n - 1)}{r - 1}$ or $S_n = \frac{a(1 - r^n)}{1 - r}$, $r \neq 1$

- If a, b, c are in G.P., then $b^2 = ac$.
- If a, G, b are in GP, then G is called **geometric** mean of a and b
- Geometric mean of two positive numbers a and b is $\sqrt{ab}$.
- If $G_1, G_2, G_3, \ldots\ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $G_n = ar^n$ where $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

- Three terms of G.P. are chosen as $\frac{a}{r}$, a, ar.
- Four terms of G.P. are chosen as $\frac{a}{r^3}, \frac{a}{r}$, a, ar^3.
- If a, b, c are in G.P. then (i) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in GP, (ii) ak, bk, ck are also in G.P., where k ≠ 0 (iii) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in G.P. where k ≠ 0, a^n, b^n, c^n are also in GP.
- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.
- Sum of infinite G.P. is possible if |r| < 1 and sum is given by $\frac{a}{1-r}$.

MIND MAP


Sequence and series
Sequence
Arrangement of number's in a definite order
Series
It is the expression $a_1 + a_2 + \ldots + a_n$ where $a_1, a_2 \ldots a_n$ is a sequence
AP
The difference between term and the term preceding to it is alway a constant. The constant called common difference = d
GP
The difference between term and the term preceding to it is alway a constant. The constant term called common radio = r
General form
a, a + d, a + 2d, a + 3d...
If a, b, c are in A.P
2b = a + c
General term
$a_n = a + (n - 1)d$
Sum of n terms
$S_n = n/2[2a + (n - 1)d]$
or
$= n/2[a + a_n]$
Arthemetic Mean (A.M.)
The number which when inserted between two numbers make the sequence A.P.
Single A.M. between a, b
$A.M. = \frac{a+b}{2}$
n. A.M. between a, b
$A_n = a + nd$
$d = \frac{b-a}{n+1}$
Relation between A.M. and G.M.
$AM \geq GM$
General form
a, ar, ar^2, ar^3 ...
General form
$a_n = ar^{n-1}$
if a, b, c are in G.P.
$b^2 = ac$
Geometric Mean (G.M.)
The number which when inserted between two numbers make the sequence G.P.
Single G.M between a, b
$G.M. = \sqrt{ab}$
n G.M.between a, b
$G_n = ar^n$
$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$
Sum of n terms
$S_n = a\left(\frac{r^n - 1}{r - 1}\right) r \neq 1$
$S_n = na \quad r = 1$
Sum of infante terms
$S_\infty = \frac{a}{1 - r} \quad r \neq 1$

VERY SHORT ANSWER TYPE QUESTIONS

1. If n^{th} term of an A.P. is 6n – 7 then write its 50^{th} term.

2. If $S_n = 3n^2 + 2n$, then write a_2

3. Which term of the sequence 3, 10, 17, ... is 136?

4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.

6. Which term of the G.P.2, 1, $\frac{1}{2}, \frac{1}{4}$ .... is $\frac{1}{1024}$?

7. If in a G.P., $a_3 + a_5 = 90$ and if r = 2 find the first term of the G.P.

8. In G.P. $2\sqrt{2}, 4, ..., 128\sqrt{2},$ find the 4^{th} term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term.

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}, ...$

11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ .... upto infinity.

12. Write the value of $0.\overline{3}$

    [Hint: $0.\overline{3} = 0.3 + 0.03 + 0.003 + ... = \frac{0.3}{1-0.1}$]

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. If 7^{th} and 13^{th} terms of an A.P. be 34 and 64 respectively, find 18^{th} term.

15. Find geometric mean of 4 and 9.

16. Find If the sum of first p terms of an A.P. is q and sum of first q terms is p, then the sum of first p + q terms.

17. Find sum to infinity of sequence $5, \frac{5}{3}, \frac{5}{9}, ...$.

18. If a, b, c are in A.P. and x, y, z are in G.P., then find the value of $x^{b-c}\ y^{c-a}\ z^{a-b}$.

19. Find two geometric means between numbers 1 and 64.

20. Write third term of sequence whose general term is $a_n = \frac{2n-3}{4}$.

SHORT ANSWER TYPE QUESTIONS

21. Write the n^{th} term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + ...$

22. Find the number of terms in the A.P. 7, 10, 13, ..., 31.

23. In an A.P., 8, 11, 14, ... find $S_n - S_{n-1}$

24. Find the sum of given terms:-

(a) 81 + 82 + 83 .... + 89 + 90

(b) 251 + 252 + 253 + ... + 259 + 260

25. (a) If a, b, c are in A.P. then show that 2b = a+c.

(b) If a, b, c are in G.P. then show that $b^2 = a \cdot c$.

26. If a, b, c are in G.P. then show that $a^2 + b^2$, ab + bc, $b^2 + c^2$ are also in G.P.

27. Find the least value of n for which

$1 + 3 + 3^2 + ... + 3^{n-1} > 1000$

28. Write the first negative term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, ... $[a_n < 0]$

29. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11^{th} term from the end.

30. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

31. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

32. Find the sum of the sequence,

72 + 70 + 68 + ………. + 40

33. If in an A.P. $\frac{a_7}{a_{10}} = \frac{5}{7}$, find $\frac{a_4}{a_7}$.

34. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

35. Solve: 1 + 6 + 11 + 16 + ... + x = 148

36. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their 10^{th} terms.

37. If the I^{st}, 2^{nd} and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

38. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{a}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term].

39. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

40. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

41. Find the sum to infinity of the series:

$$1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \text{......} \infty$$

42. If $A = 1 + r^a + r^{2a} + \text{...........}$ up to infinity, then express r in terms of 'a' and 'A'.

43. Find the sum of first terms of the series 0.7 + 0.77 + 0.777 + …..

44. If $x = a + \frac{a}{r} + \frac{a}{r^2} + \text{...........} + \infty$; $y = b - \frac{b}{r} + \frac{b}{r^2} - \text{...........} \infty$ and

$z = c + \frac{c}{r^2} + \frac{c}{r^4} + \text{........} \infty$ Prove that $\frac{xy}{z} = \frac{ab}{c}$.

45. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

46. Prove that $0.003\overline{1} = \frac{7}{225}$.

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

47. If a, b, c are in G.P. that the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

48. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b+c, c+a, a+b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

49. If the numbers a^2, b^2 and c^2 are given to be in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

50. Show that: $0.3\overline{56} = \frac{353}{990}$

51. The n^{th} term of a G.P. is 128 and the sum of its n term is 255. If its common ratio is 2, find the first term.

52. The fourth term of a G.P. is 4. Find product of its first seven terms.

53. If A_1, A_2, A_3, A_4 are four A.M's between $\frac{1}{2}$ and 3, then prove $A_1 + A_2 + A_3 + A_4 = 7$.

54. If S_n denotes the sum of first n terms of an A.P. If $S_{2n} = 5S_n$, then prove $\frac{S_{6n}}{S_{3n}} = \frac{17}{4}$.

LONG ANSWER TYPE QUESTIONS

55. Prove that the sum of n numbers between a and b such that the resulting series becomes A.P. is $\frac{n(a+b)}{2}$.

56. If a, b, c are in G.P., then prove that $\frac{1}{a^2-b^2}-\frac{1}{b^2-c^2}=-\frac{1}{b^2}$.

[**Hint :** Put b = ar, c = ar^2]

57. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

58. If a is A.M. of b and c and c, G_1, G_2, b are in G.P., then prove that $G_1^3+G_2^3=2abc$

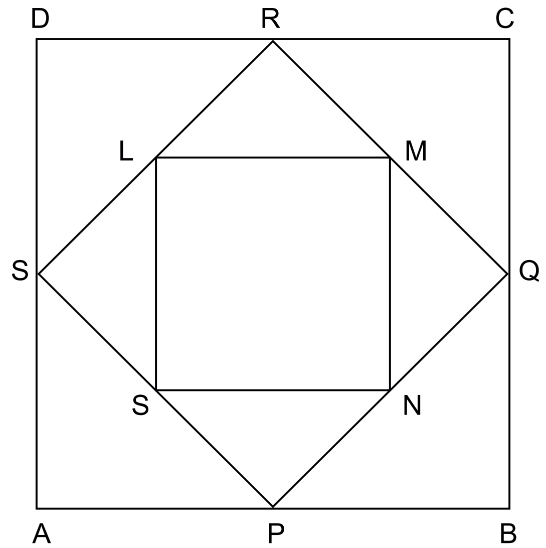
59. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

60. Find the sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ terms. Hint: $\left[1-\frac{1}{2}+1-\frac{1}{4}+1-\frac{1}{8}+1-\frac{1}{16}+....\right]$

61. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

62. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 91.

63. The side of given space is 10 cm. The mid points at its, sides are joined to form a new square. Again the mid point of the sides of this new square are joined to form another square. This process is contined indefinitely. Based on the information answer the following questions.



(i) The side of first square is 10 cm what is the side of IInd square formed.

(ii) What is the sum of area's of all the square formed?

(iii) What is the sum of perimeters of all the square formal?

CASE STUDY TYPE QUESTIONS

64. Abhishek buys Kisan Vikas Patra (KVP) from post office every year. Each year he exceeds the value of KVP by ₹1000 from last year's purchase. After 5 years he finds that the total value of KVP purchased by him is ₹40,000.00.

Based on the above information answer the following :-

i. The sequence of amount of KVP forms a/an

(a) Arithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the amount of KVP purchased by him initially.

(a) ₹7000 (b) ₹8000

(c) ₹6000 (d) ₹7500

iii. What will be the total amount of KVP purchased by him after 10 years?

(a) ₹1,20,000 (b) ₹1,05,000

(c) ₹1,40,000 (d) ₹1,35,000

iv. What is the amount of KVP purchased by him in the 8^{th} year?

(a) ₹14,000 (b) ₹15,000

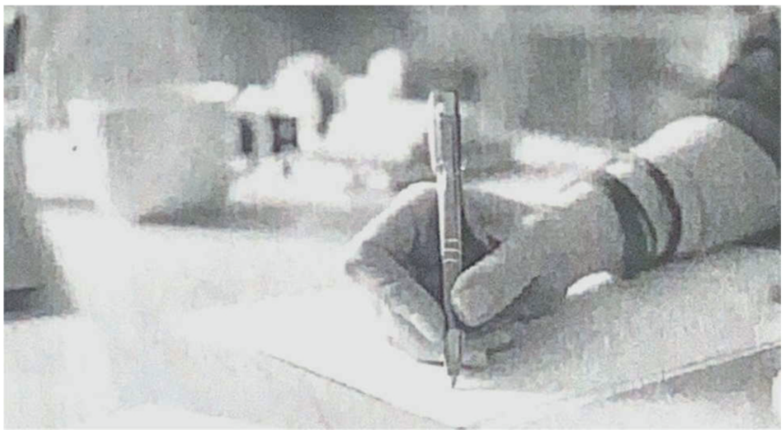
(c) ₹13,000 (d) ₹12,000

v. If he buys KVP every year for 10 years, how much will he spend in the purchase of last 4 KVP?

(a) ₹65,000 (b) ₹54,000

(c) ₹75,000 (d) None of these

65. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail it to four different persons with the instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paisa to mail one letter, anwer the following questions.



i. The sequence of letters mailed in each set forms a/an.

(a) Arithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the number of letters mailed in the 4^{th} set.

(a) 64 (b) 16

(c) 256 (d) 1024

iii. Find the total number of letters mailed in the first 5 sets.

(a) 1364 (b) 1650 (c) 1236 (d) 1368

iv. Find the amount spent on the postage when 8^{th} set of letters is mailed?

(a) ₹46,930 (b) ₹54,930 (c) ₹87,380 (d) ₹43,690

v. Find the amount spent on the mailing of 9^{th} set?

(a) ₹1,74,762 (b) ₹1,31,072

(c) ₹1,54,536 (d) None of these

Multiple Choice Questions

66. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5, then the number of side is -

(a) 8 (b) 10

(c) 9 (d) 6.

67. α and β are the roots of the equation $x^2 - 3x + a = 0$ and γ and δ are the roots of the equation $x^2 - 12x + b = 0$. If α, β, γ and δ form an increasing G.P., then (a, b)-

(a) (3, 12) (b) (12, 3)

(c) (2, 32) (d) (4, 16).

68. If A be the arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then -

(a) S = nA (b) A = nS

(c) A = S (d) None of these.

69. If n geometric means be inserted between a and b, then the n^{th} geometric mean will be-

(a) $a\left[\frac{b}{a}\right]^{\frac{n}{n-1}}$ (b) $a\left[\frac{b}{a}\right]^{\frac{n-1}{n}}$

(c) $a\left[\frac{b}{a}\right]^{\frac{n}{n+1}}$ (d) $a\left[\frac{b}{a}\right]^{\frac{1}{n}}$.

70. If the arithmetic and geometric means of two numbers are 10 and 8 respectively, then one number exceeds the other number by-

(a) 8 (b) 10

(c) 12 (d) 16.

71. The first and last terms of A.P. are 1 and 11. If the sum of its term is 36, then the number of terms will be-

(a) 5 (b) 6

(c) 7 (d) 8.

72. If the first, second and last term of an A.P. are a, b and 2a respectively, then its sum is -

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) None of these.

73. If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of this G.P.is -

(a) $\frac{p-q}{q-r}$ (b) $\frac{q-r}{p-q}$

(c) pqr (d) None of these.

74. If A be one A.M. and p, q be two GM's between two numbers, then 2A is equal to-

(a) $\frac{p^3+q^3}{pq}$ (b) $\frac{p^3-q^3}{pq}$

(c) $\frac{p^2+q^2}{2}$ (d) $\frac{pq}{2}$.

75. In a G.P. if the $(m + n)^{th}$ term is p and $(m - n)^{th}$ term is q, then its m^{th} term is -

(a) O (b) pq

(c) $\sqrt{pq}$ (d) $\frac{1}{2}(p+q)$.

76. If S be the sum, P the product, R be the sum of reciprocals of n terms of G.P. then P^2 is equal to

(a) $\frac{S}{R}$ (b) $\frac{R}{S}$

(c) $\left(\frac{R}{S}\right)^n$ (d) $\left(\frac{S}{R}\right)^n$

77. The n^{th} term of a G.P. is 128 and the sum of its n terms is 225. If its common ratio is 2, then its first term is

(a) 1 (b) 3

(c) 8 (d) none of these

78. If second term of a G.P. is 2 and the sum of its infinite term is 8, then its first term is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) 4

79. The two geometric means between the numbers 1 and 64 are

(a) 1 and 64 (b) 4 and 16

(c) 2 and 16 (d) 8 and 16

80. The product $(32), (32)^{1/6}, (32)^{1/36}$ .... to ∞ is equal to

(a) 64 (b) 16

(c) 32 (d) 0

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(b) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(c) Assertion is correct , reason is incorrect.
(d) Assertion is incorrect, reason is correct.

81. **Assertion:** Value of a_{17}. Whose n^{th} term is $a_n = 4n - 3$, is 65.

Reason: Value of a_9, whose nth term is $a_n = (-1)^{n-1} . n^3$.

82. **Assertion:** If the third term of a G.P. is 4, then the product of its first five terms is 4^5.

Reason: Product of first five terms of a G.P. is given as a (ar) (ar^2) (ar^3) (ar^4).

83. **Assertion:** If a, b, c are in A.P. then b + c, c + a, a + b are in A.P.

Reason: If a, b, c are in A.P., then 10^a, 10^b, 10^c are in G.P.

84. **Assertion:** If $\frac{2}{3}$, k, $\frac{5}{8}$ are in A.P., then the value of k is $\frac{31}{48}$.

Reason: Three numbers a, b, c are in A.P. iff 2b = a + c

85. **Assertion:** For x = ± 1, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

Reason: Three numbers a, b, c are in G.P. if $b^2 = ac$.

ANSWERS

1.	293	2.	11
3.	20th	4.	0
5.	4n + 5	6.	12th
7.	$\frac{9}{2}$	8.	64
9.	3	10.	$20\left(1-\frac{1}{2^8}\right)$
11.	5	12.	$\frac{1}{3}$
13.	$\frac{2}{3}$	14.	89
15.	6	16.	–(p + q)
17.	15/2	18.	1

19.	4 and 16	20.	3/4
21.	$\frac{2n+1}{(n+6)(n+10)^2}$	22.	9
23.	3n + 5	27.	n = 7
24.	855, 2555		
28.	$-\frac{1}{4}$	29.	102, 367
30.	33	31.	7799
32.	952	33.	$\frac{3}{5}$
34.	11	35.	36
36.	33:17	37.	$\frac{(b+c-2a)(a+c)}{2(b-a)}$
39.	5, 10, 20, ...; or 20, 10, 5,	40.	18, 6, 2; or 2, 6, 18
41.	6	42.	$\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$
43.	$\frac{7}{81}\left[9n-1+10^{-n}\right]$	45.	$\frac{15}{7}\left(2^n-1\right)$
51.	1	52.	16384
57.	16, 4	59.	$19, \frac{38}{3}, \frac{76}{9}, ...$
60.	$n + 2^{-n} - 1$	61.	$r=2+\sqrt{3}$
62.	1, 3, 9		

63.	i. $5\sqrt{2}$ cm	ii. 200 cm^2	iii. $(80+40\sqrt{2})$ cm		
64.	i. (a)	ii. (c)	iii. (b)	iv. (c)	v. (b)
65.	i. (b)	ii. (c)	iii. (a)	iv. (d)	v. (b)

66.	(c)	67.	(c)	68.	(a)
69.	(c)	70.	(c)	71.	(b)
72.	(c)	73.	(b)	74.	(a)
75.	(c)	76.	(d)	77.	(a)
78.	(d)	79.	(b)	80.	(a)
81.	(b)	82.	(a)	83.	(b)
84.	(a)	85.	(a)		

CHAPTER - 9

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

  $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

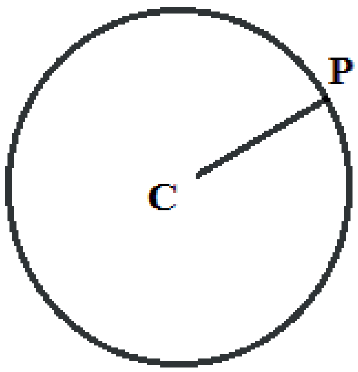
- Let the vertices of a triangle ABC are $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$.Then area of triangle

  $$ar(\Delta ABC) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

  Note: Area of a triangle is always positive. If the above expression is zero, then a triangle is not possible. Thus the points are collinear.

- **LOCUS:** When a variable point P(x,y) moves under certain condition then the path traced out by the point P is called the locus of the point.

  For example: Locus of a point P, which moves such that its distance from a fixed point C is always constant, is a circle.

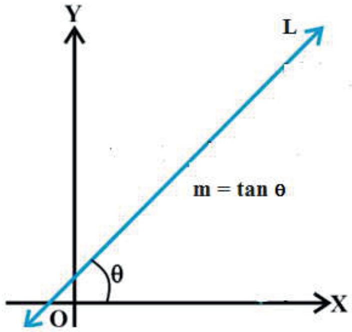


CP = constant

- A line is also defined as the locus of a point satisfying the condition $ax + by + c = 0$ where a, b, c are constants.

- **Slope of a straight line:**

  If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m



  $m = \tan\theta, \theta \neq 90°$

  If $0° < \theta < 90°$ then $m > 0$ and

  $90° < \theta < 180°$ then $m < 0$

  Note-1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined

  Note-2: Slope of any horizontal line i.e.|| to x-axis is zero.Slope of a vertical line i.e.|| to y-axis is not defined.

- Three points A, B and C lying in a plane are collinear, if slope of AB = Slope of BC.

- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

- Interecept: There are two types of intercepts x-intercept and y-intercept. The x-intercept is the x-coordinate of the point where line cut x axis while y-intercept is the coordinate of the point where line cut y axis.

- Two lines are parallel to each other if and only if their slopes are equal.

  i.e., $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$.

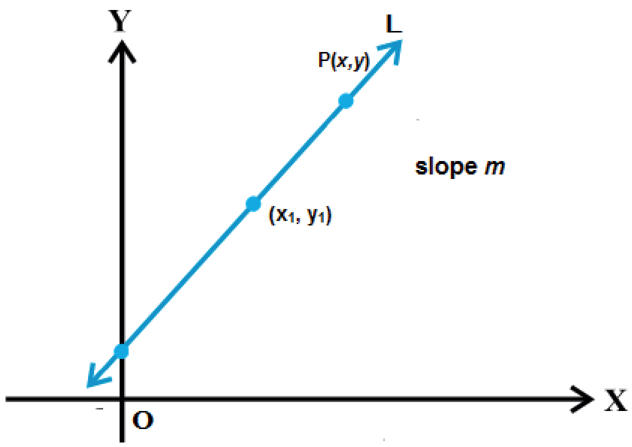
- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

  i.e., $l_1 \perp l_2 \Leftrightarrow m_1 m_2 = -1 \Leftrightarrow m_2 = \frac{-1}{m_1}$.

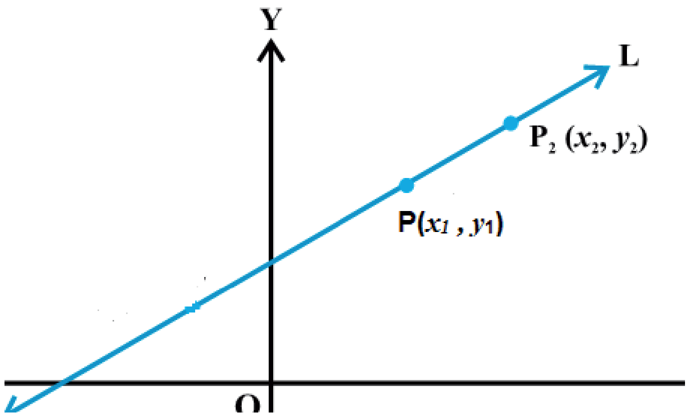
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$ and obtuse angle is $\phi = 180° - \alpha$ or $\pi - \alpha$

- **Point slope form:**

  Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$



- **Two Point Form:**



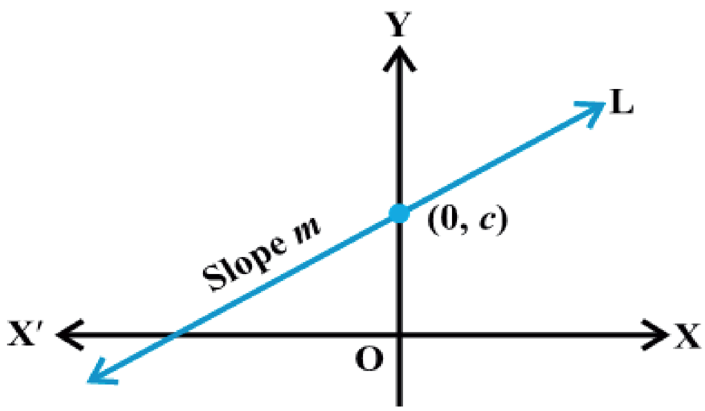
Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

- **Slope intercept form(y-intercept):**

  Equation of a line having slope m and y-intercept 'c' is given by

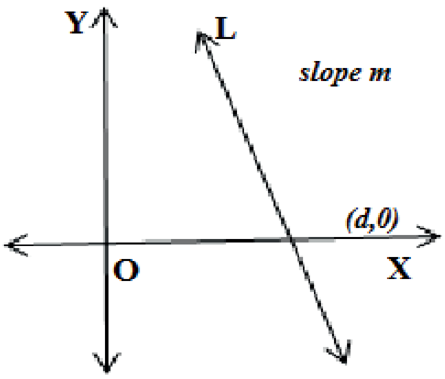
  $y = mx + c$



- **Slope intercept form (x-intercept):**

  Equation of a line having slope m and x-intercept d is given by

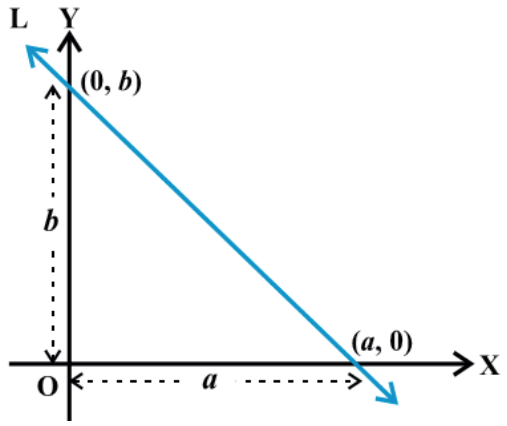
  $y = m (x–d)$



- **Intercept Form:**

  Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

  $$\frac{x}{a}+\frac{y}{b}=1$$



- **General Equation of a line:**

  Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non-zero.

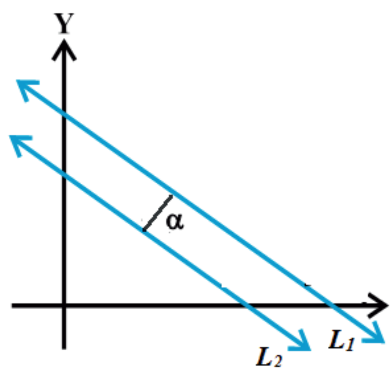
  Slope = $\frac{-A}{B}$ and y-intercept = $\frac{-C}{B}$, x-intercept = $\frac{-C}{A}$.

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

  $$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

- Distance between two parallel lines Ax + By + C_1 = 0 and Ax + By + C_2 = 0 is given by

  $$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$



MIND MAP

STRAIGHT LINE

1. If θ is the angle at which a straight line is inclined to be +ve direction of x-axis and $0^\circ \le 0 < 180^\circ, 0 \ne 90^\circ$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m doesn't exist, but the line is parallel to y-axis. If $\theta = 0^\circ$, then $m = 0$ and the line is parallel to x-axis.
2. If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1 \ne x_2$ are points on straight line, then the slope m of the line is given by $m = \frac{(y_1 - y_2)}{(x_1 - x_2)}$

If m_1 and m_1 are the slopes of two intersecting lines $(m_1 m_2 = -1)$ and 0 be the acute angle between them them $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

1. When two lines are parallel their slopes are equal. Thus, any line parallel to $y = mx + c$ is of the type $y = mx + d$, where d is any parameter
2. Two lines $ax + by + c = 0$ and $a^2x + b^2x + c^2 = 0$ are parallel if $\frac{a}{a^2} = \frac{b}{b^2} \ne \frac{c}{c^2}$
3. The distance between two paralle lines with equations $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $d = d\left|\frac{c_2 - c_2}{\sqrt{a^2 + b^2}}\right|$

1. When two lines of the slope m_2 and m_2 are at right angles, the Product of their slope is –1, i.e., $m_2 m_2 = -1$. Thus, any line perpendicular to $y = mx + c$ is of the form, $y = \frac{-1}{m}x + d$ where d is any parameter.
2. Two lines $ax + by + c = 0$ and $a'x + b'x + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus, any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

1. **POINT SLOPE FORM :** $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is 'm' and passes through the point (x_1, y_1).
2. **SLOPE INTERCEPT FORM :** $y = mx + e$ is the equationof a straight line whose slope is 'm' and makes an intercept c on the y-axis.
3. **TWO POINT FORM :** $y - y_1 = \frac{y_1 - y_2}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passed through (x_1, y_1) & (x_2, y_2).
4. **INTERCEPT FORM :** $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on x and y axis respectively.
5. **NORMAL/PERPENDICULAR FORM :** $x\cos\alpha + y\sin x = p$ (where $p > 0, \le 0\ \alpha < 2n$) is the equation of a straight line whre the length of the perpendicular from origin O on the line is p and the perpendicular makes an angle α with +ve x-axis.
6. **GENERAL FORM :** $ax + by + c = 0$ is the equation of a straight line in general form. In this case, slope of line $= -\frac{a}{b}$

1. The image of a point (x_1, y_1) about a line $ax + by + c = 0$ is:
   $\frac{x - x_1}{a} - \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$
2. Similarly, foot of perpendicular from a point on the line is:
   $\frac{x - x_1}{a} - \frac{y - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{\sqrt{a^2 + b^2}}$

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Area of triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_2, y_2) is $\frac{1}{2}|x_1(y_1 - y_2) + x_2(y_2 - y_1) + x_3(y_1 - y_2)|$

The length of the perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is:
$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b_2}}\right|$

The $P(x, y)$ divided the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then $x = \frac{mx_2 \pm nx_1}{m \pm n}$; $y_1 = \frac{my_1 \pm ny_1}{m \pm n}$

Note:
- (+ve) sign tells the division is internal, but (–ve) signs tells, the division is external.
- If $m = n$, then P is the mid-point of the line segment joining A & B.

VERY SHORT ANSWER TYPE QUESTIONS

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.

2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?

3. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6, q). Find p and q.

4. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1)?

5. Show that the points (a, 0), (0, b) and (3a,– 2b) are collinear.

6. Find the equation of straight line cutting off an intercept –1 from *y* axis and being equally inclined to the axes.

7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

8. Find k so that the line 2x + ky – 9 = 0 may be perpendicular to 2x + 3y – 1 = 0

9. Find the acute angle between lines x + y = 0 and y = 0

10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

11. Find the equation of a line with slope 1/2 and making an intercept 5 on y-axis.

12. Find Equation of line which is parallel to y-axis and at distance 5 units from y-axis.

13. Find the length of perpendicular from a point (1, 2) to a line 3x + 4y + 5 = 0.

SHORT ANSWER TYPE QUESTIONS

14. Determine the equation of line through a point (–4, –3) and parallel to *x*-axis.

15. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices a triangle or not?

16. If a vertex of a triangle is (1, 1) and the midpoints of two sides through this vertex are (–1, 2) and (3, 2). Then find the centroid of the triangle.

17. If the medians through A and B of the triangle with vertices A(0, b), B(0, 0) and C(a, 0) are mutually perpendicular. Then show that $a^2 = 2b^2$.

18. If the image of the point (3, 8) in the line px + 3y – 7 = 0 is the point (–1, –4), then find the value of p.

19. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines x + 2y = 5 and x – 3y + 5 = 0

20. The line 2x – 3y = 4 is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

21. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line y = 2x + c. Find c and remaining two vertices.

22. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

23. Find the equation of a line with slope -1 and whose perpendicular distance from the origin is equal to 5.

24. If a vertex of a square is at $(1, -1)$ and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

25. What is the value of y so that line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$?

26. In what ratio, the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$?

27. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and -6 respectively.

28. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.

29. Find the coordinates of the orthocentre of a triangle whose vertices are $(-1, 3)$ $(2, -1)$ and $(0, 0)$. [Orthocentre is the point of concurrency of three altitudes].

30. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

31. If the image of the point $(2, 1)$ in a line is $(4, 3)$ then find the equation of line.

32. The vertices of a triangle are $(6,0)$, $(0,6)$ and $(6,6)$. Find the distance between its circumcenter and centroid.

LONG ANSWER TYPE QUESTIONS

33. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

34. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

35. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

36. Find the equations of the straight lines which cut off intercepts on x-axis twice that on y-axis and are at a unit distance from origin.

37. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

38. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

39. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

40. If the slope of a line passing through to point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

41. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x - y - 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

42. Find points on the line $x + y + 3 = 0$ that are at a distance of 5 units from the line $x + 2y + 2 = 0$

43. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L.

44. Two equal sides of an isosceles triangle are given by the equation $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side pass through the point (1,–10). Determine the equation of the third side.

45. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2 AC. If the coordinates of D and M are (1,1) and (2,–1) respectively. Then find the coordinates of A.

46. Find the area enclosed within the curve $|x| + |y| = 1$.

47. Find the coordinates of the circumcentre of the triangle whose vertices are (5,7), (6,6) and (2, -2).

48. Find the equation of a straight line, which passes through the point (a, 0) and whose $\perp$ distance from the point (2a, 2a) is a.

49. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2} + b^{-2} = p^{-2} + q^{-2}$.

CASE STUDY TYPE QUESTIONS

50. A person is standing at a point A of a triangular park ABC whose vertices are A(2, 0), B(3, 4) and C(5, 6).

Based on the above information answer the following :-

i. He wants to reach BC in least time. Find the equation of the path he should follow.

(a) $2x + y = 3$ (b) $2x + 3y = 4$

(c) $x + y = 2$ (d) $x + 4y = 7$

ii. Find the shortest distance travelled by him to reach BC -

(a) $\frac{5}{2}\sqrt{2}$ units (b) $\frac{3}{2}\sqrt{2}$ units

(c) $\frac{4}{3}\sqrt{2}$ units (d) $\frac{7}{3}\sqrt{2}$ units

iii. Suppose he meets BC at a point D. Find the coordiantes of the point D.

(a) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{7}{2}, \frac{5}{2}\right)$

iv. Find the area of the triangular park ABC.

(a) 5 sq units (b) 10 sq units

(c) 3 sq units (d) None of these

v. Find the coordinator of the centroid of the triangular park ABC?

(a) $\left(\frac{5}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{10}{3}, \frac{10}{3}\right)$

(c) $\left(\frac{7}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{2}{3}, \frac{8}{3}\right)$

51. If A and B are two points (2, – 3) and (6, – 5) respectively. If C is the point between A and B such that it divides the line AB in 1 : 3 ratio

Based on the above information, answer the following Questions

(i) Find the distance between A and B
(ii) Find eq of AB
(iii) What are the co-ordinates of C?
(iv) Find The Length AC
(v) Find the slope of line BC.

Multiple Choice Questions

52. The angle between the straight lines $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$ is-

(a) 90° (b) 60°

(c) 75° (d) 30°

53. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then which one of the following is correct?

(a) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ (b) $\frac{1}{p^2}=\frac{1}{a^2}-\frac{1}{b^2}$

(c) $\frac{1}{p}=\frac{1}{a}+\frac{1}{b}$ (d) $\frac{1}{p}=\frac{1}{a}-\frac{1}{b}$.

54. What is the equation of the line passing through (2, –3) and parallel to y-axis?

(a) $y = -3$ (b) $y = 2$

(c) $x = 2$ (d) $x = -3$

55. If the lines $3x + 4y + 1 = 0$, $5x + \lambda y + 3 = 0$ and $2x + y - 1 = 0$ are concurrent, then λ is equal to -

(a) –8 (b) 8

(c) 4 (d) –4.

56. The x-intercept and the y-intercept of the line $5x - 7 = 6y$, respectively are –

(a) $\frac{7}{5}$ and $\frac{7}{6}$ (b) $\frac{7}{5}$ and $\frac{-7}{6}$

(c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $\frac{-5}{7}$ and $\frac{6}{7}$.

57. If p be the length of the perpendicular from the origin on the straight line $x + 2y = 2q$, then what is the value of q?

(a) 1/p (b) p

(c) p/2 (d) $\frac{\sqrt{5}p}{2}$

58. A straight line through P(1, 2) is such that its intercept between the axes is bisected at P. Its equation is-

(a) $x + y = -1$ (b) $x + y = 3$

(c) $x + 2y = 5$ (d) $2x + y = 4$.

59. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?

(a) 1 (b) 3

(c) 6 (d) 0.

60. The triangle formed by the lines x + y = 0, 3x + y = 4 and x + 3y = 4 is -

(a) Isosceles (b) Equilateral

(c) Right angled (d) None of these.

61. What is the foot of the perpendicular from the point (2, 3) on the line x + y – 11 = 0?

(a) (1, 10) (b) (5, 6)

(c) (6, 5) (d) (7, 4).

62. A line cutting off intercept –3 from the Y-axis and the tangent at angle to the X-axis is $\frac{3}{5}$, its

(a) $5y - 3x + 15 = 0$ (b) $3y - 5x + 15 = 0$

(c) $5y - 3x - 15 = 0$ (d) None of the above

63. The equation of straight line passing throught the point (3, 2) and perpendicular to the line $y = x$ is

(a) $x - y = 5$ (b) $x + y = 5$

(c) $x + y = 1$ (d) $x - y = 1$

64. The tangent of angle between the line whose intercepts on the axes are a, $-b$ and b, $-a$ respectively, is

(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{b^2 - a^2}{2}$

(c) $\frac{b^2 - a^2}{2ab}$ (d) None of these

65. The equation of the lines which pass throught the point (3, – 2) and are inclined at 60° to the line $\sqrt{3}x + y = 1$ is

(a) $y + 2 = 0,\ \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(b) $x - 2 = 0, \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$

(c) $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(d) None of the above

66. The coordinates of the foot of perpendiculars from the point (2, 3) on the line $y = 3x + 4$ given by

(a) $\left(\frac{37}{10}, \frac{-1}{10}\right)$ (b) $\left(\frac{-1}{10}, \frac{37}{10}\right)$

(c) $\left(\frac{10}{37}, -10\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}\right)$

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

67. **Assertion:** If θ is the inclination of a line *l*, then the slope or gradient of the line *l* is tan θ.

Reason: The slope of a line whose inclination is 90°, is not defined.

68. **Assertion:** The inclination of the line *l* may be acute or obtuse.

Reason: Slope of x-axis is zero and slope of y-axis is not defined.

69. **Assertion:** Slope of the line passing throught the points (3, –2) and (3, 4) is 0.

Reason: If two lines having the same slope pass throught a common point, then these lines will coincide.

70. **Assertion:** If A (–2, –1), B (4, 0), C(3, 3) and D(–3, 2) are the vertices of a parallelogram, then mid-point of AC = Mid-point of BD

Reason: The points A, B and C are collinear $\Leftrightarrow$ Area of $\Delta ABC = 0$.

71. **Assertion:** Pair of lines x + 2y – 3 = 0 and – 3x – 6y + 9 = 0 are coincident.

Reason: Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

ANSWERS

1. (1, 2)

2. k = 3 (Use Slope formmula)

3. p = 2, q = –5

4. 3 : 2 (internally)

6. $y = x - 1$ and $y = -x - 1$.

7. $x + y = 7$

8. $-\frac{4}{3}$

9. $\frac{\pi}{4}$

10. $\frac{2\pi}{3}$

11. $y = \frac{x}{2} + 5$

12. x = 5

13. 16/5

14. y + 3 = 0

15. No (Use slope formula)

16. $\left(1, \frac{7}{3}\right)$

18. 1 (Hint: Mid-point – of a point and its image is on the line)

19. $\frac{10}{\sqrt{26}}$ 20. (1, –5)

21. c = –4, (2,0), (4, 4) (Hint: diagonals of a rectangle bisect each other)

22. 49 square units (Hints: given lines are parallel)

23. $x + y + 5\sqrt{2} = 0$, $x + y - 5\sqrt{2} = 0$ (Hints: first find slope intercept form then convert normal form)

24. 4 square units (Hints: given vertex is not on the line)

25. y = 9

26. 1 : 2 (Use section formula)

27. 2x – 3y – 6 = 0 and – 3x + 2y – 6 = 0

28. 4 square units (Hints: find vertices)

29. (–4, –3)

30. x + 2y = 1

31. x + y – 5 = 0

32. $\sqrt{2}$ (Hints: circumcentre is the mid-point of hypotenuse of right angle Δ)

33. 3x – 4y + 15 = 0

34. 4x + 7y – 11 = 0, 7x – 4y + 25 = 0

7x – 4y – 3 = 0 (Hints: (–3, 1) and (1, 1) are opposite Vertices)

35. $x - 2y + 3 = 0$, $2x + y - 14 = 0$,

$x - 2y + 13 = 0$, $2x + y - 4 = 0$

$3x - y - 1 = 0$, $x + 3y - 17 = 0$ (Hints: angle between side and diagonal is 45°)

36. $x + 2y + \sqrt{5} = 0$, $x + 2y - \sqrt{5} = 0$

37. $x = y$ (Hint: Given diagonal does not passes the point of intersection of given sides)

38. $107x - 3y - 92 = 0$

39. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$

40. $(-1, -1)$ or $(7, 5)$

41. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$

42. $(1,-4)$, $(-9,6)$

43. $x + 5y = \pm 5\sqrt{2}$

44. $x - 3y - 31 = 0$, $3x + y + 7 = 0$

45. $\left(1, \frac{-3}{2}\right)$ or $\left(3, \frac{-1}{2}\right)$

46. $\sqrt{3}$ (Hint: Use modulus functions property)

47. (2, 3) (Hint: Circumcentre is equidistant from the vertices of triangle)

48. $3x - 4y - 3a = 0$ and $x - a = 0$

50. i. (c) ii. (b) iii. (b) iv. (c) v. (b)

51. i. $2\sqrt{5}$ ii. 2y + x + 4 = 0 iii. $\left(3, \frac{-7}{2}\right)$

iv. $\frac{\sqrt{5}}{2}$ v. $\frac{-1}{2}$

52. (a) 53. (a)

54. (c) 55. (b)

56. (b) 57. (b)

58. (d) 59. (c)

60. (a) 61. (b)

62. (a) 5y – 3x + 15 = 0

63. (b) x + y = 5

64. (c) $\left(\frac{b^2 - a^2}{2ab}\right)$

65. (a) y + 2 = 0, $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

66. (b) $\left(-\frac{1}{10}, \frac{37}{10}\right)$

67. (b)68. (b)

69. (d)

70. (b)

71. (a)

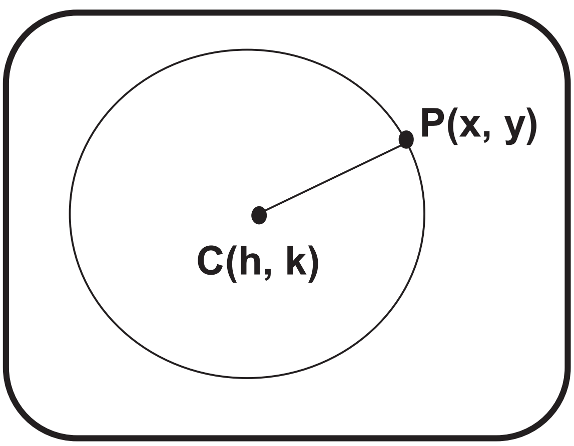
CHAPTER - 10

CONIC SECTIONS

KEY POINTS

- The curves obtained by slicing the cone with a plane not passing through the vertex are called conic sections or simply conics.
- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions.
- A conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed straight line.
- The fixed point is called focus, the fixed straight line is called directrix, and the constant ratio is called eccentricity, which is denoted by '*e*'.
- **Circle:** It is the set of all points in a plane that are equidistant from a fixed point in that plane

  Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$ where Centre (h, k), radius = r



C (h, k)

CP = CONSTANT = r

❖ **Parabola:** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line

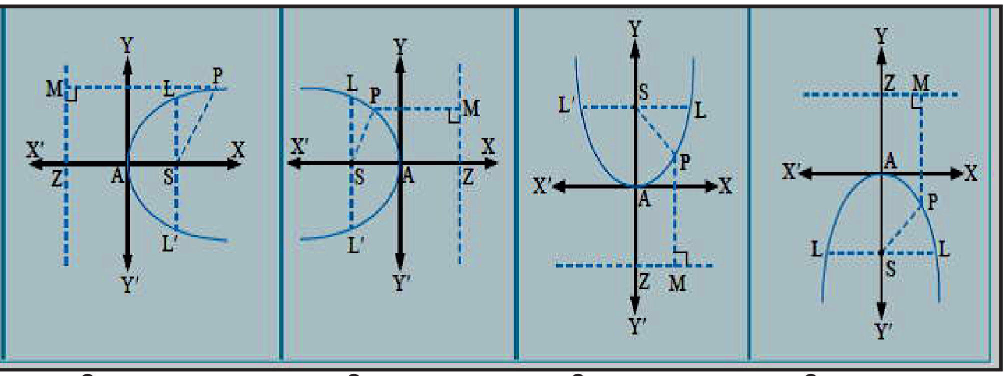
$$\frac{c}{a} = 1$$

$$e = 1$$

	$y^2 = 4ax$ *Parabola towards right*	$y^2 = -4ax$ *Parabola towards left*	$x^2 = 4ay$ *Parabola opening upwards*	$x^2 = -4ay$ *Parabola opening downwards*
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$

Note: In the standard equation of parabola, a > 0.

Figure



$y^2 = 4ax$ $\quad y^2 = -4ax$ $\quad x^2 = 4ay$ $\quad x^2 = -4ay$

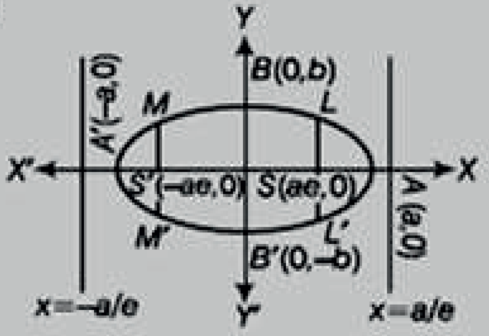
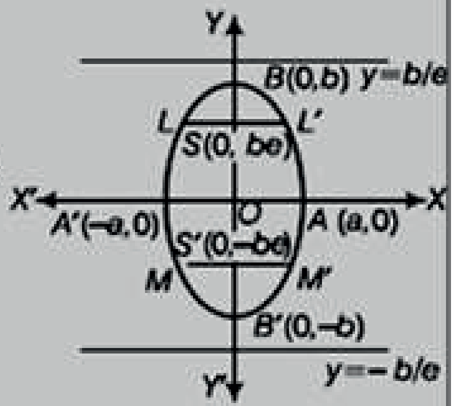
Note: In the figure above, A represents the vertex, S represents the Focus, LL' represents the Latus Rectum and Line MZ represents the Directrix to the parabola.

- **Latus Rectum:** A chord through focus perpendicular to axis of parabola is called its latus rectum.

- **Ellipse:** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points.

$$\frac{c_1 + c_2}{a} > 1$$

$$e = 1$$

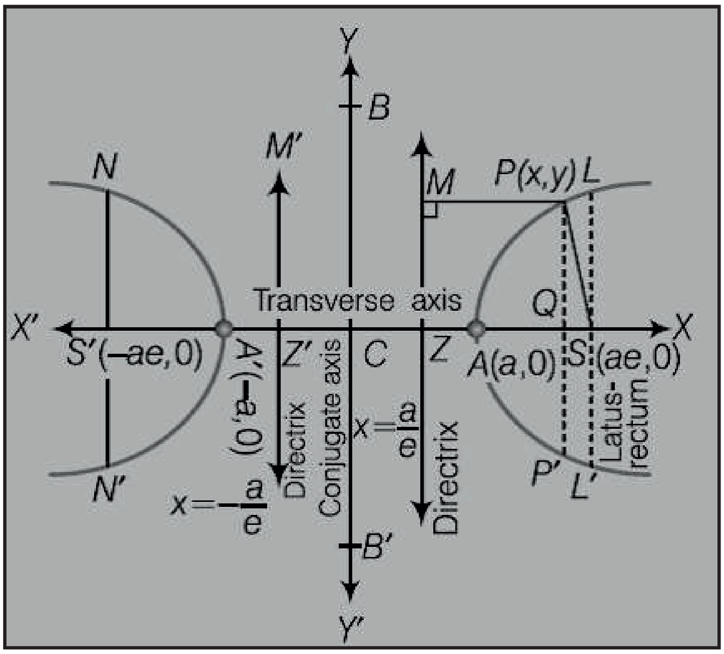
Standard equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a > b)$ (Horizontal form of an ellipse)	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 (a < b)$ (Vertical form of an ellipse)
Shape of the ellipse		
Centre	(0, 0)	(0, 0)
Equation of major axis	$y = 0$	$x = 0$
Equation of minor axis	$x = 0$	$y = 0$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Eccentricity	$e = \sqrt{\frac{a^2-b^2}{a^2}}$	$e = \sqrt{\frac{b^2-a^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Note: If $e = 0$ for an ellipse then $b = a$ and equation of ellipse will be converted in equation of the circle. Its eq. will be $x^2 + y^2 = a^2$. It is called auxiliary circle. For auxiliary circle, diameter is equal to length of major axis and $e = 0$.

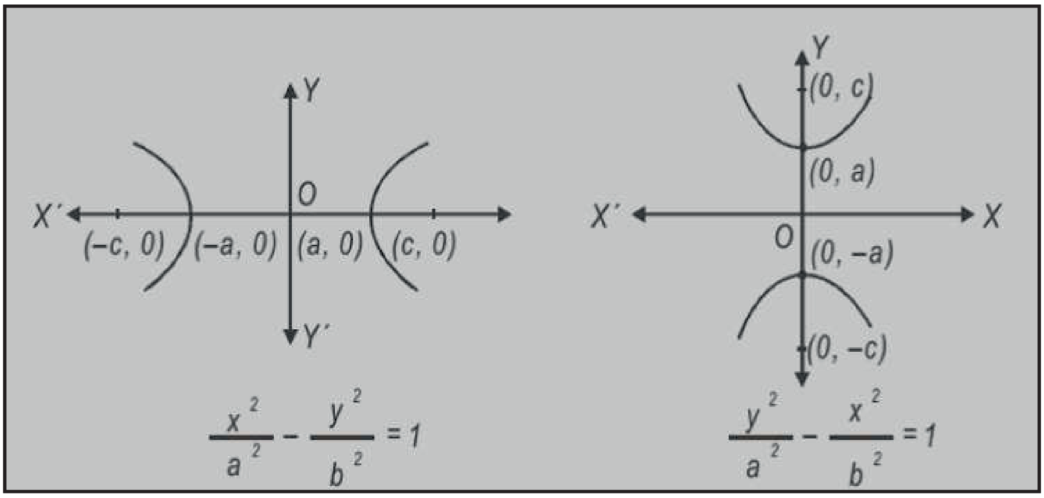
- **Latus rectum:** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola:** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.

	Hyperbola	**Conjugate hyperbola**
Standard equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$
Centre	(0, 0)	(0, 0)
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Eccentricity	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$	$e = \sqrt{\frac{a^2 + b^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

- **STANDARD HYPERBOLA:**



- **STANDARD HYPERBOLA (CONJUGATE HYPERBOLA):**



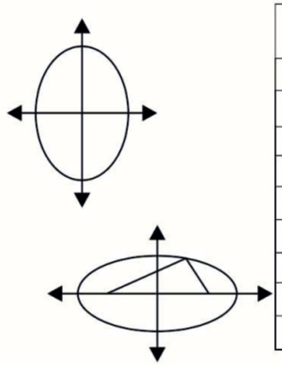
- **Latus Rectum:** Chord through foci perpendicular to transverse axis is called latus rectum.

  If $e = \sqrt{2}$ for hyperbola, then hyperbola is called rectangular hyperbola.

  For $e = \sqrt{2}$ then b = a and eq. of its hyperbola will be $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$.

MIND MAP

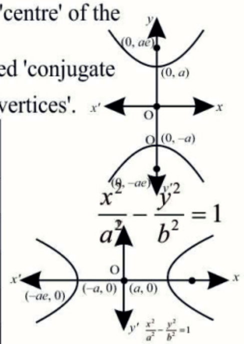
- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is constant.
- The two fixed points are called the 'foci' of the ellipse.
- The midpoint of line segment joining foci is called the 'centre' of the ellipse.
- The line segment through the foci of the ellipse is called 'major axis'.
- The line segment through centre & perpendicular to major axis is called minor axis.



Forms of ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
	$a > b$	$a > b$
Equation of major axis	$y = 0$	$x = 0$
Length of major axis	$2a$	$2a$
Equation of Minor axis	$x = 0$	$y = 0$
Leright of Minor axis	$2b$	$2b$
Directrices	$\frac{2b^2}{a}$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Equation of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Length of latus rectum	(0, 0)	(0, 0)
Centre		

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The two fixed points are called the 'foci' of the hyperbola.
- The mid-point of the line segment joining the foci is called the 'centre' of the hyperbola.
- The line through the foci is called 'transverse axis'.
- Line through centre and perpendicular to transverse axis is called 'conjugate axis'.
- Points at which hyperbola intersects transverse axis are called 'vertices'.

Forms of the hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = a$	$y = a$
Length of transverse axis	$2a$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm ae)$
Equation of latus rectum	$x = \pm ae$	$x = \pm ae$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Centre	(0, 0)	(0, 0)

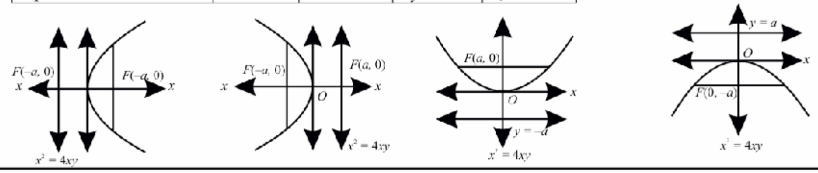


Here, $b^2 = a^2(e^2 - 1)$, $e > 1$

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane. Fixed line is called 'directrix' of parabola. Fixed point F is called the 'focus'. A line through focus & perpendicular to directrix is called 'axis'. Point axis intersection of parabola with axis is called 'vertex'.

Main facts about the parabola

Forms of Parabolas	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ax$	$x^2 = -4ax$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directix	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	$(a, 0)$	$(-a, 0)$	$(0, -a)$	$(0, -a)$
Lenght of latus rectum	$4a$	$4a$	$4a$	$4a$
Equations of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$



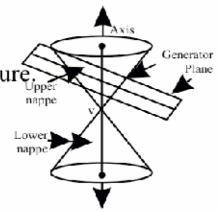
Circles, ellipses, parabolas and hyperbolas are known as conic sections because they can be obtained as intersections of plane with a double napped right circular cone α. From the given figure,

(i) Section will represent circle, if $\beta = 90^o$

(ii) Section will represent an Ellipse, if $\alpha < \beta < \pi/2$

(iii) Section will represent a parabola if if $\alpha = \beta$

(iv) Section will represent a hyperbola if $0 \le \beta < \alpha$



A circle is a set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the 'centre' of the circle and the distance from the centre to a point on the circle is called the 'radius' of the circle.

The equation of a circle with centre (h, k) and the radius r is $(x-h)^2 + (y-k)^2 = r^2$

The general equation of circle in $x^2 + y^2 + 2gx + 2fy + c = 0$ its centre is $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the centre of the circle $3x^2 + 3y^2 + 6x - 12y - 6 = 0$.

2. Find the radius of the circle $3x^2 + 3y^2 + 6x - 12y - 15 = 0$.

3. Find the equation of circle whose end points of one of its diameter are $(-2, 3)$ and $(0, -1)$.

4. If parabola $y^2 = px$ passes through point $(2, -3)$, then, find the length of latus rectum.

5. Find the coordinates of focus of parabola $3y^2 = 8x$.

6. Find the equation of the circle which passes through the point $(4, 6)$ and has its centre at $(1, 2)$.

7. Find the equation of the ellipse having foci $(0, 3)$, $(0, -3)$ and minor axis of length 8.

8. Find the length of the latus rectum of the ellipse $3x^2 + y^2 = 12$.

9. Find the eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci.

10. If the lines $5x + 12y = 3$ and $10x + 24y - 58 = 0$ are tangents to a circle, then find the radius of the circle.

11. Find the length of major and minor axis of the following ellipse, $16x^2 + 25y^2 = 400$.

12. Find the eqn. of hyperbola satisfying given conditions foci $(\pm 5, 0)$ and transverse axis is of length 8.

13. Find the coordinates of points on parabola $y^2 = 8x$ whose focal distance is 4.

14. Find the distance between the directrices to the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1.$

15. If the eccentricity of the ellipse is zero. Then show that ellipse will be a circle.

16. If the eccentricity of the hyperbola is $\sqrt{2}$. Then find the general equation of hyperbola.

17. A circle is circumscribed on an equilateral Triangle ABC where AB = 6 cm. The area of the Circumcircle is Kπ cm^2. Find the value of K.

SHORT ANSWER TYPE QUESTIONS

18. Find equation of an ellipse having vertices (0, ± 5) and foci (0, ± 4).

19. If the distance between the foci of a hyperbola is 16 and its eccentricity is 2, then obtain the equation of a hyperbola.

20. Find the equation for the ellipse that satisfies the given condition Major axis on the *x*-axis and passes through the points (4, 3) and (6, 2).

21. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of diameter.

22. Find the equation of the ellipse with foci at (± 5, 0) and x = 18 as one of the directrices.

23. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, find the equation of the hyperbola if its eccentricity is 2.

24. Find the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which passes through the points (3, 0) and $(3\sqrt{2}, 2)$.

25. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

26. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

27. Find the equation of a circle whose centre is at (4, –2) and $3x - 4y + 5 = 0$ is tangent to circle.

28. If equation of the circle is in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$ then prove that its centre and radius will be (–g, –f) and $\sqrt{g^2 + f^2 - c}$ respectively. (Hint: Complete the square and compare with standard formula)

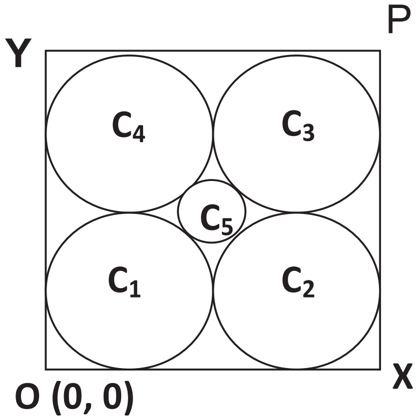
29. If the end points of a diameter of circle are (x_1, y_1) and (x_2, y_2) then show that equation of circle will be $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$. (Hint: Angle in semicircle is of 90°)

30. Find the equation of the circle which touches the lines $x = 0$, $y = 0$ and $x = 2c$ and $c > 0$.

31. Find the equation of the set of all points the sum of whose distance from A(3,0) and B(9,0) is 12 unit. Identify the curve thus obtained.

32. Find the equation of the set of all points such that the difference of their distance from (4,0) and (–4,0) is always equal of 2 unit. Identify the curve thus obtained.

33. If OXPY is a square of Side 4 cm in First Quadrant, where O is the origin. (OY and OX lie on y-axis and x-axis respectively). Find the equation of the circle C_1, C_2, C_3, $C_4$and C_5.



LONG ANSWER TYPE QUESTIONS

34. Prove that the points (1, 2), (3, – 4), (5, – 6) and (11, – 8) are concyclic.

35. A circle has radius 3 units and its centre lies on the line y = x – 1. If it is passes through the point (7, 3) then find the equations of the circle.

36. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, – 9). Find its centre and radius.

37. Find the equation of circle having centre (1, – 2) and passing through the point of intersection of the lines 3x + y = 14 and 2x + 5y = 18.

38. Show that the points A(5,5), B(6,4), C(–2,4) and D(7,1) all lies on the circle. Find the centre, radius and equation of circle.

39. Find the equation of the ellipse in which length of minor axis is equal to distance between foci. If length of latus rectum is 10 unit and major axis is along the x axis.

40. Find the equation of the hyperbolas whose axes (transverse and conjugate axis) are parallel to x axis and y axis and centre is origin such that Length of latus rectum length is 18 unit and distance between foci is 12 unit.

41. Prove that the line $3x + 4y + 7 = 0$ touches the circle $x^2 + y^2 – 4x – 6y – 12 = 0$. Also find the point of contact.

42. Find the equations of tangents to the circle

(a) $x^2 + y^2 – 2x – 4y – 4 = 0$ which are parallel to $3x – 4y – 1 = 0$

(b) $x^2 + y^2 – 4x – 6y – 12 = 0$ which are perpendicular to $4x + 3y = 7$

43. Find the equation of Circle in each of the following cases:

(a) Touches both the coordinate axes in first quadrant and having radius = 1 unit

(b) Touches both the coordinate axes in second quadrant and having radius = 2 units

(c) Touches both the coordinate axes in third quadrant and having radius = 3 units

(d) Touches both the coordinate axes in fourth quadrant and having radius = 4 units

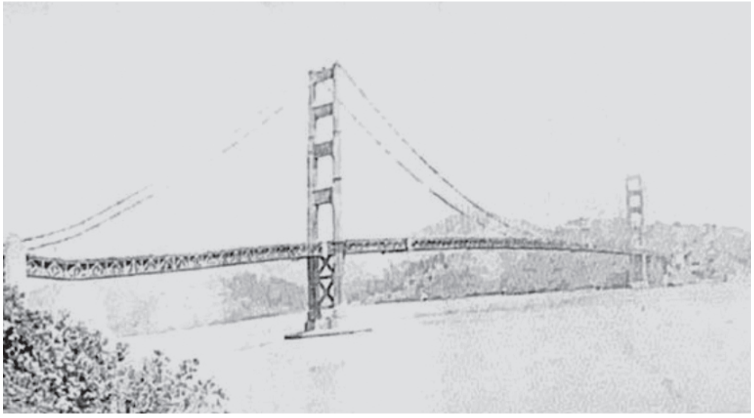
(e) Touches the x-axis at origin and having radius = 5 units

(f) Touches the y-axis at origin and having radius = 6 units

CASE STUDY TYPE QUESTIONS

44. A beam is supported at its ends by supports which are 12m apart. Since the load is concentrated at its centre, there is a

deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola.



Based on the above information answer the following :-

i. How for from the centre is deflection of 1cm?

(a) $2\sqrt{6}$ m (b) $3\sqrt{6}$ m

(c) $2\sqrt{3}$ m (d) $4\sqrt{3}$ m

ii. What will be the equation of parabola?

(a) $x^2 = 240000y$ (b) $x^2 = 120000y$

(c) $x^2 = 160000y$ (d) $x^2 = 100000y$

iii. At a distance of 2m from the centre, what will be the deflection of the beam?

(a) $\frac{3}{2}$ (b) $\frac{8}{3}$

(c) $\frac{4}{3}$ (d) $\frac{1}{5}$

iv. What is the length of latus rectum of the parabola?

(a) 100000 (b) 120000

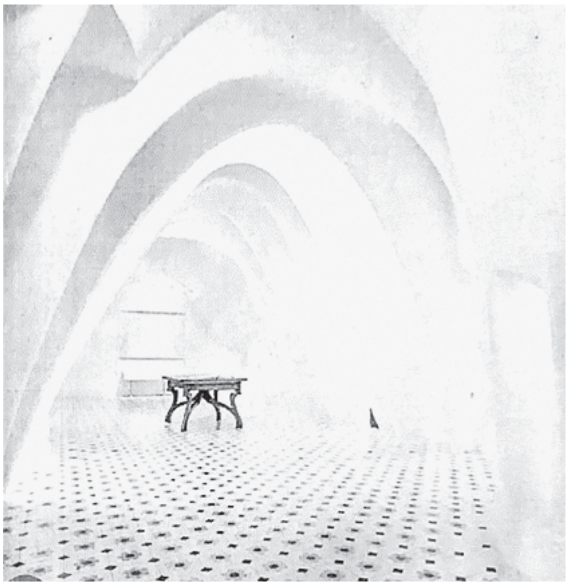
(c) 130000 (d) 140000

v. What is the difference of deflection of beam at a distance of 1m and 2m from the centre?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{3}{7}$

45. A window is in the shape of parabola with a triangle inscribed in it. The triangle is formed in such a way that the verticles of triangle coincides with vertex of parabola and end points of latus rectum. The equation of parabola is given by $x^2 = 24y$.



What are the vertices of triangle

i. What are the verticle of triangle

ii. Find the length of altitude of the triangle -

iii. Find the area of the triangle?

iv. Find the length of the longest side of the triangle?

v. Find the length of latus rectum of the parabola?

Multiple Choice Questions

46. The equation of the circle which passes through the points of intersection of the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 6y = 0$ and has its centre at (3/2, 3/2) is -

(a) $x^2 + y^2 + 3x + 3y + 9 = 0$

(b) $x^2 + y^2 + 3x + 3y = 0$

(c) $x^2 + y^2 - 3x - 3y = 0$

(d) $x^2 + y^2 - 3x - 3y + 9 = 0$.

47. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ -

(a) (4,9) (b) (9,4)

(c) (7,4) (d) (4,7).

48. Value of p, for which the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ represent a circle of radius 5 units is -

(a) 3 (b) – 3

(c) both (a) & (b) (d) Neither (a) nor (b).

49. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is 'e' then the value of '5e' is -

(a) 3 (b) 4

(c) 2 (d) 1.

50. The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is (a, b) then (2a + 3 b) is -

(a) 0 (b) 2

(c) 3 (d) 5.

51. The radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is -

(a) 1 (b) 2

(c) 3 (d) 5.

52. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 8y$ to the ends of its latus rectum is -

(a) 4 sq. units (b) 8 sq. units

(c) 12 sq. units (d) 16 sq. units.

53. Match the following:

	COLUMN 1 Conic		**COLUMN 2 Eccentricity**
A	CIRCLE	P	$e < 1$
B	PARABOLA	Q	$e > 1$
C	ELLIPSE	R	$e = 0$
D	HYPERBOLA	S	$e = 1$

Which one of the following is true?

A → P, B→ Q, C→ R, D → S

A → S, B→ Q, C→ R, D → P

A → Q, B→ S, C→ R, D → P

A → R, B→ S, C→ P, D → Q

54. At what point on the parabola $x^2 = 9y$ is the abscissa three times that of ordinate

(a) (1, 1) (b) (3, 1)

(c) (–3, 1) (d) (–3, –3)

55. The equation of parabola with vertex at origin and axis on x-axis and passing through point (2, 3) is

(a) $y^2 = 9x$ (b) $y^2 = \frac{9x}{2}$

(c) $y^2 = 2x$ (d) $y^2 = \frac{2x}{9}$

56. If the centroid of an equilateral triangle is (1, 1) and its one vertex is (–1, 2) then the equation of its circumcirle is

(a) $x^2 + y^2 - 2x - 2y - 3 = 0$

(b) $x^2 + y^2 + 2x - 2y - 3 = 0$

(c) $x^2 + y^2 + 2x + 2y - 3 = 0$

(d) none of these

57. If the circle $x^2 + y^2 = a$ and $x^2 + y^2 - 6x - 8y + 9 = 0$ touch externally, then a =

(a) 1 (b) –1

(c) 21 (d) 16

58. The area of the triangle formed by the line joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is

(a) 12 sq units (b) 16 sq units

(c) 18 sq units (d) 24 sq units

59. The eccentricity of the ellipse, if the distance between the foci is equal to the length of the latus rectum is

(a) $\frac{\sqrt{5}-1}{2}$ (b) $\frac{\sqrt{5}+1}{2}$

(c) $\frac{\sqrt{5}-1}{4}$ (d) None of these

60. If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ then, the relation between, e_1 and e_2 is

(a) $3e_1^2 + e_2^2 = 2$ (b) $e_1^2 + 2e_2^2 = 3$

(c) $2e_1^2 + e_2^2 = 3$ (d) $e_1^2 + 3e_2^2 + 2$

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices. Only one of which is the correct answer. You have to select of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct, reason is a correct explanation for asserction.

(b) Assertion is correct, reason is correct, reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

61. Parabola is symmetric with respect to the axis of the parabola.

Assertion: If the equation has a term y^2, then the axis of symmetry is along the x-axis.

Reason: If the equation has a term x^2, then the axis of symmetry is along the x-axis.

62. Let the centre of an ellipse is at (0, 0)

Assertion: If major axis is on the y-axis and ellipse passes thorught the points (3, 2) and (1, 6), then the equation of ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$

Reason: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is an equation of ellipse if major axis is along y-axis. (if a > b)

63. **Assertion:** Centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is (3, – 2)

Reason: The coordinates of the centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are ($-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of y)

64. **Assertion:** Radius of the circle $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8}=0$ is 1.

Reason: Radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$\sqrt{\left(\frac{1}{2}\text{coeff. of } x\right)^2+\left(\frac{1}{2}\text{coeff. of } y\right)^2-\text{constant term}}$$

ANSWERS

1. (–1, 2)
2. $2\sqrt{5}$ Units
3. $x^2 + y^2 + 2x - 2y - 3 = 0$ (Hint: Mid-point of diameter is center)
4. 4.5 units
5. $\left(\frac{2}{3}, 0\right)$
6. $(x - 1)^2 + (y - 2)^2 = 25$
7. $\frac{x^2}{16}+\frac{y^2}{25}=1$
8. $\frac{4\sqrt{3}}{3}$
9. e = $2\sqrt{3}$

10. 2 units (Hint: distance between two parallel tangents is the length of diameter.)

11. Length of Major Axis = 10, Length of Major Axis = 8

12. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

13. (2, ±4)

14. 18 (Hint: Distance between two directrices is $\frac{2a}{e}$)

16. $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$

17. K = 12

18. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

19. $x^2 - y^2 = 32$ or $y^2 - x^2 = 32$

20. $\frac{x^2}{52} + \frac{y^2}{13} = 1$

21. (1, 2)

22. $\frac{x^2}{90} + \frac{y^2}{65} = 1$

23. $\frac{x^2}{4} - \frac{y^2}{12} = 1$

24. $e = \frac{\sqrt{13}}{3}$

25. $e = \frac{\sqrt{3}}{2}$

26. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

27. $x^2 + y^2 - 8x + 4y - 5 = 0$

30. $x^2 + y^2 - 2cx \pm 2cy + c^2 = 0$

31. $3x^2 + 4y^2 = 36$, Ellipse

32. $15x^2 - y^2 = 15$, Hyperbola

33. $C_1: (x - 1)^2 + (y - 1)^2 = 1$

$C_2: (x - 3)^2 + (y - 1)^2 = 1$

C_3: $(x-3)^2 + (y-3)^2 = 1$

C_4: $(x-1)^2 + (y-3)^2 = 1$

C_5: $(x-2)^2 + (y-2)^2 = \left(\sqrt{2}-1\right)^2$

35. $x^2 + y^2 - 8x - 6y + 16 = 0$ or $x^2 + y^2 - 14x - 12y + 76 = 0$

36. $x^2 + y^2 - 14x - 6y - 111 = 0$ Centre (7, 3), Radius = 13 units

37. $(x-1)^2 + (y+2)^2 = 25$

38. $x^2 + y^2 - 4x - 2y - 20 = 0$ Centre(2, 1), Radius = 5 units

39. $x^2 + 2y^2 = 100$

40. $3x^2 - y^2 = 27$

41. Point of contact = (–1, –1)

42. (a) $3x - 4y - 10 = 0$ or $3x - 4y + 20 = 0$

(b) $3x - 4y + 31 = 0$ or $3x - 4y - 19 = 0$

43. (a) $(x-1)^2 + (y-1)^2 = 1$ (b) $(x+2)^2 + (y-2)^2 = 4$

(c) $(x+3)^2 + (y+3)^2 = 9$ (d) $(x-4)^2 + (y+4)^2 = 16$

(e) $x^2 + (y \pm 5)^2 = 25$ (f) $(x \pm 6)^2 + y^2 = 36$

44. i. (a) ii. (b) iii. (b) iv. (b) v. (c)

45. i. (0, 0), (±12, 6) ii. 6 units iii. 72 sq units iv. 24 units v. 24 units

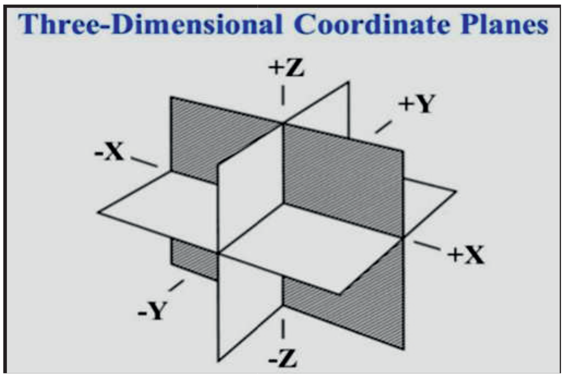
46. (c)	47. (d)	48. (c)	49. (b)
50. (a)	51. (d)	52. (b)	53. (d)
54. (b)	55. (b)	56. (a)	57. (a)
58. (c)	59. (a)	60. (c)	61. (c)
62. (b)	63. (a)	64. (a)	

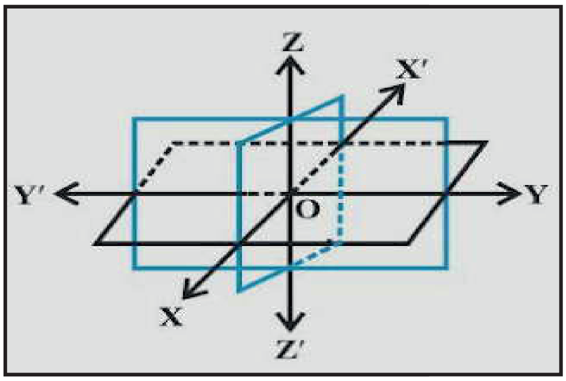
CHAPTER - 11

INTRODUCTION TO THREE-DIMENSIONAL COORDINATE GEOMETRY

KEY POINTS

- Three mutually perpendicular lines X′OX, Y′OY and Z′OZ in space **constitute rectangular coordinate system** which in turn divide the space into eight parts known as **octants** and the lines are known as **Coordinate axes**.





❖ **Coordinate axes:** XOX', YOY', ZOZ' are respectively called x-axis, y-axis and z-axis.

❖ **Coordinate planes:** XOY, YOZ, ZOX or XY, YX, ZX planes

❖ **Octants:** XOYZ, X'OYZ, X'OY′Z, XOY'Z, XOYZ', X'OYZ′, X′OY'Z' and XOY′Z' denoted as I, II, ….. VIII octant respectively.

❖ Coordinates of a points lying on x-axis, y-axis and z-axis are of the form (x,0,0), (0,y,0),(0,0,z) respectively.

❖ The signs of coordinates in eight octants are as follows:

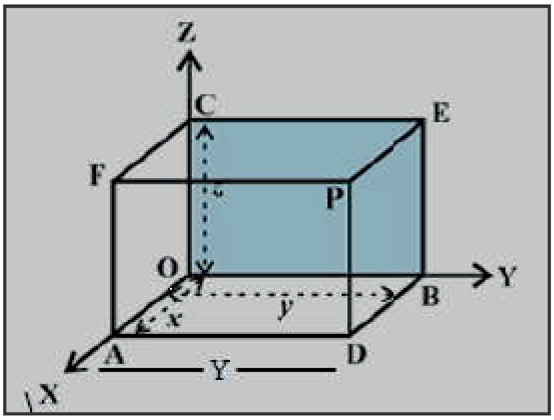
(i) (+ + +) (iii) (- - +) (v) (+ + -) (vii) (- - -)

(ii) (- + +) (iv) (+ - +) (vi) (- + -) (viii) (+ - -)

❖ Coordinates of a points lying on xy-plane, yz-plane and xz-plane are of the form (x,y,0), (0,y,z),(x,0,z) respectively.

❖ The reflection of the point (x, y, z) in xy-plane, yz-plane and xz-plane is (x, y, –z), (–x, y,z) and (x, –y, z) respectively.

❖ Absolute value of the Coordinates of a point P (x, y, z) represents the perpendicular distances of point P from three coordinate planes YZ, ZX and XY respectively.



● The distance between the point A(x_1, y_1, z_1) and B(x_2, y_2, z_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

VERY SHORT ANSWER TYPE QUESTIONS

1. What will be the image of (–1, 2, –3) in XZ plane.

2. What will be the image of (–1, 2, –3) in YZ plane.

3. In which octant The Point P (–5, 4, –3), lies?

4. If a < 0, b > 0 & c < 0, in which octant the Point P (a, b, –c) lies.

5. Find the perpendicular distance of the point P(- 6, 7, –8) from xy-plane.

6. Find the perpendicular distance of the point P(- 3, 5, –12) from x-axis.

7. Find the perpendicular distance of the point P(- 3, 4, –5) from z-axis.

8. Find the coordinates of foot of perpendicular from (3, 7, 9) on y-axis.

9. If the distance between the points (a, 2, 1) and (1, –1, 1) is 5, then find the sum of all possible value of a.

10. Name the axis formed by intersection of two planes xy-plane and yz-plane.

11. Find Distance of the point (3, 4, 5) from the origin (0, 0, 0).

12. If $(c - 1) > 0$, $(a + 2) < 0$ and $b > 0$ then the point P(a, –b, c) lies in which octant?

13. What are the coordinates of the vertices of a cube whose edge is 2 unit, one of whose vertices coincides with the origin and the three edges passing through the origin coincides with the positive direction of the axes through the origin?

14. Let A, B, C be the feet of perpendiculars from point P(1, –2, –3) on the xy-plane, yz-plane and xz-plane respectively. Find the coordinates of A, B, C.

15. If a parallelepiped is formed by planes drawn through the point (5,8,10) and (3,6,8) parallel to the coordinates planes, then find the length of the diagonal of the parallelepiped.

16. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13,10 and 8 unit.

17. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.

18. Find the point on y–axis which is equidistant from the point (3,1,2) and (5, 5, 2).

19. Determine the point in yz plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

20. Find the length of the medians of the triangle with vertices A(0, 0, 3), B(0, 4, 0) and C (5, 0, 0).

21. If the extremities (end points) of a diagonal of a square are (1,– 2, 3) and (2,–3, 5) then find the length of the side of square.

22. Three consecutive vertices of a parallelogram ABCD are A(6,–2,4) B(2, 4,–8), C(–2, 2, 4). Find the coordinates of the fourth vertex.

23. If the points A(1, 0, –6), B(3, p, q) and C(5, 9, 6) are collinear, find the value of p and q.

24. Show that the point A(1, 3, 0), B(–5, 5,, 2), C(–9, –1, 2) and D(–3, –3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle. (Hint: diagonals are not equal)

25. Describe the vertices and edges of the rectangular parallelepiped with one vertex (3, 5, 6) placed in the first octant with one vertex at origin and edges of parallelepiped lie along x, y and z-axis.

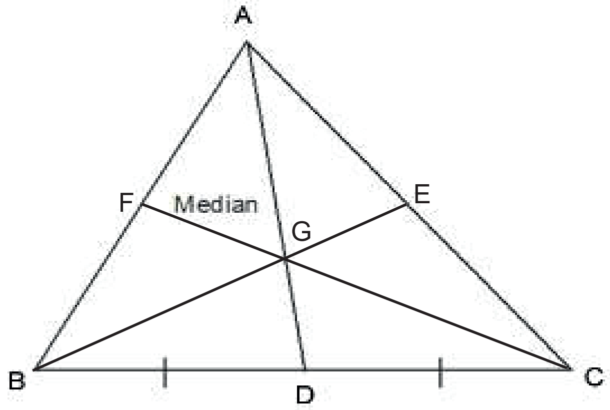
26. Find the coordinates of the point which is equidistant from the point (3, 2, 2), (–1, 2, 2), (4, 5, 6) and (2, 1, 2).

27. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) form a right angled isosceles triangle.

28. Show that the points (5, -1, 1), (7, -4, 7), (1, -6, 10) and (-1, -3, 4) are the vertices of a rhombus.

CASE STUDY TYPE QUESTIONS

29. Consider a ΔABC with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$. AD, BE and CF are medians of ΔABC.



Based on the above information, answer the following questions:-

i. Coordinates of Point D are?

(a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ (b) $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$

(c) $\left(\frac{x_3+x_1}{2}, \frac{y_3+y_1}{2}, \frac{z_3+z_1}{2}\right)$ (d) None of these

ii. A point G divides AD in 2 : 1, the coordinates of G are

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}, \frac{z_1+2z_2}{3}\right)$

(c) $\left(\frac{x_2+2x_1}{3}, \frac{y_2+2y_1}{3}, \frac{z_2+2z_1}{3}\right)$

(d) None of these

iii. For ΔABC, G is

(a) Incentre (b) Circumcentre

(c) Centroid (d) Orthocentre

iv. G divides BE in ratio

(a) 1 : 2 (b) 2 : 1

(c) 3 : 1 (d) 1 : 3

v. If ΔABC is equilateral, then coordinates of circumcentre are

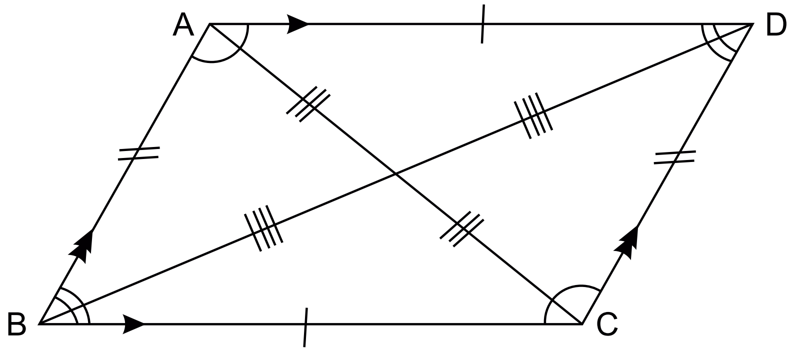
(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}, \frac{z_1+z_2+z_3}{2}\right)$

(c) $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$

(d) None of these

30. ABCD is a field in shape of parallelogram coordinate of A, B and C are (3, –1, 2), (1, 2, –4) and (–1, 1, 2) resp.



Based on the above information answer the following :-

i Coordinates of mid point of AC be

ii. Coordinates of D be

iii. Length of side BC is

iv. Coordinates of centroid G of ΔABC be

v. Length of AC is

Multiple Choice Questions

31. A point on Z-plane which is equidistant from the points (1, -1, 0), (2, 1, 2), (3, 2, -1) is

(a) $\left(\frac{1}{5}, 0, \frac{31}{10}\right)$ (b) $\left(\frac{1}{10}, 0, \frac{31}{5}\right)$

(c) $\left(\frac{31}{10}, 0, \frac{1}{5}\right)$ (d) $\left(\frac{31}{5}, 0, \frac{1}{10}\right)$

32. Lengths of medians of triangle ABC with vertices

A(0, 0, 2), B(0, 4, 0) and C(8, 0, 0) are:

(a) $2\sqrt{6}, \sqrt{33}, \sqrt{69}$ (b) 2, 4, 8

(c) 8, 4, 2 (d) $2\sqrt{5}, 10, 2\sqrt{17}$

33. A point on y-axis which is at a distance of $\sqrt{10}$ from the point (1, 2, 3) is

(a) (2, 0, 2) (b) (0, 2, 2)

(c) (2, 2, 2) (d) (0, 2, 0)

34. The locus of a point for which y = 0, z = 0 is

(a) x-axis (b) y-axis

(c) z-axis (d) y and z-axes

35. A line is parallel to xy-plane if all points on the line have equal

(a) x-coordinates (b) y-coordinates

(c) z-coordinates (d) x and y-coordinate

36. x-axis is the intersection of two planes

(a) xy and xz (b) yz and zx

(c) xy and yz (d) none of these

37. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(a) 5 (b) ± 5

(c) – 5 (d) None of these

38. The point (2, 3, –4) lies in the

(a) First octant (b) Second octant

(c) Fifth octant (d) Seventh octant

39. x = a. represents a plane parallel to

(a) xy – plane (b) yz-plane

(c) xz-plane (d) none of these

40. The distance between the point (a, b, c) and (0, 0, -c) is

(a) $\sqrt{a^2+b^2}$ (b) $\sqrt{a^2+b^2+c^2}$

(c) $\sqrt{a^2+b^2+2c^2}$ (d) $\sqrt{a^2+b^2+4c^2}$

Direction: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below:

(a) Assertion is correct, reason is correct; reason is not a explanation for assertion.
(b) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

41. **Assertion:** If three vertices of a parallelogram ABCD are A(3, –1, 2) B(1, 2, –4) and C(–1, 1, 2), then the fourth vertex is (1, –2, 8).

Reason: Diagonals of a parallelogram bisect each other and mid-point of AC and BD coincide.

42. Assertion: The distance of a point P(x, y, z) from the origin O(0, 0, 0) is given by OP = $\sqrt{x^2 + y^2 + z^2}$.

Reason: A point is on the x-axis. Its y-coordinate and z-coordinate are 0 and 0 respectively.

43. **Assertion:** Coordinates (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

Reason: Opposite sides of a parallelograom are equal and diagonals are not equal.

44. **Assertion:** If P(x, y, z) is any point in the space, then x, y and z are perpendicular distance from YZ, ZX and XY-planes, respectively.

Reason: If three planes are drawn parallel to YZ, ZX and XY-planes such that they intersect X, Y and Z-axes at (x, 0, 0), (0, y, 0) and (0, 0, z), then the planes meet in space at a point P(x, y, z).

45. **Assertion:** The distance between the points P(1, – 3, 4) and Q(–4, 1, 2) is $\sqrt{5}$ units.

Reason: PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

ANSWERS

1.	(–1, –2, –3)	2.	(1, 2, –3)
3.	Octant VI (OX' YZ')	4.	Octant II (OX' YZ)
5.	8 units	6.	13 units
7.	5 units	8.	(0, 7, 0)
9.	5 + (–3) = 2	10.	Y-axis
11.	$5\sqrt{2}$	12.	Octant III (OX' Y'Z)

13. (2, 0, 0) , (2, 2, 0) , (0, 2, 0) , (0, 2, 2) , (2, 0, 2), (0, 0, 2) , (2, 2, 2)

14. (4, –3, 0) , (0, –3, –5) , (4, 0, –5)

15. $2\sqrt{3}$

16.	$\sqrt{333}$	18.	(0, 5, 0)
19.	(0, 1, 3)	20.	$\frac{\sqrt{77}}{2}, \frac{7\sqrt{2}}{2}, \frac{5\sqrt{5}}{2}$
21.	$\sqrt{3}$	22.	(2, –4, 16)

23. $p=\frac{9}{2}, q=0$

25. (0, 0, 0), (3, 0, 0), (3, 5, 0), (0, 5, 0), (0, 5, 6)

(0, 0, 6), (3, 0, 6), (3, 5, 6), $\sqrt{61}$, $\sqrt{45}$, $\sqrt{34}$

26. (1, 3, 5)

27. [Hint: length of three side of triangle is $3\sqrt{2}, 3\sqrt{2}, 6$]

28. [Hint: each side= 7 units]

29. i. (b) ii. (a) iii. (c) iv. (b) v. (a)

30. i. (1, 0, 2) ii. (1, –2, 8) iii. $\sqrt{41}$ iv. (1, 2/3, 0) v. $2\sqrt{5}$

31.	(c)	32.	(a)	33.	(d)
34.	(a)	35.	(c)	36.	(a)
37.	(b)	38.	(c)	39.	(b)
40.	(d)	41.	(a)	42.	(b)
43.	(a)	44.	(d)	45.	(d)

CHAPTER - 12

LIMITS AND DERIVATIVES

KEY POINTS

- To check whether limit of f(x) as x approaches to a exists i.e., $\lim_{x \to a} f(x)$ exists, we proceed as follows.

  (i) Find L.H.L at x = a using L.H.L. = $\lim_{h \to 0} f(a-h)$.

  (ii) Find R.H.L at x = a using R.H.L. = $\lim_{h \to 0} f(a+h)$.

  (iii) If both L.H.L. and R.H.L. are finite and equal, then limit at x = a i.e., $\lim_{x \to a} f(x)$ exists and equals to the value obtained from L.H.L or R.H.L else we say "limit does not exist".

- $\lim_{x \to a} f(x) = l$, if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = l$

▶ **ALGEBRA OF LIMITS:** Let f, g be two functions such that $\lim_{x \to c} f(x) = l$, and $\lim_{x \to c} g(x) = m$.

  - $\lim_{x \to c}[\alpha f(x)] = \alpha \lim_{x \to c} f(x) = \alpha l$, for all $\alpha \in R$
  - $\lim_{x \to c}[f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m$
  - $\lim_{x \to c}[f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = l \cdot m$
  - $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{l}{m},\ m \neq 0,\ g(x) \neq 0$

- $\lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l},\ l \neq 0,\ f(x) \neq 0$

- $\lim_{x \to c} [f(x)]^n = [(\lim_{x \to c} f(x)]^n = l^n$, for all $n \ \varepsilon \ N$

▶ SOME IMPORTANT RESULTS ON LIMITS:

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{\tan x}{x} = 1$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$
- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$
- $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$
- $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$
- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x)$

▶ SOME IMPORTANT RESULTS ON DERIVATIVE:

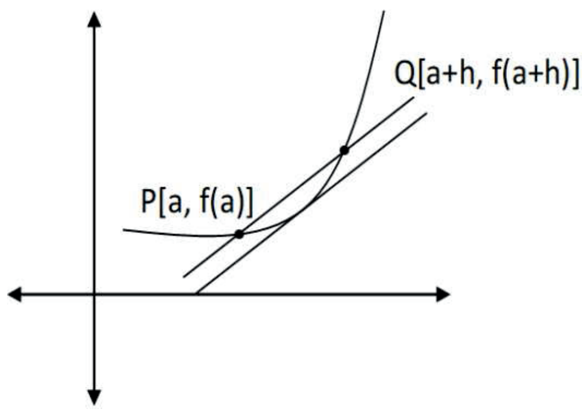
• $\frac{d}{dx}(\sin x) = \cos x$	• $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
• $\frac{d}{dx}(\cos x) = -\sin x$	• $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$
• $\frac{d}{dx}(\tan x) = \sec^2 x$	• $\frac{d}{dx}(\text{cosecx}) = -\text{cosec} x \cdot \cot x$

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(a) = 0,\ a = \text{constant}$

- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(a^x) = a^x \cdot \log a$

▶ **Logarithm Properties:** $a^b = c$ (exponential form)
$\text{Log}_a c = b$, (log form) a > 0, c > 0

- $\log_e(A \cdot B) = \log_e A + \log_e B$
- $\log_a a = 1$
- $\log_e\left(\frac{A}{B}\right) = \log_e A - \log_e B$
- $\log_b a = \frac{1}{\log_a b}$
- $\log_e(A^m) = m \cdot \log_e A$
- $\log_b a = \frac{\log_c a}{\log_c b}$ (base change formula)
- $\log_a(1) = 0$
- $a^{m\log_a k} = k^m$
- $\log_B(A) = x$, then $B^x = A$
- Let y = f(x) be a function defined in some neighbourhood of the point 'a'. Let P[a, f(a)] and Q[a + h, f(a + h)] are two points on the graph of f(x) where h is very small and h ≠ 0.



$$\text{Slope of } PQ = \frac{f(a+h)-f(a)}{h}$$

- If $\lim_{h\to 0}$ point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

  $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ (if exists) is called derivative of f(x) at the point 'a'. It is denoted by f'(a).

ALGEBRA OF DERIVATIVES:

- $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$, where c is a constant
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Product Rule:

- $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

Quotient Rule:

- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$
- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h,k) is given by $\frac{dy}{dx}\Big|_{(h,k)}$ and is denoted by 'm'

MIND MAP

Limits and Derivatives

Limits of Polynomial function

A function *f* is said to be a polynomial function of degree *n*.

$f(x) = a_0 + a_1x + a_2x^2 + - a_nx^n$

where ais are real numbers such that $a_n \neq 0$ for some natural number *n*.

Limit

Limit is used when we have to find value of a function near to some value. To express the limit of a function, we represent it as:

$\lim_{y \to c}$

Derivative at a point

He derivative of *f* at '*a*' is defined by

$$\lim_{(h \to 0)} \frac{f(a+h) - f(a)}{h}$$

Voided this limit exists. Derivative of *f*(*x*) '*a*' is denoted by *f*'(*a*)

$$f'(a) \lim_{(h \to 0)} \frac{f(a+h) - f(a)}{h}$$

Limits of Trigonometric Function

$\lim_{x \to 0} \sin x = 0$

$\lim_{x \to 0} \cos x = 1$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 0} \frac{\tan x}{x} = 1$

$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\lim_{x \to 0} \frac{\log_e(1+x)}{x} = 1$

$\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \log_e a$

$\lim_{x \to 0} \frac{x^n - a^n}{x - a} = na^{n-1}$

$\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$

LEFTHAND LIMIT (LHL) at '*a*'

$= \lim_{x \to 0} - f(n) = \lim_{n \to 0} f(a-h)$

RIGHT HAND LIMIT (RHL) at '*a*'

$= \lim_{x \to 0} + f(n) = \lim_{x \to 0} f(a+h)$

Computing Limits

- $\lim_{x \to a}[p(x) + g(x)] = \lim_{x \to a} p(x) + \lim_{x \to a} g(x)$
- $\lim_{x \to a}[p(x) - g(x)] = \lim_{x \to a} p(x) - \lim_{x \to a} g(x)$
- For every real number k, $\lim_{x \to a}[kp(x)] = k \lim_{x \to a} p(x)$
- $\lim_{x \to a}[p(x)q(x)] = \lim_{x \to a} p(x) \times \lim_{x \to a} q(x)$
- $\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{\lim_{x \to a} p(x)}{\lim_{x \to a} q\hat{A} \times \hat{A}}$

Rules of Differentation

$\frac{d}{dx}(cu) = c.u.'$

$\frac{d}{dx}(u^n) = n.u^{n\ 1}.u'$

$\frac{d}{dx}(u \pm v) = u' \pm v'$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.u' - u.v'}{v^2}$

$\frac{d}{dx}(u.v)\ u.v' + v.u'$

$\frac{d}{dx}(\log_a u)\frac{1}{u1na}.u'$

$\frac{d}{dx}(a^u) = a^u.\ \text{In}\ a.u'$

$\frac{d}{dx}(\sin u) = \cos u.u' \frac{d}{dx}(\text{coses}\ u) = -\csc u \cot .u'$

$\frac{d}{dx}(\cos u) = -\sin u.u' \frac{d}{dx}(\sec u) = \sec u \tan u.u'$

$\frac{d}{dx}(\tan u)\sec^2 u.u' \frac{d}{dx}(\cot u) = -\text{coses}\ u \cot u.u'$

$\frac{d}{dx}(arc \sin u) = \frac{1}{\sqrt{1-u^2}}.u' \frac{d}{dx}(arc \cos u) = \frac{-1}{\sqrt{1-u^2}}.u'$

$\frac{d}{dx}(arc \tan u) = \frac{1}{1+u^2}.u' \frac{d}{dx}(arc \cot u) = \frac{1}{1+u^2}.u'$

$\frac{d}{dx}(arc\ \text{coses}\ u) = \frac{1}{|u|\sqrt{u^2-1}}.u' \frac{d}{dx}(arc \sec u) = \frac{-1}{|u|\sqrt{u^2-1}}.u'$

VERY SHORT ANSWER TYPE QUESTIONS

1. Evaluate $\lim_{x \to 3} \frac{4x+3}{x-2}$

2. Evaluate $\lim_{x \to 2} \frac{x^2+x-2}{x-1}$

3. Evaluate $\lim_{x \to 2} \frac{x^4-16}{x-2}$

4. Evaluate $\lim_{x \to 0} \frac{(1+x)^8-1}{x}$

5. Evaluate $\lim_{x \to 0} \frac{\sin^2 3x}{x^2}$

6. Evaluate $\lim_{x \to 0} \frac{\sin^3 x/2}{x^3}$

7. Evaluate $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

8. Evaluate $\lim_{x \to 0} \frac{2^x-1}{x}$

9. Evaluate $\lim_{x \to 0} \frac{5^x-1}{3^x-1}$

10. Evaluate $\lim_{x \to 0} \frac{3^{-2x}-1}{x}$

11. Evaluate $\lim_{x \to 0} \frac{\log(1-3x)}{x}$

12. Evaluate $\lim_{x \to 0} \frac{7^x-1}{\tan x}$

13. Differentiate $f(x) = x^2 + \cos x$

14. If $y = (x+1)(x-2)$, find $\frac{dy}{dx}$

15. If $y = \frac{x^5}{x-3}$, find $\frac{dy}{dx}$

Short Answer Type Questions

16. Evaluate $\lim_{x \to 0} \frac{(1+x)^m - 1}{(1+x)^n - 1}$

17. Evaluate $\lim_{x \to 0} \frac{(\sin 2x) + 3x}{2x + (\tan 3x)}$

18. Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{1 - \cos 4x}$

19. If y = $\sin^2 x . \cos^3 x$, then $\frac{dy}{dx}$.

20. If y = sin 2x.cos3x, then $\frac{dy}{dx}$.

21. Differentiate $\frac{\sin x}{x}$ with respect to x.

22. Differentiate $x^3 + 3^3 + 3^x$ with respect to x.

23. Differentiate $\sin^2(x^3 + x - 1) + \frac{1}{\sec^2(x^3 + x - 1)}$ with respect to x.

24. Differentiate $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$ with respect to x.

25. Differentiate $\frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{a-c} + x^{b-c}}$ w.r.t to x.

26. Find the derivative of x using first principle method.

27. If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then find the value of k.

28. Find the derivative of $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$ with respect to x.

29. Differentiate $\frac{x^8 - 1}{x^4 - 1}$ with respect to x.

Differentiate the following with respect to x using First principle method. (For Q. 30 – 35)

30. $\frac{1}{x}$

31. $\sqrt{x}$

32. $\cos(x + 1)$

33. $\sqrt{\sin x}$

34. $\frac{2x + 3}{x + 1}$

35. $x \cos x$

Long Answer Type Questions

Evaluate the following Limits: (For Q. 36 – 58)

36. $\lim_{x \to \infty} \frac{2x^8 - 3x^2 + 1}{x^8 + 6x^5 - 7}$

37. $\lim_{x \to 2} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$

38. $\lim_{x\to 0} \frac{1-\cos 2x}{x \cdot \tan 3x}$

39. $\lim_{x\to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

40. $\lim_{x\to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{\frac{\pi}{6} - x}$

41. $\lim_{x\to 0} \frac{\sin x}{\tan x^0}$ (where x^0 represents x degree)

42. $\lim_{x\to 9} \frac{x^{\frac{3}{2}} - 27}{x^2 - 81}$

43. $\lim_{x\to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$

44. $\lim_{x\to 0} \frac{\cos ax - \cos bx}{1-\cos x}$

45. $\lim_{x\to a} \frac{\cos x - \cos a}{\cot x - \cot a}$

46. $\lim_{x\to \pi} \frac{1+\sec^3 x}{\tan^2 x}$

47. $\lim_{x\to 1} \frac{x-1}{\log_e x}$

48. $\lim_{x\to e} \frac{x-e}{(\log_e x)-1}$

49. $\lim_{x \to 2} \left[\frac{4}{x^3 - 2x^2} + \frac{1}{2 - x} \right]$

50. $\lim_{x \to a} \left[\frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}} \right]$

51. $\lim_{x \to 0} \frac{\sin(2 + x) - \sin(2 - x)}{x}$

52. $\lim_{x \to 0} \frac{1 - \cos x \cdot \sqrt{\cos 2x}}{\sin^2 x}$

53. $\lim_{x \to 0} \frac{6^x - 2^x - 3^x + 1}{\log(1 + x^2)}$

54. $\lim_{x \to 0} \frac{\tan x - \sin x}{\sin^3 x}$

55. $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$

56. Find the values of a and b if $\lim_{x \to 2} f(x)$ and $\lim_{x \to 4} f(x)$ exists where

$$f(x) = \begin{cases} x^2 + ax + b, & 0 \le x < 2 \\ 3x + 2, & 2 \le x \le 4 \\ 2ax + 5b, & 4 < x < 8 \end{cases}$$

57. Differentiate the following w.r.t.

(a) $\frac{(x - 1)(x - 2)(x - 3)}{x^2 - 5x + 6}$

(b) $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$

(c) $\dfrac{x\sin x+\cos x}{x\sin x-\cos x}$

(d) $x\cdot\sin x\cdot e^{x}$

58. Prove the following statements

(a) If $y=\dfrac{x}{x+2}$, then $\dfrac{dy}{dx}=\dfrac{(1-y)y}{x}$

(b) If $y=e^{x}\cos x$, then $\dfrac{dy}{dx}=\sqrt{2}\,e^{x}\cos\left(x+\dfrac{\pi}{4}\right)$

(c) If $y=\dfrac{1-x}{1+x}$, then $\dfrac{dy}{dx}=\dfrac{-2}{(1+x)^{2}}$

(d) If $xy=4$, then $x\left(\dfrac{dy}{dx}+y^{2}\right)=3y$

CASE STUDY TYPE QUESTIONS

59. Mr. Pradeep has a rectangular plot, which is used for growing vegetables. Perimeter of plot is 50m. Length and width of plot are x m and y m respectively.

Based on the above information, answer the following questions:-

i. Relation between x and y is

(a) x + y = 50 (b) x + y = 100

(c) x + y = 25 (d) x = y

ii. Area function, A(x) =

(a) $x^2 - 5$ (b) $25x - x^2$ (c) $x^2 - 25x$ (d) 25 – x

iii. Derivative of A(x) w.r.t. x [A'(x)] =

(a) 2x (b) –2x (c) 25 – 2x (d) 2x – 25

iv. Value of x for which A'(x) = 0 is

(a)25 (b) 12.5 (c) 5 (d) 0

v. Value of A(x) at x = 12.5 is

(a) 625 (b) 250 (c) 156.25 (d) 144.25

60. Consider the following functions.

$$u(x) = \sqrt{x}, \quad v(x) = \cot x, \quad f(x) = u(x) \times v(x)$$

$$g(x) = \frac{u(x)}{v(x)} \text{ and } h(x) = \frac{v(x)}{u(x)}$$

Based on the above information answer the following :-

i. Derivative of $u(x)$ is

ii. Derivative of $v(x)$ is

iii. Derivative of $f(x)$ is

iv. Derivative of $g(x)$ is

v. Derivative of $h(x)$ is

Multiple Choice Questions

> **Note:** Q.61 – Q.70 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

61. $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$ is -

(a) 1 (b) 2

(c) –1 (d) does not exist.

62. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80,$ then n is -

(a) 2 (b) 3

(c) 4 (d) 5.

63. If $L = \lim_{x \to 1} \frac{x^4 - 1}{x^3 - 1},$ then 3L is -

(a) 2 (b) 3

(c) 4 (d) None of these.

64. $\lim_{x \to 0} \frac{(1+x)^{16} - 1}{(1+x)^4 - 1}$ is -

(a) 0 (b) 4

(c) 8 (d) 16.

65. $\lim_{x \to 1} \frac{x + x^2 + x^3 + x^4 - 4}{x - 1}$ is -

(a) 0 (b) 4

(c) 10 (d) Does not exist.

66. $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$ is -

(a) 0 (b) 1

(c) 2 (d) 4.

67. $\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 1

68. $\lim_{x\to 0}(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 1

69. If $y=\sin^4 x+\cos^4 x,$ then $\frac{dy}{dx}=$

(a) $4\sin^3 x + 4\cos^3 x$ (b) $4\sin^3 x - 4\cos^3 x$

(c) $-\sin 4x$ (d) 0.

70. If $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$ then $\frac{dy}{dx}$ is

(a) y^2 (b) $1 + y^2$

(c) $y^2 - 1$ (d) $1 - y^2$

71. $\lim_{x\to\infty}\frac{\sin x}{x}$ equals.

(a) 0 (b) ∞

(c) 1 (d) does not exists

72. $\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ is equal to

(a) $\sqrt{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) 0

73. If f(x) = $\frac{x-4}{2\sqrt{x}}$, then f′(1) is

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$

(c) 1 (d) 0

74. If f(n) = $\frac{x^n - a^n}{x-a}$, then f′ (a) is

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) does not exist

75. If $y = \frac{\sin(x+9)}{\cos x}$ then $\frac{dy}{dx}$ at x = 0 is

(a) cos 9 (b) sin 9

(c) 0 (d) 1

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

76. **Assertion:** $\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b}$

Reason: $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{b}{a} (a, b \neq 0)$

77. **Assertion:** $\lim_{x \to 0}(\operatorname{cos es} x - \cot x) = 0$

Reason: $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = -1$

78. **Assertion:** If a and b are non-zero constants, then the derivative of f(x) = ax + b is a.

Reason: If a, b and c are non-zero constants, then the derivative of f(x) = ax^2 + bx + c is ax + b.

79. Let a_1, a_2, a_3, ... a_n be fixed real numbers and define a function f(x) = (x – a_1) (x – a_2) .... (x – a_n), then

Assertion: $\lim_{x \to a_1} f(x) = 0$

Reason: $\lim_{x \to a} (x) = (a - a_1)(a - a_2)...(a - a_n)$, for some a = a_1, a_2, ... a_n.

80. **Assertion:** Suppose f is real valued function, the derivative of f at x is given by f'(x) is given by f'(x) = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Reason: If y = f(x) is the function, then derivative of 'f' at any x is denoted by f'(x).

ANSWERS

1.	15	2.	4
3.	32	4.	8
5.	9	6.	$\frac{1}{8}$
7.	$\frac{1}{2}$	8.	$\log 2$

9. $\log_3 5$

10. –2 log 3

11. –3

12. log 7

13. 2x – sinx

14. 2x – 1

15. $\frac{4x^5 - 15x^4}{(x-3)^2}$

16. m/n

17. 1 (Hint: divide by x)

18. $\frac{1}{4}$ (Hint: $\cos 2\theta = 1 - 2\sin^2\theta$)

19. $\cos^2 x.\sin x(2\cos^2 x - 3\sin^2 x)$

20. 2cos2x.cos3x – 3sin2x.sin3x

21. $\frac{x\cos x - \sin x}{x^2}$

22. $3x^2 + 3x.\log 3$

23. 0 (Hint: Use $\sin^2\theta + \cos^2\theta = 1$)

24. 0 (Hint: firstly simplify and then differentiate)

25. 0 (Hint: Simplify and then differentiate)

26. 1

27. $\frac{8}{3}$ (Hint: Use $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$)

28. $8x^7$ (Hint: Simplify using identify $(a + b)(a - b) = a^2 - b^2$)

29. $4x^3$ (Hint: Simplify using identify $(a + b)(a - b) = a^2 - b^2$)

30. $\frac{-1}{x^2}$

31. $\frac{1}{2\sqrt{x}}$

32. –sin(x + 1)

33. $\frac{1}{2}\cot x\sqrt{\sin x}$

34. $\frac{-1}{(x+1)^2}$

35. $\cos x - x\sin x$

36. 2

37. $\frac{1}{2}$

38. $\frac{2}{3}$

39. $\sqrt{2}$

40. – 2

41. $\frac{180^0}{\pi}$

42. $\frac{1}{4}$

43. $\frac{5(a+2)^{\frac{3}{2}}}{2}$

44. $b^2 - a^2$

45. $\sin^3 a$

46. $\frac{-3}{2}$

47. 1

48. e

49. –1

50. $\frac{2}{3\sqrt{3}}$

51. 2cos2

52. $\frac{3}{2}$

53. (log 2) (log 3)

54. $\frac{1}{2}$

55. 2

56. a = –3, b = – 2

57. (a) 1

(b) $8x + 8x^{-9}$

(c) $\frac{-2(x+\sin x \cdot \cos x)}{(x\sin x - \cos x)^2}$

(d) $e^x (x\sin x + x\cos x + \sin x)$

59. i. (c) ii. (b) iii. (c) iv. (b) v. (c)

60. i. $\frac{1}{2}\sqrt{x}$ ii. $-\text{coses}^2 x$ iii. $\frac{-2x\,\text{coses}^2 x + \cot x}{2\sqrt{x}}$

iv. $\dfrac{2x\,\text{cosec}^2 x + \cot x}{2\sqrt{x}\cot^2 x}$ v. $\dfrac{-(2x\,\text{cosec}^2 x + \cot x)}{2x^{3/2}}$

61.	(c)	62.	(d)	63.	(c)
64.	(b)	65.	(c)	66.	(c)
67.	(b)	68.	(a)	69.	(c)
70.	(d)	71.	(a)	72.	(c)
73.	(a)	74.	(d)	75.	(a)
76.	(c)	77.	(c)	78.	(c)
79.	(a)	80.	(b)		

CHAPTER - 13

STATISTICS

KEY POINTS

Range of Ungrouped Data and Discrete Frequency Distribution.

- RANGE = Largest observation – smallest observation.

Range of Continuous Frequency Distribution.

- Upper Limit of Highest Class – Lower Limit of Lowest Class.
- **Mean deviation for ungrouped data or raw data:**

$$M.D.\ (\text{about mean}) = \frac{\sum |x_i - \bar{x}|}{n}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D.\ (\text{about median}) = \frac{\sum |x_i - M|}{n}, \text{ where } M \text{ is the Median.}$$

- **Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution):**

$$M.D.\ (\text{about mean}) = \frac{\sum f_i |x_i - \bar{x}|}{N}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D.\ (\text{about mean}) = \frac{\sum f_i |x_i - M|}{N}, \text{ where } M \text{ is the Median.}$$

Note: $N = \sum f_i$

- Variance is defined as the mean of the squares of the deviations from mean.

- Standard deviation 'σ' is positive square root of variance.

  $\sigma = \sqrt{\text{Variance}}$

- **Variance 'σ^2' and standard deviation (SD) σ for ungrouped data**

  $$\sigma^2 = \frac{1}{n}\sum (x_i - \overline{x})^2 \Rightarrow \boxed{S.D. = \sigma = \sqrt{\frac{1}{n}\sum (x_i - \overline{x})^2}}$$

- **Standard deviation of a discrete frequency distribution**

  $$S.D. = \sigma = \sqrt{\frac{1}{N}\sum f_i (x_i - \overline{x})^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - (\sum f_i x_i)^2}$$

- **Short cut method to find variance and standard deviation**

  $$\text{Variance} = \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

  $$S.D. = \sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$

  where $y_i = \frac{x_i - A}{h}$, A = Assumed mean

- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k^2 times of the original value and standard deviation becomes k times of the original value.

- If each observation is increased by k, where k is positive or negative, then variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.

- The mean of first ‘n; natural number is $\frac{n+1}{2}$.
- The mean of first ‘n’ even natural numbers = (n + 1)

VERY SHORT ANSWER TYPE QUESTIONS

1. The sum of the squares of deviation for 10 observations taken from their mean 50 is 250. Find Standard Deviation.
2. The sum of the squares of deviation for 10 observations taken from their mean 25 is 500. Find Variance.
3. If the variance of 14, 18, 22, 26, 30 is ‘k’, then find the variance of 28, 36, 44, 52, 60.

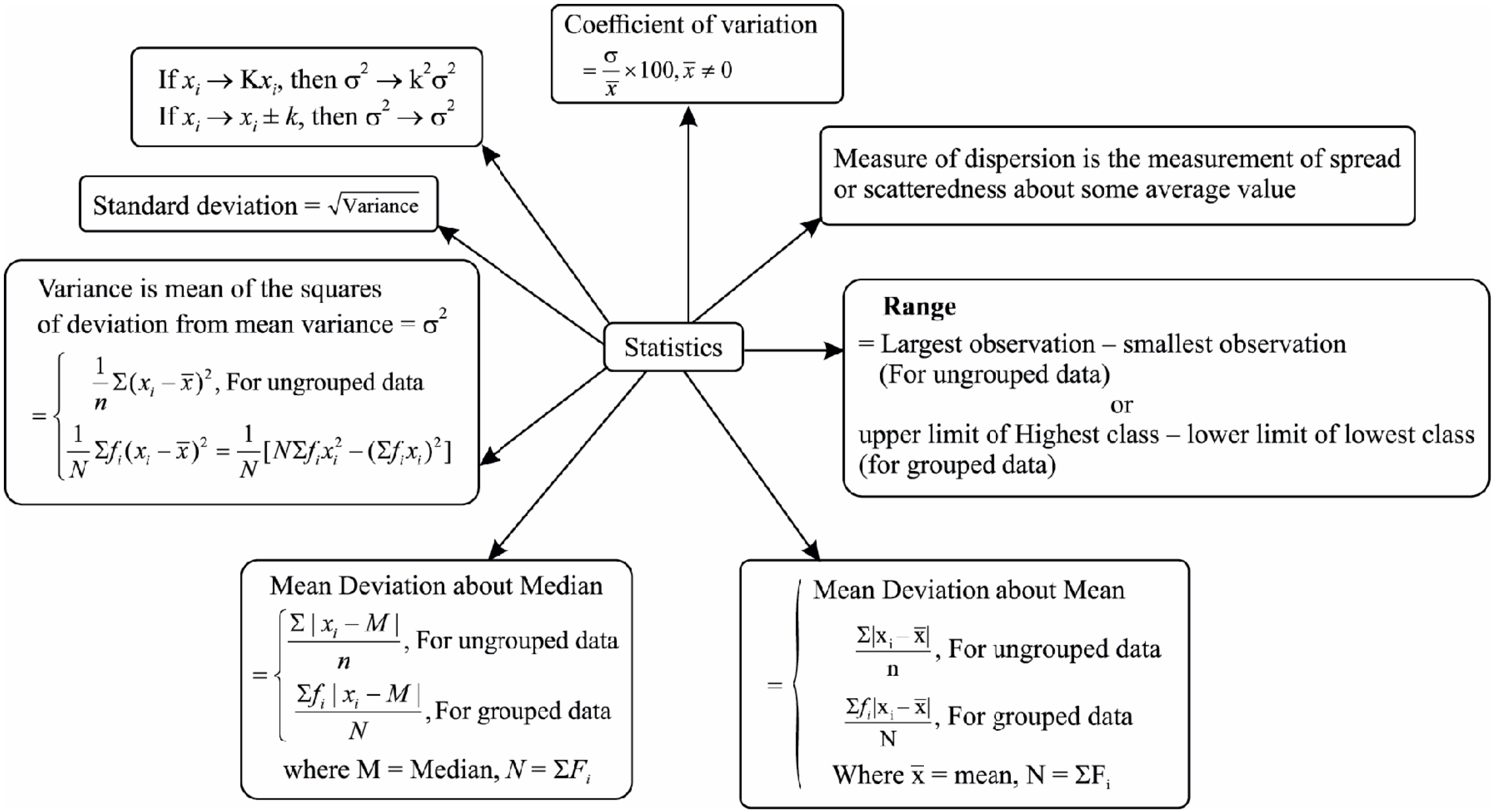
SHORT ANSWER TYPE QUESTIONS

4. Find the Variance of First 10 Natural Numbers.
5. Find the Variance of First 5 Multiples of 6.
6. Find the Standard Deviations of First 10 Even Natural numbers.
7. Find the Standard deviation for the following data:

   10, 20, 30, 40, 50, 50, 60, 70, 80, 90
8. Find the variance for the following Data:

Class-Interval	Frequency
0 - 10	1
10 - 20	2
20 – 30	3
30 – 40	3
40 – 50	1

MIND MAP


Statistics
Coefficient of variation
$= \frac{\sigma}{\bar{x}} \times 100, \bar{x} \neq 0$
If $x_i \to Kx_i$, then $\sigma^2 \to k^2\sigma^2$
If $x_i \to x_i \pm k$, then $\sigma^2 \to \sigma^2$
Standard deviation = $\sqrt{\text{Variance}}$
Variance is mean of the squares of deviation from mean variance = σ^2
$= \begin{cases} \frac{1}{n}\Sigma(x_i - \bar{x})^2, \text{ For ungrouped data} \\ \frac{1}{N}\Sigma f_i(x_i - \bar{x})^2 = \frac{1}{N}[N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2] \end{cases}$
Measure of dispersion is the measurement of spread or scatteredness about some average value
Range
= Largest observation – smallest observation (For ungrouped data)
or
upper limit of Highest class – lower limit of lowest class (for grouped data)
Mean Deviation about Median
$= \begin{cases} \frac{\Sigma |x_i - M|}{n}, \text{ For ungrouped data} \\ \frac{\Sigma f_i |x_i - M|}{N}, \text{ For grouped data} \end{cases}$
where M = Median, $N = \Sigma F_i$
Mean Deviation about Mean
$= \begin{cases} \frac{\Sigma|x_i - \bar{x}|}{n}, \text{ For ungrouped data} \\ \frac{\Sigma f_i|x_i - \bar{x}|}{N}, \text{ For grouped data} \end{cases}$
Where $\bar{x}$ = mean, N = ΣF_i

LONG ANSWER TYPE – I QUESTIONS

9. In a series of '2p' observations, half of the observations are equal 'a' each and remaining half equal (–a) each. If the standard deviation of the observations is 2, then find the value of |a|.

10. In the following Distribution

x	f
A	2
2A	1
3A	1
4A	1
5A	1
6A	1

Where A is positive integer, has a variance of 160. Determine the value of A.

11. Find the mean deviation from mean of first ten terms of an Arithmetic Progression (A.P.) with first term is 2 and Common difference is 3.

12. Find the Variance and Standard Deviation of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

13. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, find the variance of the numbers so obtained.

14. Two sets each of 20 obvservations, have the same standard deviation 5. The first set has a mean 17 and the second a mean 22. Determine the SD of the set obtained by combining the given two sets.

15. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observationsare 1, 2 and 6. Find the other two observations.

16. Calculate the possible values of 'x' if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

17. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

18. Suppose a population A has 100 observations 101, 102,......, 200. Another population B has 100 observations 151, 152,......., 250. If V_A and V_B represent the variances of the two populations respectively then find the ratio of $V_A$and V_B.

LONG ANSWER TYPE – II QUESTIONS

19. Calculate the mean deviation about mean for the following data.

X	2	4	6	8	10	12	14	16
f	2	2	4	5	3	2	1	1

20. If for a distribution $\sum(x-5)=3,\ \sum(x-5)^2=43$ and the total number of item is 18, find the mean and standard deviation.

21. Calculate the mean deviation about median for the following data:

X	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	9

22. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test :

X	0	1	2	3	4	5
f	p – 2	p	p^2	$(p + 1)^2$	2p	p + 1

where p is positive integer. Determine the mean and standard deviation of the marks.

23. Calculate the mean deviation about mean

Class Interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
f	2	3	8	14	8	3	2

24. Mean and standard deviation of 100 observations were found to be 40 and 10 respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively. Find correct standard deviation.

25. Calculate the mean deviation about mean for the following data:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	8	15	16	6

26. Calculate the mean deviation about median for the following data:

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
f	8	10	10	16	14	2

27. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results:

Number of observations = 25,
mean = 18.2 seconds
Standard deviation = 3.25 seconds

Further another set of 15 observations x_1, x_2, ..., x_{15}, also in seconds is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524.$

Calculate the standard deviation based on all 40 observations.

28. Find the mean deviation about mean of the following data:

Class Interval	f
20 - 29	5
30 - 39	12
40 - 49	15
50 - 59	20
60 - 69	18
70 - 79	10
80 - 89	6
90 - 99	4

CASE STUDY TYPE QUESTIONS

29. Following data represents the salaries of 11 employees in a firm

10000, 12000, 15000, 13000, 11000, 12000, 12000, 14000, 10000, 13000, 12000.

i. Find the mean salary.

(a) 11181.82 (b) 12181.82

(c) 13181.82 (d) 10000.82

ii. What is the median salary?

(a) 12000 (b) 11000

(c) 12181.82 (d) 11181.82

iii. When arranged in ascending order, which entry gives the median salary?

(a) 6^{th} (b) 5^{th} (c) 4^{th} (d) 7^{th}

iv. The mean deviation about the median salary is

(a)1190.99 (b) 1000 (c) 1100 (d) 1090.91

v. What is the range of salaries?

(a)4500 (b) 4000 (c) 5000 (d) 6000

30. Following are the prices of shares X and Y (of ten days) :

Days	X	Y
1	35	108
2	54	107
3	52	105
4	53	105
5	56	106
6	58	107
7	52	104

8	50	103
9	51	104
10	49	101

i. What is the mean price of the share X during these 10 days?

(a) 52 (b) 51 (c) 50.3 (d) 51.5

ii. What is the mean price of the share Y during these 10 days?

(a) 105 (b) 106 (c) 104 (d) 107

iii. What is the standard deviation of the price of share X?

(a) 5.01 (b) 6.75 (c) 5.92 (d) 7.25

iv. What is the standard deviation of the price of share Y?

(a) 1.75 (b) 2.87 (c) 1.25 (d) 2

v. If a person wants to invest in shares (X or Y) whose price remain more stable. He should invest in

(a) X

(b) Y

(c) Both are equally stable. So, he can invest in anyone

(d) Insufficient data to decide

Multiple Choice Questions

Note: Q.31–Q.46 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

31. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new mean -

(a) 12 (b) 16

(c) 24 (d) 48.

32. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new standard deviation -

(a) 4 (b) 8

(c) 16 (d) 32.

33. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new mean-

(a) 25 (b) 29

(c) 30 (d) 35.

34. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new variance -

(a) 4 (b) 14

(c) 16 (d) 25.

35. Match the following:

If the mean of $x_1, x_2, \ldots\ldots, x_{20}$ is 10.

	Column-1		Column-2
A	mean of $2x_1, 2x_2, \ldots\ldots\ldots, 2x_{20}$	P	0
B	mean of $(-3x_1 + 32), (-3x_2 + 32), \ldots\ldots, (3x_{20} +32)$	Q	2
C	mean of $(x_1 + 2), (x_2 + 2), \ldots\ldots\ldots, (x_{20} + 2)$	R	12
D	mean of $(x_1 - 10), (x_2 - 10), \ldots\ldots, (x_{20} - 10)$	S	20

(a) A → P, B→ Q, C→ R, D → S

(b) A → S, B→ Q, C→ R, D → P

(c) A → Q, B→ S, C→ R, D → P

(d) A → S, B→ Q, C→ P, D → R

36. If mean of first n natural numbers is $\frac{5n}{9}$, then n =

(a) 5 (b) 4

(c) 9 (d) 10

37. Find the mean of 6, 7, 10, 12, 13, 4, 8, 12

(a) 9 (b) 10

(c) 12 (d) 13

38. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

(a) 2.23 (b) 2.57

(c) 3.23 (d) 3.57

39. The following information relates to a sample of size 60 : $\Sigma x^2 = 18000,\ \Sigma x = 960$

The variance is

(a) 6.63 (b) 16

(b) 22 (d) 44

40. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$

(c) $\sqrt{6}$ (d) 6

41. The variance of n observation $x_1, x_2, \ldots\ldots, x_n$ is given by

(a) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})$ (b) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_1 - \overline{x})^2$

(c) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + \overline{x})$ (d) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + \overline{x})^2$

42. Variance of the numbers 3, 7, 10, 18, 22 is equal to

(a) 12 (b) 64

(c) $\sqrt{49.2}$ (d) 49.2

43. The mean deviation from the mean of the following data:

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	5	8	15	16	6

is

(a) 10 (b) 10.22

(c) 9.86 (d) 9.44

44. The mean of the numbers, a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?

(a) a = 0, b = 7 (b) a = 5, b = 2

(c) a = 1, b = 6 (d) a = 3, b = 4

45. Find the mean deviation about the mean for the data: 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5

(a) 5.2 (b) 6.2

(c) 7.2 (d) 8.2

46. Find the mean deviation about the mean for the data:

x_1	5	10	15	20	25
f_i	7	4	6	3	5

(a) 6.32 (b) 7.32

(c) 8.32 (d) 9.32

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct;l reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

47. **Assertion:** Mean of deviations = $\frac{\text{Product of deviations}}{\text{No. of observations}}$

Reason: To find the dispersion of values of x from mean $\bar{x}$, we take absolute measure of dispersion.

48. Let x_1, x_2, ... x_n be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.

Assertion: Variance of $2x_1$, $2x_2$,...... $2x_n$ is $4\sigma^2$.

Reason: Arithmetic mean of $2x_1$, $2x_2$, ... $2x_n$ is $4\bar{x}$.

49. **Assertion:** The range is the difference between two extreme observations of the distribution.

Reason: The variance of a variate X is the arithmetic mean of the squares of all deviations of X from the arithmetic mean of the observations.

50. **Assertion:** The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is 2.57.

Reason: For individual observation.

Mean deviation $(\bar{X}) = \frac{\sum |x_i - \bar{x}|}{n}$

ANSWERS

1. 5
2. 50
3. 4 k
4. 8.25
5. 72
6. $\sqrt{33}$
7. $10\sqrt{6}$
8. 129
9. 2
10 A = 7
11. $\frac{15}{2}$
12. Variance = $\frac{(n^2-1)}{12}d^2$

    Standard Deviation = $d\sqrt{\frac{(n^2-1)}{12}}$
13. 8.25
14. 5.59 Hint: $\left[S.D. = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2(\bar{x}_1 - \bar{x}_2)^2}{n_1+n_2}}\right]$
15. 4, 9
16. 3, $\frac{7}{3}$ Hint: $\bar{x} = 4 + \frac{x}{2}$ S.D. $= \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$
17. 6.5, 2.5 Hint: [correct mean $= \frac{126+12-21}{18}$]

    [Correct $\Sigma x_i^2 = 1170 - 21^2 + 12^2 = 873$]

18. 1 : 1 $\left[\begin{array}{l}\text{Hint: } V_A = \dfrac{\frac{1}{2}(1^2 + 3^2 + ... + 99^2)}{100} \\ V_B = \dfrac{\frac{1}{2}(99^2 + 97^2 + ...1^2)}{100}\end{array}\right]$

19. 2.8

20. Mean = 5.17,
Standard Deviation = 1.53

21. 10.1

22. Mean = 2.8,
Standard deviation = 1.12

23. 10

24. 10.24 $\left[\begin{array}{l}\text{Hint: correct mean} = 39.3 \\ \text{Correct } \Sigma x_i^2 = 164938 \\ \text{Correct } \sigma^2 = \dfrac{164938}{100} - (39.3)^2\end{array}\right]$

25. 9.44

26. 12.25

27. 3.87

28. 109.19

29. i. (b) ii. (a) iii. (a) iv. (d) v. (c)

30. i. (b) ii. (a) iii. (c) iv. (d) v. (b)

31. (d)

32. (c)

33. (d)

34. (c)

35. (b)

36. (c)

37. (a)

38. (b)

39. (d)

40. (a)

41. (b) $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$

42. (d) Mean = 12, Variance = 49.2

43. (d) 9.44

44. (d) a = 3, b = 4 [Hint: a + b = 7, $a^2 - 7a + 12 = 0$]

45. (b) 6.2 [Hint: Mean = 10]

46. (a) 6.32 [Hint: Mean = 14]

47. (d)

48. (c)

49. (b)

50. (a)

CHAPTER - 14

PROBABILITY

KEY POINTS

- **Random Experiment:** If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.
- **Sample Space:** The collection or set of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space (set) is called **a sample point**.
- **Event:** A subset of the sample space associated with a random experiment is called an event.
- **Elementary or Simple Event:** An event which has only one sample point is called a simple event.
- **Compound Event:** An event which has more than one sample point is called a Compound event.
- **Sure Event:** If an event is same as the sample space of the experiment, then event is called sure event. In other words an event which is certain to happen is sure event.
- **Impossible Event:** Let S be the sample space of the experiment, $\phi \subset S$, ϕ is called impossible event. In other words an event which is impossible to happen is the impossible event.
- **Exhaustive and Mutually Exclusive Events:** If Events E_1, E_2, E_3........E_n are n events of a sample space S such that
  (i) $E_1 U\ E_2 U E_3 U$........ $U E_n = S$ then Events E_1, E_2, E_3........E_n are called exhaustive events.

(ii) $E_i \cap E_j = \phi$ for every $i \neq j$ then Events $E_1, E_2, E_3 \ldots\ldots E_n$ are called mutually exclusive.

- **Probability of an Event:** For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

$$P(A) = \frac{n(A)}{n(S)}$$

where n(A) is number of elements in A and n(S) is number of elements in set S and $0 \leq P(A) \leq 1$

(a) If A and B are any two events then

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A \text{ and } B)$

(b) A and B are mutually exclusive events, then

$P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B) = 0$ for mutually exclusive events)

(c) $P(A) + P(\bar{A}) = 1$ or P(A) + P(not A) = 1

(d) P (Sure event) = P(S) = 1

(e) P (impossible event) = $P(\phi) = 0$

(f) $P(A - B) = P(A) - P(A \cap B) = P(A \cap \bar{B})$

(g) $P(B - A) = P(B) - P(A \cap B) = P(\bar{A} \cap B)$

(h) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

(i) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

- **Addition theorem for three events**

  Let A, B and C be any three events associated with a random experiment, then

  $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- **Axiomatic Approach to Probability:**

  Let S be a sample space containing elementary outcomes w_1, w_2, ..., w_n

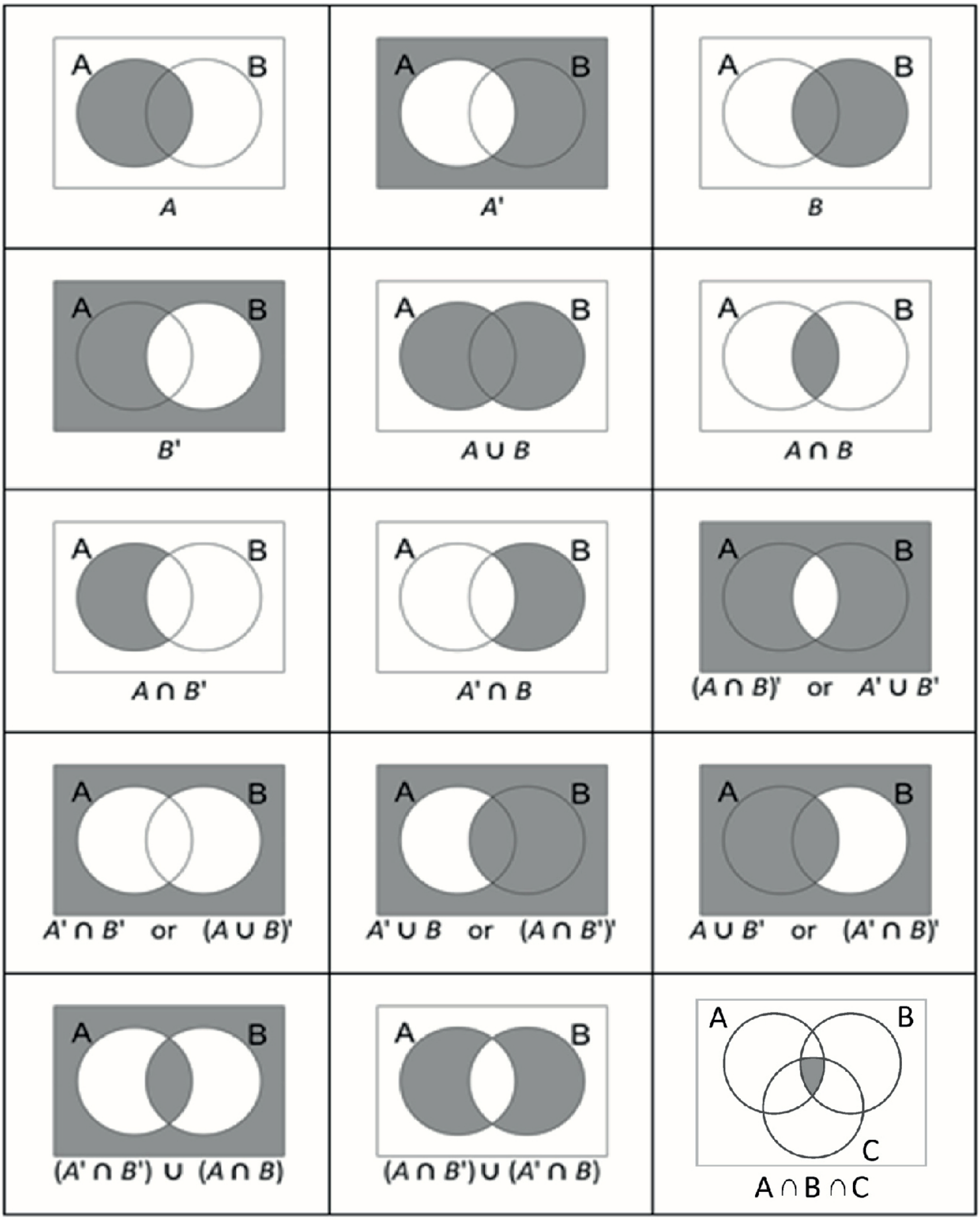
  i.e. S = $\{w_1, w_2, ..., w_n\}$

  (i) $0 \leq P(w_i) \leq 1, \text{ for all } w_i \in S$

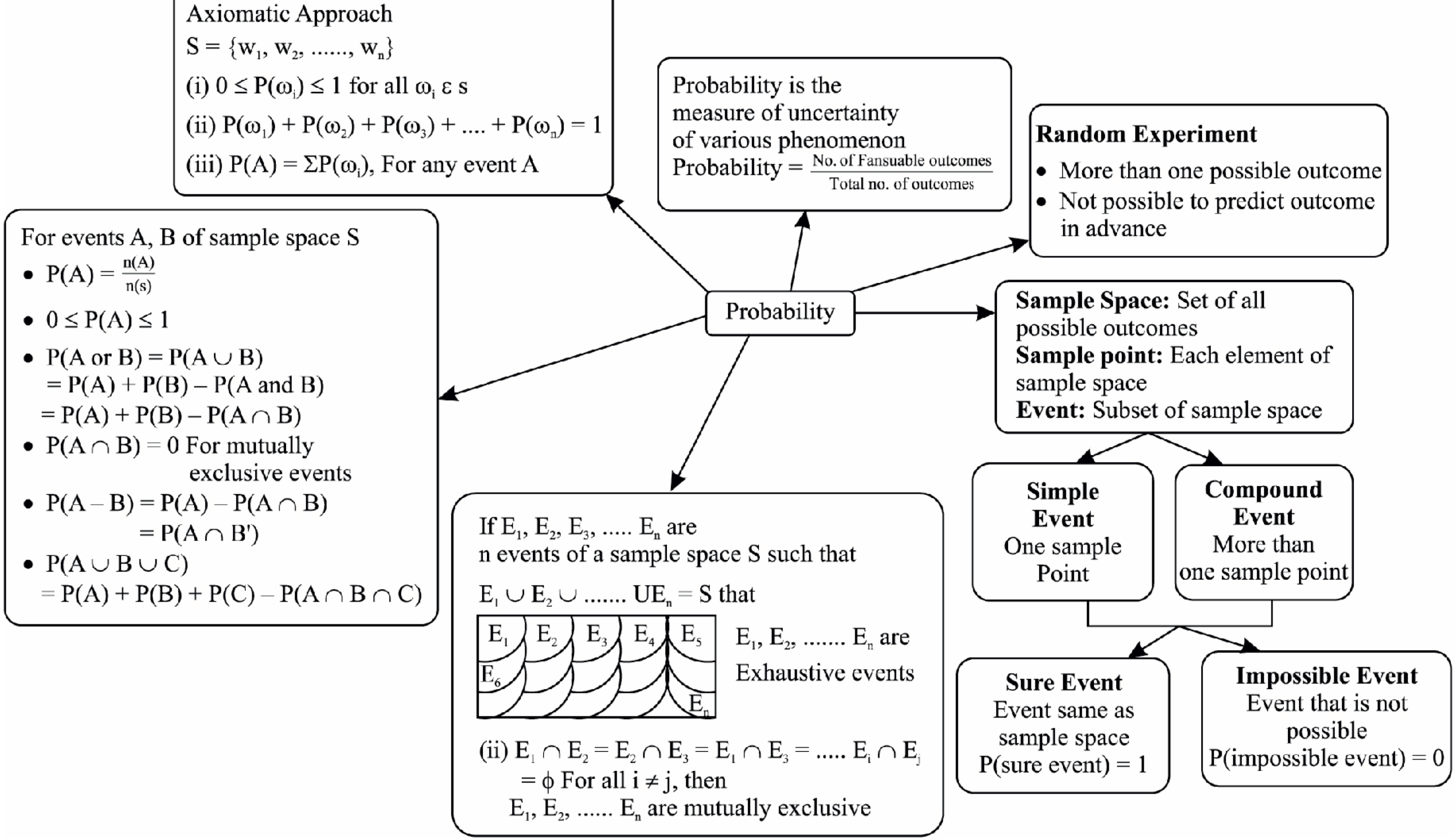
  (ii) $P(w_1) + P(w_2) + P(w_3) + \ldots\ldots + P(w_n) = 1$

  (iii) P (A) = $\sum P(w_i)$, for any event A containing elementary events w_i.

▶ VENN DIAGRAM OF DIFFERENT SETS



MIND MAP


Probability
Axiomatic Approach
$S = \{w_1, w_2, \ldots\ldots, w_n\}$
(i) $0 \le P(\omega_i) \le 1$ for all $\omega_i \varepsilon s$
(ii) $P(\omega_1) + P(\omega_2) + P(\omega_3) + \ldots. + P(\omega_n) = 1$
(iii) $P(A) = \Sigma P(\omega_i)$, For any event A
Probability is the measure of uncertainty of various phenomenon
Probability = $\frac{\text{No. of Fansuable outcomes}}{\text{Total no. of outcomes}}$
Random Experiment
• More than one possible outcome
• Not possible to predict outcome in advance
Sample Space: Set of all possible outcomes
Sample point: Each element of sample space
Event: Subset of sample space
Simple Event
One sample Point
Compound Event
More than one sample point
Sure Event
Event same as sample space
P(sure event) = 1
Impossible Event
Event that is not possible
P(impossible event) = 0
For events A, B of sample space S
• $P(A) = \frac{n(A)}{n(s)}$
• $0 \le P(A) \le 1$
• $P(A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B) - P(A \text{ and } B)$
$= P(A) + P(B) - P(A \cap B)$
• $P(A \cap B) = 0$ For mutually exclusive events
• $P(A - B) = P(A) - P(A \cap B)$
$= P(A \cap B')$
• $P(A \cup B \cup C)$
$= P(A) + P(B) + P(C) - P(A \cap B \cap C)$
If $E_1, E_2, E_3, \ldots.. E_n$ are n events of a sample space S such that
$E_1 \cup E_2 \cup \ldots\ldots. UE_n = S$ that
E_1 E_2 E_3 E_4 E_5 E_6 E_n
$E_1, E_2, \ldots\ldots. E_n$ are Exhaustive events
(ii) $E_1 \cap E_2 = E_2 \cap E_3 = E_1 \cap E_3 = \ldots.. E_i \cap E_j$
$= \phi$ For all $i \ne j$, then
$E_1, E_2, \ldots\ldots E_n$ are mutually exclusive

VERY SHORT ANSWER TYPE QUESTIONS

1. Describe the Sample Space for the following experiments:

   A coin is tossed twice and number of heads is recorded.

2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up.
4. A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
5. Two balls are drawn at random in succession without replacement from a box containing 1 red and 3 identical white balls.
6. A coin is tossed n times. Find the number of element in its sample space.
7. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?
8. What is the probability that a given two-digit number is divisible by 15?
9. If P(A U B) = P(A) + P(B), then what can be said about the events A and B?
10. If P(A U B) = P(A ∩ B), then find relation between P(A) and P(B).

SHORT ANSWER TYPE QUESTIONS

11. Let A and B be two events such that P(A) = 0.3 and P(A U B) = 0.8, find P(B) if P(A∩B) = P(A) P(B).
12. Three identical dice are rolled. Find the probability that the same number appears on each of them.
13. In an experiment of rolling of a fair die. Let A, B and C be three events defined as under:

    A : a number which is a perfect square

B : a prime number

C : a number which is greater than 5.

Is A, B, and C exhaustive events?

14. Punching time of an employee is given below:

DAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
TIME (AM)	10:35	10:20	10:22	10:27	10:25	10:40

If the reporting time is 10:30 a.m, then find the probability of his coming late.

15. A game has 18 triangular blocks out of which 8 are blue and rest are red and 19 square blocks out of which 7 are blue and rest are yellow. One piece is lost. Find the probability that it was a square of blue colour.

16. A card is drawn from a pack of 52 cards. Find the probability of getting:
    (i) a jack or a queen
    (ii) a king or a diamond
    (iii) a heart or a club
    (iv) either a red or a face card.
    (v) neither a heart nor a king
    (vi) neither an ace nor a jack
    (vii) a face card

17. In a leap year find the probability of
    (i) 53 Mondays and 53 Tuesdays
    (ii) 53 Mondays and 53 Wednesday
    (iii) 53 Mondays or 53 Tuesdays
    (iv) 53 Mondays or 53 Wednesday

18. In a non-leap year, find the probability of
    (i) 53 Mondays and 53 Tuesdays.

(ii) 53 Mondays or 53 Tuesdays.

19. Three candidates A, B, and C are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. A is thrice as likely to win as B and B is twice as likely as to win as C. Find the respective probability of A, B and C to win the cup.

LONG ANSWER QUESTIONS

20. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I′s come together.

21. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.

22. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

23. Find the probability of at most two tails or at least two heads in a toss of three coins.

24. A, B and C are events associated with a random experiment such that

$P(A) = 0.3$,

$P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$ and

$P(A \cap B \cap C) = 0.09$. If

$P(A \cup B \cup C) \geq 0.75$ Then prove that $P(B \cap C)$ lies in the interval

$[0.23, 0.48]$.

25. $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probability of three mutually exclusive events. Then find the set of all values of p.

26. An urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from

urn B and placed in urn A. Now if one ball is drawn at random from urn A then find the probability that it is found to be red.

27. If three distinct numbers are chosen randomly from the first 100 natural numbers, then find the probability that all three of them are divisible by both 2 and 3.

28. S = {1, 2, 3, ………, 30}, A = {x : x is multiple of 7}, B = { x : x is multiple of 5},C = {x : x is a multiple of 3}.

    If x is a member of S chosen at random find the probability that

    (i) $x \in A \cup B$

    (ii) $x \in B \cap C$

    (iii) $x \in A \cap \bar{C}$

29. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.

30. If A and B are any two events having $P(A \cup B) = \frac{1}{2}$ and $P(\bar{A}) = \frac{2}{3}$, then find the $P(\bar{A} \cap B)$.

31. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?

32. A typical PIN (Personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

33. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.

34. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that

    (i) it will get at least one of the two awards

    (ii) it will get only one of the awards.

35. A girl calculates that the probability of her winning the first prize in a lottery is 0.02. If 6000 tickets were sold, how many tickets has she bought?

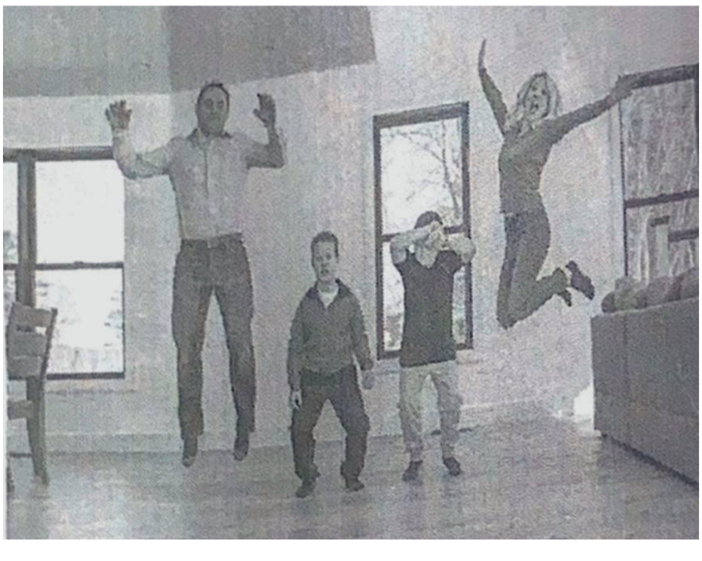
36. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9?

37. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.

38. If A,B and C are three mutually exclusive and exhaustive events of an experiment such that
$3P(A) = 2P(B) = P(C)$, then find the value of P(A).

CASE STUDY TYPE QUESTIONS

39. To make a healthy routine and to do some physical exercise during lockdown a family decided to roll a dice and based on the outcomes, they will decide activities to be done.



- If the outcome is 2, 4 or 6, they will do 30 minutes walk on the roof.

- If it shows 1 or 3 on the dice, 15 minutes meditation to be done.
- If outcome is 5, then they will toss a coin. If it shows "Head", the family will do 5 minutes of rope skipping. If there is "Tail", family will do 20 minutes of Yoga.

i. How many elements are there in the sample space?

ii. What is the probability of doing walking?

iii. What is the probability of doing rope skipping?

iv. What is the probability of doing yoga or meditation?

v. Two activities having the same probability are

40. In a class of 60 students, hobbies were discussed. 30 liked reading, 32 liked singing and 24 liked about reading and singing.

i. Find the probability that the student liked reading or singing.

(a) $\frac{17}{30}$ (b) $\frac{19}{30}$ (c) $\frac{23}{30}$ (d) $\frac{29}{30}$

ii. How many students neither like reading nor singing?

(a) 30 (b) 28 (c) 22 (d) 38

iii. Find the probability that the student neither like singing nor reading?

(a) $\frac{11}{30}$ (b) $\frac{13}{30}$

(c) $\frac{7}{30}$ (d) $\frac{1}{30}$

iv. Find the probability that a student like singing but not reading?

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{1}{15}$ (d) $\frac{2}{15}$

v. Find the probability that a student like reading only.

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{10}$ (d) 0

Multiple Choice Questions

41. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is -

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{9}$ (d) $\frac{1}{30}$.

42. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(a) $\frac{1}{16}$ (b) $\frac{16}{25}$

(c) $\frac{1}{65}$ (d) $\frac{1}{25}$.

43. The probability that a non-leap year selected at random will have 52 Sundays is -

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

44. The probability that a non-leap year selected at random will have 53 Sundays is -

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

45. The probability that a leap year selected at random will have 54 Sundays is

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

46. Three unbiased coins are tossed. If the probability of getting at least 2 tails is p, Then the value of 8p -

(a) 0 (b) 1

(c) 3 (d) 4.

47. Four unbiased coins are tossed. If the probability of getting odd number of tails is p, then the value of 16p -

(a) 1 (b) 2

(c) 4 (d) 8

48. From 4 red balls, 2 white balls and 4 black balls, four balls are selected. The probability of getting 2 red balls is p, then the value of 7p -

(a) 1 (b) 2

(c) 3 (d) 4

49. If A and B are mutually exclusive events, then

(a) $P(A) \leq P(\overline{B})$ (b) $P(A) \geq P(\overline{B})$

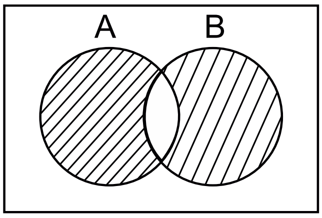
(c) $P(A) < P(\overline{B})$ (d) None of these

50. The probability that atleast one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1.6

51. In the following Venn diagram circles A and B represent two events:



The probability of the union of shade region will be

(a) $P(A) + P(B) - 2P(A \cap B)$ (b) $P(A) + P(B) - P(A \cap B)$

(c) $P(A) + P(B)$ (d) $2P(A) + 2P(B) - P(A \cap B)$

52. A bag contains 10 balls, out of which 6 balls are white and the others are non-white. The probability of getting a non-white is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

53. Two dice are thrown together. The probability of getting the sum of digits as a multiple of 4 is:

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{5}{9}$

54. The probability of getting sum more then 8 when a pair of dice are thrown is:

(a) $\frac{7}{36}$ (b) $\frac{5}{18}$

(c) $\frac{7}{18}$ (d) $\frac{5}{36}$

55. If A and B are two events, such that

$P(A \text{ u } B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(\bar{A}) = \frac{2}{3},$

Then P(B) is given by:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{9}$ (d) $\frac{2}{9}$

56. A die is rolled. Let E be the event ‘die shows 4 and F be the event ‘die shows an even number’’. Then events E and F are :

(a) mutually exclusive

(b) exhaustive

(c) mutually exclusive and exhaustive

(d) None of these

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four

Alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

57. **Assertion:** Probability of getting a head in a toss of an unbiased coin is $\frac{1}{2}$.

Reason: In a simultaneous toss of two coins, the probability of getting 'no tails' is $\frac{1}{4}$.

58. **Assertion:** In tossing a coin, the exhaustive number of cases is $2 \times 2 = 4$.

Reason: If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$.

59. **Assertion:** If $A \cap B = \phi$, then $P(A \cap B) = 0$

Reason: For mutually exclusive events A and B, $P(A \cap B) = 0$

60. Consider a single throw of die and two events.

A = the number is even = {2, 4, 6}

B = the number is a multiple of 3 = {3, 6}

Assertion: $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$

Reason: $P(\bar{A} \cap \bar{B}) = 1 - \frac{1}{6} = \frac{5}{6}$

ANSWERS

1. {0, 1, 2}

2. {R, B}

3. {T, HT, HHT, ...}

4. {HR_1, HR_2, HB_1, HB_2, HB_3, TH, TT}

5. {RW, WW, WR}

6. 2^n [eg: 2^1 = {H, T}, 2^2 = {HH, HT, TH, TT} , 2^3 = {HHH, ... TTT}]

7. $\frac{8}{21}$

8. $\frac{1}{15}$

9. Mutually Exclusive

10. P(A) = P(B) [Possible only if A = B]

11. $\frac{5}{7}$ [Hint: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$]

12. $\frac{1}{36}\left[=\frac{6}{216}\right]$

13. Yes, A, B and C are Exhaustive Events [A = {1, 4}, B = {2, 3, 5}, C = {6}

14. $\frac{1}{3}$

15. $\frac{7}{37}$

16. (i) $\frac{2}{13}$ [Hint: $P(J \cup Q) = P(J) + P(Q) - P(J \cap Q)$]

(ii) $\frac{4}{13}$

(iii) $\frac{1}{2}$

(iv) $\frac{8}{13}$ (v) $\frac{9}{13}$ $[P(\overline{H \cup K}) = 1 - P(H \cup K)]$

(vi) $\frac{11}{13}$ $[P(\overline{A \cup J}) = 1 - \{P(A) + P(J) - P(A \cap J)]$

(vii) $\frac{3}{13}$ [J, Q, K of diamond, heart, club and spade are 12 face cards]

17. (i) $\frac{1}{7}$ (ii) 0

(iii) $\frac{2}{7}+\frac{2}{7}-\frac{1}{7}=\frac{3}{7}$ (iv) $\frac{2}{7}+\frac{2}{7}-0=\frac{4}{7}$

18. Non leap year = 52 weeks and 1 day

(i) 0 (both together not possible)

(ii) $\frac{1}{7}+\frac{1}{7}-0=\frac{2}{7}$

19. [Hint: P(C) = x, P(B) = 2P(C) = 2x P(A) = 3P(B) = = 6x]

$\frac{2}{3}, \frac{2}{9}, \frac{1}{9}$

20. $\frac{1}{5}\left[\text{Hint}: \frac{9!}{10!/2!} = \frac{9!}{5 \times 9!}\right]$ 21. 3 $\left[\text{Hint}: \frac{{}^{5}C_{2}}{{}^{5+x}C_{2}} = \frac{5}{14}\right]$

22. $\frac{23}{28}$ $\left[\text{Hint}: \frac{{}^{8}C_{3} - {}^{5}C_{3}}{{}^{8}C_{3}}\right]$ 23. $\frac{7}{8}$ [Hint: $P(A \cup B)$]

24. $0.23 \le P(B \cap C) \le 0.48$ [Hint: $0.75 \le P(A \cup B \cup C) \le 1$, $0.75 \le 1.23 - x \le 1$]

25. $\frac{1}{3} \le p \le \frac{1}{2}$

$$\left[\begin{array}{l}\text{Hint}: 0 \le P(A) \le 1,\ 0 \le P(B) \le 1 \ \text{ Multually Exclusive} \\ \qquad\qquad 0 \le P(C) \le 1 \quad 0 \le P(A)+P(B)+P(C) \le 1\end{array}\right]$$

26. $\frac{32}{55}$

$$\left[\begin{array}{l}\text{Hint}: \\ \text{Case I}: A \xrightarrow{\text{Red}} B, B \xrightarrow{\text{Red}} A, \frac{6}{10} \times \frac{5}{11} \times \frac{6}{11} \\ \text{Case II}: A \xrightarrow{\text{Red}} B, B \xrightarrow{\text{Black}} A, \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} \\ \text{Case III}: A \xrightarrow{\text{Black}} B, B \xrightarrow{\text{Red}} A, \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} \\ \text{Case IV}: A \xrightarrow{\text{Black}} B, B \xrightarrow{\text{Black}} A, \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10}\end{array}\right]$$

27. $\frac{4}{1155}$ $\left[\text{Hint}: \frac{{}^{16}C_3}{{}^{100}C_3}\right]$

28. (i) $\frac{1}{3}$ $\left[\frac{n(A \cup B)}{n(s)} = \frac{n(A)+n(B)-n(A \cap B)}{n(s)}\right]$

(ii) $\frac{1}{15}$ $\left[\text{Hint}: B \cap C = \{15, 30\}\right]$

(iii) $\frac{1}{10}$ $\left[\text{Hint}: A \cap \bar{C} = \{7, 14, 28\}\right]$

29. $\frac{3}{10}$ $\left[\text{Hint}: P(A \cup B) = P(A)+P(B)-P(A \cap B) = \frac{25}{100} + \frac{10}{100} - \frac{5}{100}\right]$

30. $\frac{1}{6}$ $\left[\text{Hint}: P(\bar{A} \cap B) = P(A \cup B) - P(A)\right]$

31. $\frac{1}{10}$ $\left[\text{Hint}: \frac{2}{^6C_3}\right]$

32. $\frac{1231}{7776}$ $\left[\text{Hint}: 1-\frac{36\times35\times34\times33}{(36)^4}\right]$

33. $\frac{63}{190}$ $\left[\text{Hint}: \frac{^9C_2}{^{20}C_2}+\frac{^7C_2}{^{20}C_2}+\frac{^4C_2}{^{20}C_2}\right]$

34. (i) 0.64 [Hint: P(D U M)]

(ii) 0.44 $\left[\begin{array}{l}\text{Hint}: \\ = P(D\cap\bar{M})\cup P(\bar{D}\cap M) \\ = P(D\cup M)-P(D\cap M)\end{array}\right]$

35. 120

36. $\frac{4}{9}$

37. $\frac{3}{5}$ $\left[\text{Hint}: = \frac{4\times{}^6C_1}{^{40}C_1}\right]$

38. $\frac{2}{11}$ $\left[\begin{array}{r}\text{Hint}: = \text{Let } P(C) = x \\ \frac{x}{3}+\frac{x}{2}+x=1\end{array}\right]$

39. $S=\left\{\frac{2,4,6}{\text{walking}},\frac{1,3}{\text{meditation}},\frac{5H}{\text{Rope skipping}},\frac{5T}{\text{Yoga}}\right\}$

i. 7 ii. $\frac{3}{7}$ iii. $\frac{1}{7}$ iv. $\frac{1}{7}+\frac{2}{7}=\frac{3}{7}$

v. Yoga and rope skipping

40. i. (b) $\frac{19}{30}$ ii. (c) 22 iii. (a) $\frac{11}{30}$ iv. (d) $\frac{2}{15}$ v. (a) $\frac{1}{10}$

41. (c) $\frac{5}{9}\left[\frac{4}{18}+\frac{6}{18}=\frac{10}{18}\right]$

42. (d) $\frac{1}{25}\left[=\frac{4}{4\times5\times5}\right]$

43. (b) 1

44. (c) $\frac{1}{7}$

45. (a) 0

46. (d) 4

47. (d) 8

48. (c) 3

49. (a) $P(A) \le P(\bar{B})$

50. (c) 1.2

51. (a) $P(A) + P(B) - 2P(A \cap B)$

52. (a) $\frac{2}{5}$

53. (c) $\frac{1}{4}$

54. (b) $\frac{5}{18}$

55. (b) $\frac{2}{3}$

56. (d) None of these (neither mutually exclusive nor exhaustive)

57. (b)

58. (d)

59. (a)

60. (c)

Solved Practice Paper

Class – XI
Session 2024-25

MATHEMATICS (Code-041)

Time Allowed : 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains – five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A
(Multiple Choice Questions)

Each question carries 1 mark

1. If $f(x) = \sqrt{x} + \frac{1}{x}$, then $f'(4) =$

   (a) $\frac{3}{4}$ (b) $\frac{9}{4}$

   (c) $\frac{3}{16}$ (d) $\frac{9}{16}$

2. Eccentricity of equilateral hyperbola is

   (a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

   (c) 2 (d) $\frac{1}{2}$

3. $\lim\limits_{x \to 0} \frac{e^{3x}-1}{x}$ is equal to

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{9}$ (d) 9

4. If A and B are two sets, then $A \cap (A \cap B)' =$

(a) $A \cap B$ (b) $A' \cap B$

(c) $A \cap B'$ (d) $A' \cap B'$

5. If $A = \{x : x \in R, x > 4\}$ and $B = \{x : x \in R, x \leq 5\}$ then $(A \cap B) =$

(a) (4, 5) (b) [4, 6]

(c) [4, 5) (d) (4, 5]

6. If A = {2, 3}, B = {1, 4, 6, 9} and R be a relation from A to B defined by x is less than y. The range of R is

(a) O (b) {1}

(c) {4, 6, 9} (d) {1, 4, 6, 9}

7. For any sets A and B; $(A \cap B) \cup (A - B) =$

(a) A (b) B

(c) A′ (d) B′

8. If $x \in R$, range of $f(x) = \frac{1}{1+x^2}$ is

(a) R (b) $(0, \infty)$

(c) (0, 1) (d) (0, 1]

9. Let A = {1, 2}, B = {3, 4}. The number of subsets of $A \times B$ is

(a) 4 (b) 8

(c) 16 (d) 32

10. $\dfrac{\cos 10° + \sin 10°}{\cos 10° - \sin 10°} =$

(a) cot 35° (b) tan 35°

(c) – cot 35° (d) – tan 35°

11. $3i^{15} - 5i^{8} + 1$ represents the following complex number

(a) 3i – 5 (b) – 3i – 5

(c) 3i – 4 (d) – 3i – 4

12. In how many ways can we form a four digit number using all the given digits 2, 3, 4, 2.

(a) 24 (b) 12

(c) 8 (d) 4

13. Find, n, if ${}^{n}C_5 = {}^{10}C_4 + {}^{10}C_5$

(a) 4 (b) 5

(c) 9 (d) 11

14. The total number of terms in the expansion of $(x + 11)^{23} - (x - 11)^{23}$ after simplification is

(a) 0 (b) 12

(c) 24 (d) 46

15. Distance of the point (7, –3, 5) from its reflection in XZ plane is

(a) 6 units (b) 25 units

(c) 49 units (d) $\sqrt{83}$ units

16. Derivation of sin x cos x with repect of x is

(a) cosec 2x (b) sec 2x

(c) cos 2x (d) sin 2x

17. For any two events A and B, if P(A) = 0.05, P(B) = 0.10 and $P(A \cap B)$ = 0.02, find $P(\overline{A} \cap \overline{B})$

(a) 0.97 (b) 0.87

(c) 0.77 (d) 0.73

18. The probability that a leap year will have 53 Mondays or 53 Tuesdays is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

ASSERTION-REASON BASED QUESTION

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer among the following choices:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): Given 4 flags of different colours, then number of different signals can be generated, if a signal requires the use of 2 flags one below the other is 12.

Reason (R): If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is m × n.

20. Assertion (A): If E_1 and E_2 are two mutually exclusive events, then $E_1 \cap E_2 = \varnothing$

Reason (R): If E_1 and E_2 are mutually exclusive and exhaustive events, then $E_1 \cap E_2 = \varnothing$ and $E_1 \cup E_2 = S$.

SECTION-B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second.

22. Prove that $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$.

OR

Find value of $\tan \frac{\pi}{8}$

23. A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

24. Four cards from a pack of 52 cards are drawn at random. Find the probability that all four cards are of same suit.

25. A card is drawn from a deck of 52 cards. Find the probability of getting a Jack or a spade or a black card.

SECTION-C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {x : x is prime}, B = {x : x is even integer}. Write

(i) $A - B$ (ii) $A \cap B'$

OR

Using Venn diagram, prove that $(A \cup B)' = A' \cap B'$

27. Find real θ such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is purely imaginary; $0° \le \theta \le 90°$

OR

If $z = x - iy$ and $z^{1/3} = p + iq$, then find the value of:

$$\frac{\frac{x}{p}+\frac{y}{q}}{p^2+q^2}$$

28. The temprerature (in celsius) in a city is considered normal when the average of three daily measurements (morning, afternoon, night) is between 19.2 and 29.8. On one day, if morning and night temperature (in celsius) are 19.48 and 19.85 repectively, find the range of afternoon temperature (in celsius) that will result in normal temperature of the day.

OR

Solve the inequality $\frac{x+8}{x+2} > 2$. Represent the solution on number line.

29. Using binomial theorem, prove that $6^n - 5n$ always leaves the remainder 1 when divided by 25.

30. Find the equation of ellipse that satisfies the following conditions: Centre at origin, major axis on the y-axis and passes through the points (3, 2) and (1, 6).

31. Find the equation of the set of points P, the sum of whose distance from A(0, 5, 0) and B(0, –5, 0) is equal to 15.

SECTION-D

(This section comprises of long answer type questions (LA) of 5 marks each)

32. Draw the graph of the following function.

$F(x) = |x - 1| + |2 + x|$ for all $-3 \leq x \leq 3$. Also find its range

OR

Draw graph of the function

$$f(x) = \begin{cases} 1+2x, x < 0 \\ 3+5x, x \geq 0 \end{cases}$$

Also find its range.

33. If A is the arithmetic mean and G_1, G_2 be two geometric mean between any two number, then prove that

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

34. Evaluate $\lim_{x \to \frac{1}{2}} \left(\frac{8x-3}{2x-1} - \frac{4x^2+1}{4x^2-1} \right)$

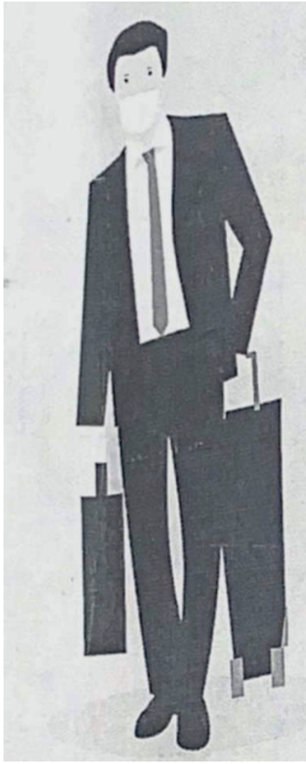
Evalute $\lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

35. Determine the mean and standard deviation for the following distribution:

Marks	Frequency
2	1
3	6
4	6
5	8
6	8
7	2
8	2
9	3
10	0
11	2
12	1
13	0
14	0
15	0
16	1

SECTION-E

(This section comprises of 3 case study/passage-based questions of 4 marks each.)



36. Daksh travelled from Delhi to Mumbai with his suitcase. After reaching to Mumbai, he forgot the password to unlock the suitcase. He remembered that there are 2 digits followed by 2 letters in the password.

    (i) Find total number of possibilities for the password, if repetition is allowed.

    (ii) How many possiblilities are there for the password, if neither numbers nor words are repeated.

    (iii) How many different passwords are possible, if Daksh remembered first two and last entry, and repetition is allowed.

OR

How many different passwords are possible, if Daksh remembered first and last entry and no entry is repeated?

37. A teacher mentioned in his class that there is some $\theta \in R$ such that $\sec\theta = \frac{-13}{12}$, where θ lies in second quadrant. Then he asks his students about some other values.

(i) What will be the value of $\tan\theta$

(ii) What will be the value of $\cos\frac{\theta}{2}$

(iii) Find value of $\sin 2\theta$

38. Nikhil and his friend Aman live in a neat and clean society in a big city. We imagine cartesian axes in the society placing central park at the origin. Nikhil's house lies on a straight lane represented by straight line $4x + 7y + 5 = 0$. Aman's house lies on another straight lane represented by straight line $2x - y = 0$

    (i) Nikhil and Aman can meet at the intersection of two lanes. Find the coordinates of the point of their meeting junction.

    (ii) Aman is in his room. If he starts from a point in his room having coordinates (1,2) and moves along the straight lane along his house $2x - y = 0$, find distance covered by him to reach the meeting junction.

Practice Paper Solution

SECTION-A

1. $f'(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{x^2}$

   $f'(4) = \dfrac{1}{2 \times 2} - \dfrac{1}{4^2}$ = $\dfrac{1}{4} - \dfrac{1}{16} = \dfrac{4-1}{16} = \dfrac{3}{16}$

   Correct option (c).

2. $e = \dfrac{c}{a} = \dfrac{\sqrt{a^2 + b^2}}{a}$

   $= \dfrac{\sqrt{2a^2}}{a}$[For equilateral hyperbola $a = b$]

   = $= \dfrac{\sqrt{2a^2}}{a} = e = \sqrt{2}$

   Correct option (a)

3. $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1}{x}$ = $\lim\limits_{\substack{x \to 0 \\ 3x \to 0}} \dfrac{3(e^{3x} - 1)}{3x}$ = 3 × 1 = 3

   Correct option (b)

4. $A \cap (A \cap B)'$

   $\Rightarrow A \cap (A' \cup B')$

   $\Rightarrow (A \cap A')$ U $(A \cap B')$

   $\Rightarrow \phi \cup (A \cap B')$

   $\Rightarrow A \cap B'$

   Correct option is (c)

5. $A = (4, \infty)$

   $B = (-\infty, 5]$

   $\Rightarrow A \cap B = (4, 5]$

   Correct option is (d).

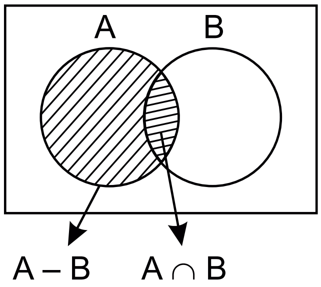
6. $R = \{(x, y) \mid x < y\}$

   = {(2, 4) (2, 6) (2, 9) (3, 4) (3, 6) (3, 9)}

   Range = {4, 6, 9}

Correct option (c).

7.



Correct option (a)

$(A \cap B) \cup (A - B) = A$

8. $y = \dfrac{1}{1+x^2}$

$y + x^2y = 1$

$x^2y = 1 - y$

$x = \sqrt{\dfrac{1-y}{y}}$

$x \in R \Rightarrow y > 0, 1 - y \geq 0$

$\Rightarrow y > 0, y \leq 1$

$\therefore \quad y \in (0, 1]$

Correct option (d)

9. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

$n(A \times B) = 2^4 = 16$

correct option (c)

10. $\dfrac{\cos 10° + \sin 10°}{\cos 10° - \sin 10°}$

$= \dfrac{\sin 80° + \sin 10°}{\sin 80° - \sin 10°}$

$= \dfrac{2 \sin 45° \cos 35°}{2 \cos 45° \sin 35°}$

$= \tan 45° \cot 35°$

$= \cot 35°$

Correct option (a)

11. $3i^{15} - 5i^8 + 1 = 3i^3 - 5 + 1 = -3i - 4$

Correct option (d)

12. 4 digits with 2 alike can be arranged in $\frac{4!}{2!} = 12$ ways

Correct option (b)

13. ${}^{n}C_{15} = {}^{10}C_4 + {}^{10}C_5$

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\therefore \quad {}^{10}C_5 + {}^{10}C_4 = {}^{11}C_5$

$\Rightarrow \quad n = 11$

Correct option (d)

14. $(x + 11)^{23} \rightarrow$ 24 terms (all positive)

$(x - 11)^{23} \rightarrow$ 24 terms (12 positive, 12 negative)

After simplification $\rightarrow$ 12 terms cancel

$\rightarrow$ 12 left in answer

Correct option (b)

15. $(7,-3,5) \xleftrightarrow[\text{XZ plance}]{\text{reflection in}} (7,3,5)$

$$d = \sqrt{(7-7)^2 + (-3-3)^2 + (5-5)^2}$$

$$= \sqrt{0+36+0} = 6 \text{ units}$$

Correct option (a)

16. $f(x) = \sin x \cos x = \frac{1}{2}\sin 2x$

$f'(x) = \frac{1}{2}(2\cos 2x) = \cos 2x$

Correct option (c)

17. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.05 + 0.10 - 0.02 = 0.13$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - 0.13$

$= 0.87$

Correct option is (b)

18. For leap year

Let P(53 Mondays) = P(M)

P(53 Tuesdays) = P(T)

$P(M \cup T) = P(M) + P(T) - P(M \cap T)$

$$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$$

Correct option is (c).

19. Assertion (A) is true. Reason (R) is true and R is correct explanation of A.

$\therefore$ Correct option is (a)

20. A is true. R is true but R is not correct explanation of B.

$\therefore$ Correct option is (b)

SECTION B

21. One minute $\rightarrow$ 360 revolutions

$\Rightarrow$ 60 second = 2π radians × 360

$\Rightarrow$ 1 second = $\left(\frac{2\pi}{60} \times 360\right)$ radians

$\Rightarrow$ 12 π radians.

22. LHS = sin 4A = sin 2(2A)

= 2 sin 2A cos 2A

= 2(2sinA cos A) ($\cos^2$A – $\sin^2$A)

= 4sinA $\cos^3$ A – 4$\sin^3$A cos A

= RHS

OR

Let $\frac{\pi}{8} = y \quad \Rightarrow \quad 2y = \frac{\pi}{4}$

$\Rightarrow$ tan 2y = 1

$\Rightarrow \quad \frac{2\tan y}{1-\tan^2 y} = 1$

Let tan y = t

$\Rightarrow \quad \frac{2t}{1-t^2} = 1$

$\Rightarrow$ 2t = 1 – t^2

$\Rightarrow$ t^2 +2t – 1 = 0

$\Rightarrow \; t = \frac{-2 \pm 2\sqrt{2}}{2}$

$\Rightarrow \quad t = -1 \pm \sqrt{2}$

$\Rightarrow \qquad \tan y = -1 \pm \sqrt{2}$

$\Rightarrow \qquad \tan \frac{\pi}{8} = -1 \pm \sqrt{2}$

$\Rightarrow \qquad \tan \frac{\pi}{8} = -1 \pm \sqrt{2}$

as $\frac{\pi}{8}$ lies in first quadrant, tan $\frac{\pi}{8}$ will be positive

$\therefore \quad \tan \frac{\pi}{8} = \sqrt{2} - 1$

23. $40 < C < 45$

$$\frac{9}{5} \times 40 < \frac{9}{5} C < \frac{9}{5} \times 45$$

$$72 < \frac{9}{5} C < 81$$

$$104 < \frac{9}{5} C + 32 < 113$$

$$104 < F < 113$$

Ans: between 104°F and 113°F

24. Total outcomes = $^{52}C_4$

Favourable outcomes = $4 \times {}^{13}C_4$

Required probability = $\frac{4 \times {}^{13}C_4}{^{52}C_4}$

$$= \frac{4 \times \frac{13!}{4!9!}}{\frac{52!}{4!48!}} = \frac{44}{4165}$$

25. $P(J \cup S \cup B) = P(J) + P(S) + P(B) - P(J \cap S) - P(S \cap B) - P(J \cap B) + P(J \cap S \cap B)$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$

SECTION-C

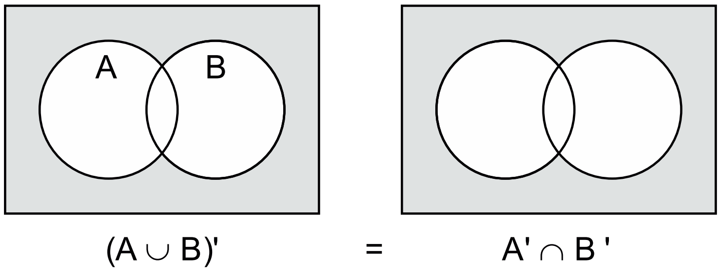
26. U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {2, 3, 5, 7}   B = {2, 4, 6, 8, 10}

(a) A – B = {3, 5, 7}   (b) B′ = {1, 3, 5, 7, 9}

$A \cap B' = \{3, 5, 7\}$

OR



27. $\dfrac{3+2i\sin\theta}{1-2i\sin\theta} \times \dfrac{1+2i\sin\theta}{1+2i\sin\theta}$

$$= \frac{3(1+2i\sin\theta)+2i\sin\theta(1+2i\sin\theta)}{1-(2i\sin\theta)^2}$$

$$= \frac{3+6i\sin\theta+2i\sin\theta+4i^2\sin^2\theta}{1-4i^2\sin^2\theta}$$

$$= \frac{3+8i\sin\theta-4\sin^2\theta}{1+4\sin^2\theta}$$

$$= \frac{3-4\sin^2\theta}{1+4\sin^2\theta} + \frac{i(8\sin\theta)}{1+4\sin^2\theta}$$

Purely imaginary = real part is zero

$\Rightarrow \dfrac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow \sin^2\theta = \dfrac{3}{4}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60°.$

OR

$z^{1/3} = p + iq$

$$z = (p + iq)^3$$
$$= (p^3 - 3pq^2) + i(3p^2q - q^3)$$
$$= x - iy$$

$\Rightarrow x = p^3 - 3pq^2$ and $y = -3p^2q + q^3$

$\Rightarrow \dfrac{x}{p} = p^2 - 3q^2$ and $\dfrac{y}{q} = -3p^2 + q^2$

$$\Rightarrow \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 = -2(p^2 + q^2)$$

$$\Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{p^2 + q^2} = -2$$

28. Let x = after noon temperature (in celsius)

19.2 < Average Normal temp < 29.8

$$19.2 < \frac{19.48 + x + 19.85}{3} < 29.8$$

$57.6 < 39.33 + x < 89.4$

$18.27 < x < 50.07$ = range of afternoon temperature (in celsius)

OR

$$\frac{x+8}{x+2} - 2 > 0 \Rightarrow \frac{x-4}{x+2} < 0$$

$\Rightarrow \ x \in (-2, 4)$

29. $6^n = (1 + 5)^n$

$6^n = 1 + 5n + 5^2\,{}^nC_2 + 5^3\,{}^nC_3 + \ldots\ldots + 5^n\,{}^nC_n$

$6^n - 5n = 1 + 25[{}^nC_2 + 5\,{}^nC_3 + \ldots\ldots. + 5^{n-2}\,{}^nC_n]$

Hence proved.

30. $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

(3, 2): $\dfrac{9}{b^2} + \dfrac{4}{a^2} = 1$

(1, 6): $\dfrac{1}{b^2} + \dfrac{36}{a^2} = 1$

Solving $a^2 = 40$, $b^2 = 10$

$$\Rightarrow \frac{x^2}{10} + \frac{y^2}{40} = 1$$

31. Let $P(x, y, z)$ be the required set of points, such that:

$$AP + BP = 15$$
$$AP = 15 - BP$$
$$(AP)^2 = (15 - BP)^2$$

$$(x-0)^2 + (5-y)^2 + (0-z)^2 = (15 - \sqrt{[(x-0)^2 + (y+5)^2 + (z-0)^2]})^2$$

$$\Rightarrow x^2 + 25 + y^2 - 10y + z^2 = [15 - \sqrt{(x^2 + y^2 + 25 + 10y + z^2)}]^2$$

$$\Rightarrow x^2 + 25 + y^2 - 10y + z^2 = 225 + [x^2 + y^2 + 25 + 10y + z^2] - 30\sqrt{x^2 + y^2 + z^2 + 10y + 25}$$

$$\Rightarrow -10y - 10y - 225 = -30\sqrt{x^2 + y^2 + z^2 + 10y + 25}$$

$$\Rightarrow 20y + 225 = 30\sqrt{x^2 + y^2 + z^2 + 10y + 25}$$

$$\Rightarrow 4y + 45 = 6\sqrt{x^2 + y^2 + z^2 + 10y + 25}$$

Squaring both sides,

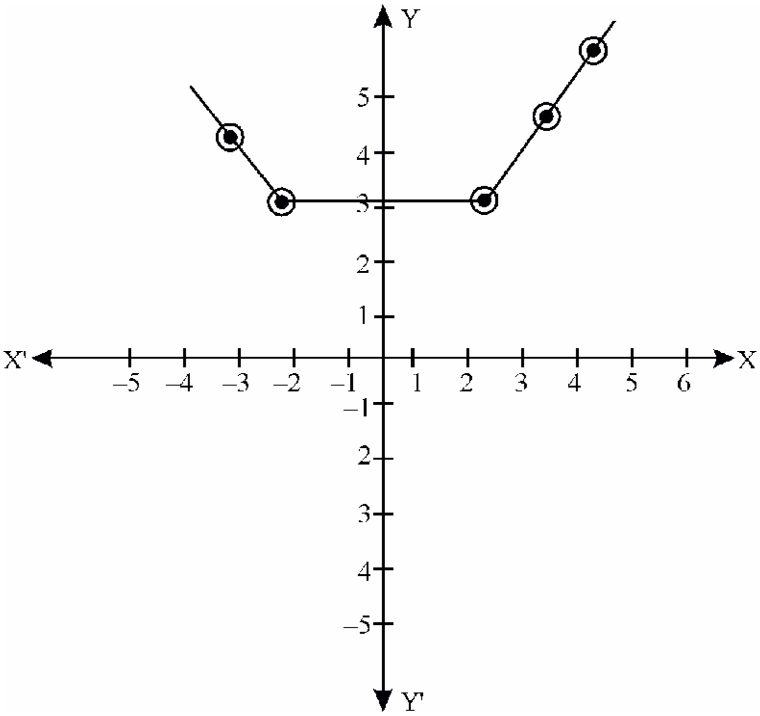
$$(4y + 45)^2 = 36[x^2 + y^2 + z^2 + 10y + 25]$$

$$16y^2 + 2025 + 360y = 36x^2 + 36y^2 + 36z^2 + 360y + 900$$

$$\Rightarrow 36x^2 + 20y^2 + 36z^2 - 1125 = 0$$

32. $f(x) = |x - 1| + |2 + x|; -3 \le x \le 3$

$$= \begin{cases} -(2x+1) & -3 \le x < -2 \\ 3 & -2 \le x < 1 \\ 2x+1 & x \ge 1 \end{cases}$$



$y = (2x + 1),\ 3 \le x < 2$

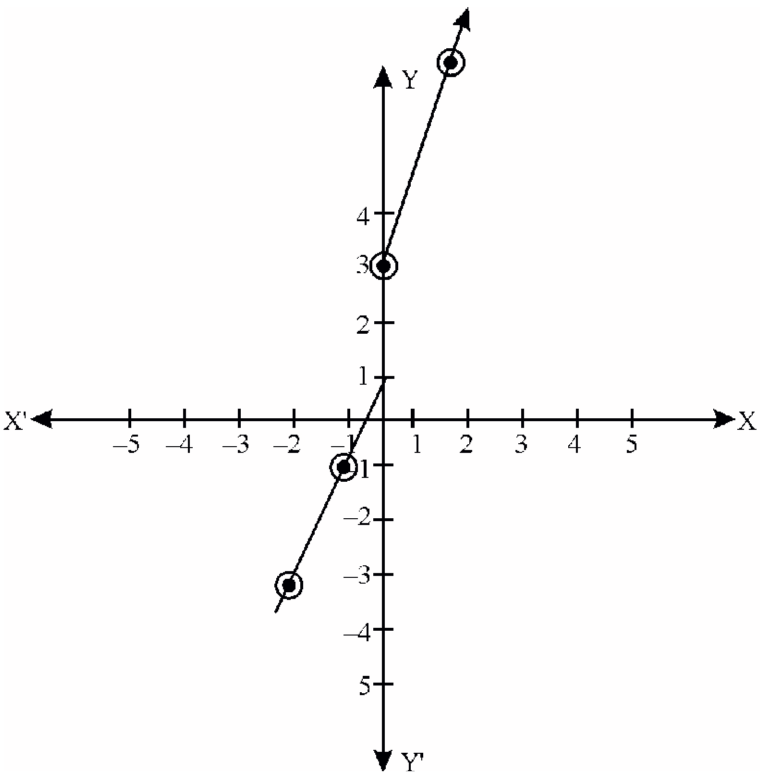
x	– 3	–2.5
y	5	4

$y = 2x + 1, x \ge 1$

x	1	2	3
y	3	5	7

From graph: Range: [3, 7]

OR



$y = 1 + 2x, x < 0$

x	–1	–2	–0.1
y	–1	–3	0.8

$y - 3 + 5x, x \ge 0$

x	0	1	0.5
y	3	8	5.5

Range: $(-0, 1) \cup [3, \infty)$

33. Let the numbers be a and b,

Then A = $\frac{a+b}{2}$ and a, G_1, G_2, b are in GP

$\Rightarrow$ 2A = $a + b$ and G_1 = ar, G_2 = ar^2, $b = ar^3$?

As $b = ar^3$

$$\Rightarrow \qquad \frac{b}{a} = r^3$$

$$\Rightarrow \qquad \left(\frac{b}{a}\right)^{1/3} = r$$

$$\therefore \qquad G_1 = a\left(\frac{b}{a}\right)^{1/3}$$

$$\text{and} \qquad G_2 = a\left(\frac{b}{a}\right)^{2/3}$$

Now $\qquad$ RHS $= \dfrac{G_1^2}{G_2} + \dfrac{G_2^2}{G_1}$

$$= \frac{a^2\left(\frac{b}{a}\right)^{2/3}}{a\left(\frac{b}{a}\right)^{2/3}} + \frac{a^2\left(\frac{b}{a}\right)^{4/3}}{a\left(\frac{b}{a}\right)^{1/3}}$$

$$= a + a\left(\frac{b}{a}\right)$$

$= a + b$

$= 2A$ = LHS

34. $\lim_{x \to \frac{1}{2}} \left(\dfrac{8x-3}{2x-1} - \dfrac{4x^2+1}{4x^2-1} \right)$

$$= \lim_{x \to \frac{1}{2}} \left[\frac{(8x-3)(2x+1)-(4x^2+1)}{4x^2-1} \right]$$

$$= \lim_{x \to \frac{1}{2}} \left[\frac{16x^2+2x-3-4x^2-1}{4x^2-1} \right]$$

$$= \lim_{x \to \frac{1}{2}} \frac{12x^2 + 2x - 4}{4x^2 - 1} = \lim_{x \to \frac{1}{2}} \left[\frac{2(6x^2 + x - 2)}{(2x)^2 - 1^2} \right]$$

$$= \lim_{x \to \frac{1}{2}} \frac{2(3x+2)(2x-1)}{(2x-1)(2x+1)} = \lim_{x \to \frac{1}{2}} \frac{2(3x+2)}{2x+1} = \frac{7}{2}$$

OR

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan x(\tan^2 x - 1)}{\cos\left(x + \frac{\pi}{4}\right)} = \lim_{x \to \frac{\pi}{4}} \frac{(-\tan x)(1 - \tan x)(1 + \tan x)}{\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4}}$$

$$\lim_{x \to \frac{\pi}{4}} \frac{(-\tan x)(1+\tan x)\left(\frac{\cos x - \sin x}{\cos x}\right)}{\frac{1}{\sqrt{2}}(\cos x - \sin x)} = \lim_{x \to \frac{\pi}{4}} \frac{(-\tan x)(1+\tan x)}{\frac{1}{\sqrt{2}}\cos x}$$

$$= \frac{(-1)(1+1)}{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = \frac{-2}{\frac{1}{2}} = -4$$

35.

x	f	fx	fx^2
2	1	2	4
3	6	18	54
4	6	24	96
5	8	40	200
6	8	48	288
7	2	14	98
8	2	16	128
9	3	27	243
10	0	0	0
11	2	22	242
12	1	12	144
13	0	0	0
14	0	0	0
15	0	0	0

16	1	16	256
	N = 40	239	1753

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{239}{40} = 5.975 \sim 6$

Standard Deviation = $\frac{1}{N}\sqrt{N\Sigma f_i x_i^2 - (\Sigma fx)^2}$

$= \frac{1}{40}\sqrt{(40)(1753) - (239)^2}$

$= \frac{1}{40}\sqrt{70120 - 57121}$

$= \frac{1}{40}\sqrt{12999} = \frac{114.01}{40} \simeq 2.85$

36. (i) $10 \times 10 \times 26 \times 26 = 67600$

(ii) $(10 \times 9) \times (26 \times 25) = 58500$

(iii) 26

or $9 \times 25 = 225$

37. (i) $\frac{-5}{12}$

(ii) $\frac{1}{\sqrt{26}}$

(iii) $\frac{-120}{169}$

38. (i) Point of intersection of $4x + 7y + 5 = 0$ and $2x - y = 0$ is $\left(\frac{-5}{18}, \frac{-5}{9}\right)$

(ii) distance between point of intersection of lines $\left(\frac{-5}{18}; \frac{-5}{9}\right)$ and (1, 2)

$= \frac{23}{18}\sqrt{5}$ units (Using distance formula)

Practice Paper-1

Class – XI
Session 2024-25

MATHEMATICS (Code-041)

Time Allowed : 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains – five sections A, B, C, D and E. Each section is compulsory. Howerver, there are internal choices in some questions.
2. Section A has 5 Very Short Answer (VSA)-type questions of 2 marks each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 mark each.
4. Section C has 6 short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A
(Multiple Choice Questions)

Each question carries 1 mark

1. Which of the following is the empty set

   (a) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$

   (b) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$

   (c) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$

   (d) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

2. The number of proper subsets of the set $\{1, 2, 3\}$ is

   (a) 8 (b) 7

   (c) 6 (d) 5

3. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to

   (a) A (b) B

(c) ϕ (d) None of these

4. Let A = {1, 2, 3}. The total number of distinct relations that can be defined over A is

(a) 2^9 (b) 6

(c) 8 (d) None of these

5. Let R be is relation on N defined by x + 2y = 8. The domain of R is

(a) {2, 4, 8} (b) {2, 4, 6, 8}

(c) {2, 4, 6} (d) {1, 2, 3, 4}

6. If $f(x) = \frac{x - |x|}{|x|}$, then $f(-1) =$

(a) 1 (b) –2

(c) 0 (d) +2

7. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta =$

(a) –4/5 but not 4/5 (b) –4/5 or 4/5

(c) 4/5 but not –4/5 (d) None of these

8. The complex number $\frac{1+2i}{1-i}$ lies in which quadrant of the complex plane

(a) First (b) Second

(c) Third (d) Fourth

9. In how many ways can 5 boys and 3 girls sit in a row so that no two girls are together

(a) 5 ! × 3 ! (b) ${}^4P_3 \times 5!$

(c) ${}^6P_3 \times 5!$ (d) ${}^5P_3 \times 3!$

10. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then the value of r is

(a) 3 (b) 4

(c) 5 (d) 8

11. If coefficient of $(2r + 3)^{th}$ and $(r - 1)^{th}$ terms in the expansion of $(1 + x)^{15}$ are equal, then value of r is

(a) 5 (b) 6

(c) 4 (d) 3

12. The equation of the straight line passing through the point (3, 2) and perpendicular to the line y = x is

(a) $x - y = 5$ (b) $x + y = 5$

(c) $x + y = 1$ (d) $x - y = 1$

13. The distance between the foci of the ellipse $3x^2 + 4y^2 = 48$ is

(a) 2 (b) 4

(c) 6 (d) 8

14. If $\lim\limits_{x \to 2} \dfrac{x^n - 2^n}{x - 2} = 80,$ where n is a positive integer, then n =

(a) 3 (b) 5

(c) 2 (d) None of these

15. $\lim\limits_{x \to 0} \dfrac{1 - \cos mx}{1 - \cos nx} =$

(a) m/n (b) n/m

(c) $\dfrac{m^2}{n^2}$ (d) $\dfrac{n^2}{m^2}$

16. Let the function f be defined by the equation $f(x) = \begin{cases} 3x & \text{if } 0 \le x \le 1 \\ 5 - 3x & \text{if } 1 < x \le 2 \end{cases},$ then

(a) $\lim\limits_{x \to 1} f(x) = f(1)$ (b) $\lim\limits_{x \to 1} f(x) = 3$

(c) $\lim\limits_{x \to 1} f(x) = 2$ (d) $\lim\limits_{x \to 1} f(x)$ does not exist

17. There are two children in a family. The probability that both of them are boys is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

18. Two dice are thrown simultaneously. The probability of getting the sum 2 or 8 or 12 is

(a) $\frac{5}{18}$ (b) $\frac{7}{36}$

(c) $\frac{7}{18}$ (d) $\frac{5}{36}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion:** If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24^{th} position.
Reason: The number of ways of arranging four distinct objects taken all a time is C(4, 4).

20. **Assertion:** A letter is chosen at random from the word NAGATATION. Then the total number of outcomes is 10.
Reason: A letter is chosen at random from the word 'ASSASSINATION' Then, the total number of outcomes is 13.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Prove that: sin 780° sin 120° + cos 240° sin 390° = $\frac{1}{2}$

22. Find the value of $\tan\frac{13\pi}{12}$

23. Solve x + 5 > 2 (x + 1), 2 – x < 3 (x + 2)

24. A coin is tossed repeatedly until a head comes up for the first time. Describe the sample space.

25. Tickets numberd from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?

OR

If A and B are two events associated with a random experiment such that P(A u B) = 0.8, P(A n B) = 0.3 and $P(\overline{A})$ = 0.5, find P(B).

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers liked product P_1 and 1450 consumers liked product P_2. What is the least number that must have liked both the products?

27. If $(x+iy)^{1/3} = a+ib, x, y, a\ b \in R.$ Show that $\frac{x}{a}+\frac{y}{b} = 4(a^2-b^2).$

OR

Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

28. A solution is to be kept between 86° and 95° F. What is the range of temperature in degree Celsius, if the Celsius (C)/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C+32.$

29. Find the coefficient of x^5 in the expansion of the product $(1 + 2x)^6 (1 - x)^7$.

OR

Find the term independent of x in the expansion of $\left(3x^2-\frac{1}{2x^3}\right)^{10}$

30. Find the equation of a circle of radius 5 whose centre lies on x-axis and passes through the point (2, 3).

OR

Find the equation of an ellipse whose major axes lie along x-axes and which passes through the points (4, 3) and (–1, 4).

31. Find the locus of the point, the sum of whose distances from the points A(4, 0, 0) and B(–4, 0 , 0) is equal to 10.

SECTION D

(This section compreses of long answer-type questions (LA) of 5 marks each)

32. Prove that: $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$

OR

Prove that: $\cos^3 A + \cos^3 (120° + A) + \cos^3 (240° + A) = \frac{3}{4}\cos 3A$

33. If the A.M. and G.M. between two numbers are in the ratio m: n, then prove that the number are in the ratio $m+\sqrt{m^2-n^2} : m-\sqrt{m^2-n^2}$.

OR

The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 and third is increased by 1, we obtain three consecutive terms of a G.P. Find the numbers.

34. Differentiate the following functions:

(i) $\frac{x^2-x+1}{x^2+x+1}$ (ii) $\frac{x\tan x}{\sec x+\tan x}$

35. Calculate the mean and standard deviation of the following distribution:

Class interval:	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70
Frequency:	2	3	8	12	16	5	2	3

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Dr Jitender Singh, Lecturer Physical Education is selecting Students for volley ball team for inter zonal sports competitions. 6 students are required for team and there are total 15 students (8 Boys and 7 Girls) available. Two students named Anurag and. Rakhi are very good in every sports. Based on above information, answer the following questions:

    (i) In how many ways this selection will be done, if Anurag and Rakhi will be already selected?

    (ii) In how many ways this selection will be done, if there will be no Girl in team?

    (iii) In how many ways this selection will be done, if there is a condition of 3 Boys and 3 Girls in Team?

OR

In how many ways this selection will be done, if Anurag and Rakhi will not be selected?

37. A person is standing at a point A of a triangular park ABC whose vertices are A(2, 0), B(3, 4) and C(5, 6). Based on the above information answer the following :

    I. He wants to reach BC in least time. Find the equation of the path he should follow.

    II. Find the shortest distance travelled by him to reach BC.

    III. Suppose he meets BC at a point D. Find the co-ordinates of the point D

OR

Find the area of the triangular park ABC.

38. To make himself self-dependent and to earn his living, a person decided to setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture. Based on the above information answer the following :

    I. If x units of hand sanitizers are manufactured per month. What is the profit function?

    II. What is the monthly cost borne by the person if he decided to manufacture 1500 units in a month?

ANSWERS

Practice Paper-1

1. (b)
2. (b)
3. (c)
4. (a)
5. (c)
6. (b)
7. (b)
8. (b)
9. (c)
10. (a)
11. (a)
12. (b)
13. (b)
14. (b)
15. (c)
16. (d)
17. (c)

18. (b)
19. (c)
20. (b)
22. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
23. (–1, 3)
24. S = {H, TH, TTH, TTHH, TTTTH, ... }
25. 2/5 OR 0.6
26. 1170
27. $\theta = n\pi, n \in z$
28. Between 30°C and 35°C
29. 171 OR $\frac{76545}{8}$
30. $x^2 + y^2 - 12x + 11 = 0$ and $x^2 + y^2 + 4x - 21 = 0$ OR $\frac{7x^2}{247} + \frac{15y^2}{247} = 1$
31. $9x^2 + 25y^2 + 25z^2 - 225 = 0$
33. 12, 7, 2, or 3, 7, 11
34. (i) $\frac{2(x^2-1)}{(x^2+x+1)^2}$ (ii) $\frac{x \sec x(\sec x - \tan x) + \tan x}{(\sec x + \tan x)}$
35. 50.35, 7.94
36. (i) 715 (ii) 28 (iii) 1960 or 1716
37. (i) x + y = 2 (ii) $3/\sqrt{2}$ (iii) (1/2, 3/2) or 3.
38. (i) 15000 + 30x (ii) Rs 60000

Practice Paper-2

Class – XI
Session 2024-25

MATHEMATICS (Code-041)

Time Allowed : 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains – five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
3. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
4. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
5. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A
(Multiple Choice Questions)

Each question carries 1 mark

1. For any two sets A and B, $A \cap (A \cup B)'$ is equal to

   (a) A (b) B

   (c) $\varnothing$ (d) $A \cap B$

2. The value of tan 1° tan 2° tan 3° .... tan 89° is

   (a) 0 (b) 1

   (c) 1/2 (d) Not Defined

3. ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then value of r is

   (a) 12 (b) 8

   (c) 6 (d) 10

4. If $a + ib = c + id$, then

(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$

(c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

5. If f(x) = $x^3 - \frac{1}{x^3}$, then f(x) +f$\left(\frac{1}{x}\right)$ is equal to

(a) $2x^3$ (b) $\frac{2}{x^3}$

(c) 0 (d) 1

6. The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is

(a) 105 (b) 15

(c) 175 (d) 185

7. The x-intercept and y-intercept of the line 5x – 7 = 6y, respectively are

(a) $\frac{7}{5}$ and $\frac{7}{6}$ (b) $\frac{7}{5}$ and $-\frac{7}{6}$

(c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $-\frac{5}{7}$ and $\frac{6}{7}$

8. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49n - 49 : n \in N\}$. Then

(a) $X \subset Y$ (b) $Y \subset X$

(c) $X = Y$ (d) $X \cap Y = \varnothing$

9. The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

(a) 50 (b) 202

(c) 51 (d) none of these

10. $\lim_{\theta \to 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta}$ is

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) – 1

11. While shuffling a pack of 52 playing cards, 2 are accidently dropped. The chances that the missing cards be of different colours is

(a) $\frac{29}{52}$ (b) $\frac{1}{2}$

(c) $\frac{26}{51}$ (d) $\frac{27}{51}$

12. The radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is

(a) 1 (b) 2

(c) 3 (d) 5

13. The domain and range of the real function F defined by $F(x) = \frac{4-x}{x-4}$ is given by

(a) Domain = R, Range = {–1, 1}

(b) Domain = R –{1}, Range = R

(c) Domain = R – {4}, Range = R – {–1}

(d) Domain = R – {–4}, Range = {–1, 1}

14. In a leap year the probability of having 53 Sundays or 53 Mondays is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

15. If f(x) = x sin x, then $f'\left(\frac{\pi}{2}\right)$ is equal to

(a) 0 (b) 1

(c) –1 (d) $\frac{1}{2}$

16. Everybody in a room shakes hands with everybody else. The total number of handshakes is 66. The total number of persons in the room is

(a) 11 (b) 12

(c) 13 (d) 14

17. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is

(a) $\left\{-1, \frac{4}{3}\right\}$ (b) $\left[-1, \frac{4}{3}\right]$

(c) $\left(-1, \frac{4}{3}\right)$ (d) $\left[-1, \frac{4}{3}\right]$

18. $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$ is

(a) 1 (b) 2

(c) –1 (d) –2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): The following assignment of probabilities to each outcome are valid.

Outcomes	W_1	W_2	W_3	W_4	W_5	W_6
Probability	1	0	0	–1	0	1

Reason (R): Sum of all assigned values of probabilities should be 1.

20. Assertion (A): The following pair of sets are equal.
A = {x : x is a letter in the word FOLLOW}
B = {y : y is a letter in the word WOLF}
Reason (R): Two sets A and B are said to be equal if they have exactly the same elements.

SECTION B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Prove that tan3x tan2x tanx = tan3x – tan2x – tanx

OR

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second.

22. The vertices of triangle PQR are P(1, 2), Q(–2, 3) and R(4, 5). Find equation of the median through the vertex R.

23. Solve the inequality, $3x - 5 < x + 7$, when

(a) x is a whole number (b) x is a real number

OR

A solution is to be kept between 40°C and 45° C. What is the range of temperature in degree Fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

24. Prove that following: $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

25. Find the points on the x-axis, whose distance from the lines $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Let P = {3, 4, 5, {1, 2}}, which of following satatement are true and which are false?

(i) $\{1, 2\} \in P$

(ii) $\{3, 4, 5\} \in P$

(c) $\phi \in P$

OR

If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 5}, B = {2, 4, 6, 7} and C = {2, 3, 4, 8}; find

(i) $(C - A)'$ (ii) $(B \cup C)'$ (iii) $(A \cap B \cap C)'$

27. Show that point A(4, –3, –1), B(5, –7, 6) and C(3, 1, –8) are collinear.

28. Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

OR

Expand using Binomial theorem $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4$, $x \neq 0$

29. Find the equation of the ellipse, with major axis along the x-axis and which passes through the points (4, 3) and (–1, 4).

OR

Find the equation of the hyperbola where foci are (0, ± 12) and the length of latus rectum is 36.

30. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

31. If $(x + i y)^3 = u + iv$ Where u, v, x, y ∈ R, then show that

$$\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Find the mean and variance of the following frequency distribution:

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

OR

The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observation are 1, 2 and 6, find the other two observations.

33. Find the value of cos $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$

OR

Prove that $\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$

34. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d from a G.P. Prove that (q + q) : (q-p) = 17:15

35. Find the derivative of following functions (where m, n, a, b, c, d are fixed non- zero constants)

(i) $(ax + b)^n (cx + d)^m$ (ii) $\frac{x}{\sin^n x}$

SECTION E

(This section comprises of 3 case study/passage-based questions of 4 marks each.

36. Anita is doing an experiment in which she has to arrange to alphabets of the word 'HARYANA' in all possible orders and notes the observations.

HARYANA

Help her to find the answers of the following questions :

(I) How many words can be made starting with 'A' ?

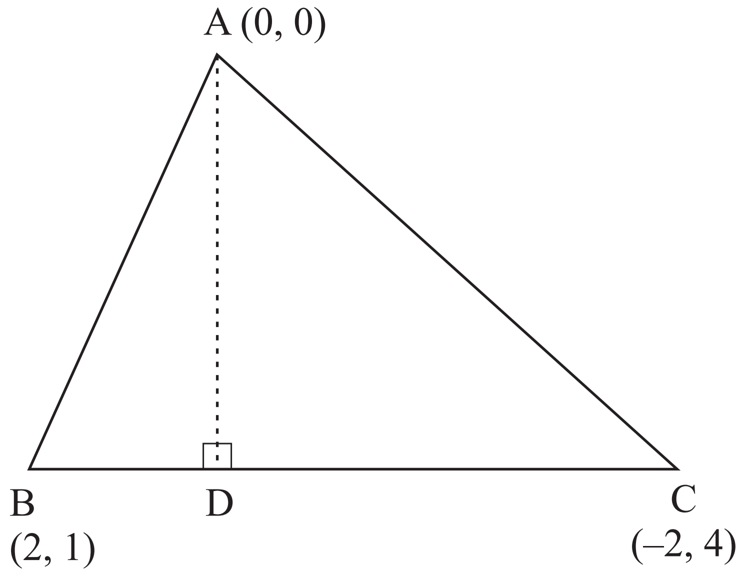
(II) How many words can be made ending at 'H' ?

(III) (a) Find the number of words, in which no two vowels occur together.

OR

(b) If all the words are written according as to the dictionary, then find 481th word.

37. Sanjay is standing at point A of a triangular park ABC, whose vertices are A (0, 0); B (2, 1) and C (–2, 4).



Based on the above information, answer the following questions :

(I) Find the area of triangular park ABC.

(II) Find the shortest distance travelled by Sanjay to reach BC.

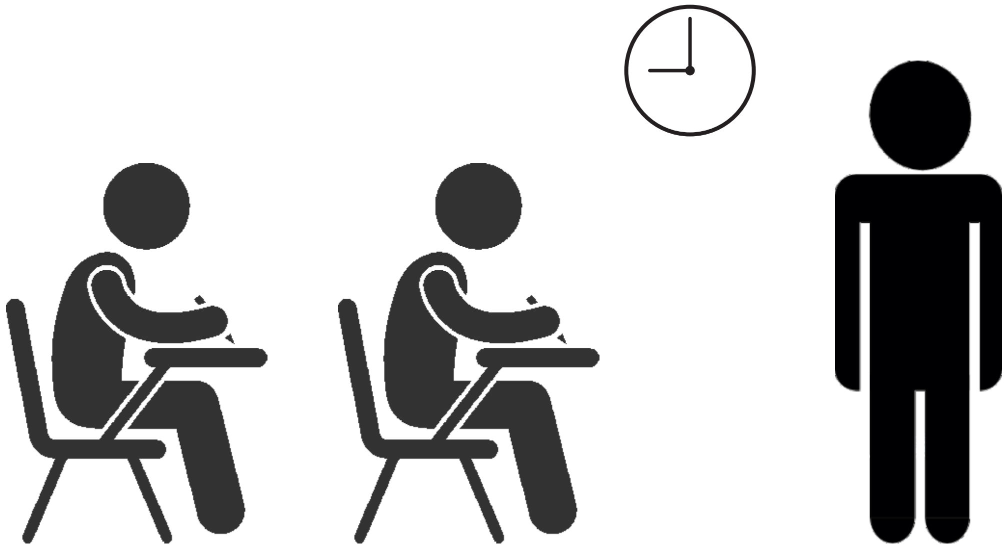
(III) (a) Sanjay wants to reach BC in least time. Find the equation of the path he should follow.

OR

(b) Suppose Sanjay reaches BC in least time and he meets BC at point D. Find the coordinates of point D.

38. Two students Ramesh and Naresh appeared in an examination.

The probability that Ramesh will quality the examination is 0.05 and that Naresh will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02.



Based on the above information, answer the following questions :

(I) Find the probability that both Ramesh and Naresh will not qualify the examination.

(II) Find the probability that only one of them will qualify the examination.

ANSWERS

Practice Paper-2

1. (c) ϕ
2. (b) 1
3. (a) 12
4. (d) $a^2 + b^2 = c^2 + d^2$
5. (c) 0
6. (d) 185
7. (b) $\frac{7}{5}$ and $-\frac{7}{6}$
8. (a) $X \subset Y$
9. (c) 51
10. (a) $\frac{4}{9}$
11. (c) $\frac{26}{51}$
12. (d) 5
13. (c) Domain = R – {4}, Range = R – {– 1}
14. (b) $\frac{3}{7}$
15. (b) 1
16. (b) 12
17. (a) $\left\{-1, \frac{4}{3}\right\}$
18. (c) – 1
19. (d) A is false but R is true
20. (a)
21. Or 12 π radians
22. $5x - 9y + 25 = 0$
23.
    (a) {0, 1, 2, 3, 4, 5}
    (b) $\{x : x \in R \text{ and } x < 6\}$ or $(-\infty, 6)$
    OR

104°F to 113° F

25. (– 2, 0) and (8, 0)

26. (i) True (ii) False (iii) False

OR

(i) {1, 2, 3, 5, 6, 7, 9, 10}

(ii) {1, 5, 9, 10}

(iii) {1, 3, 4, 5, 6, 7, 8, 9, 10}

28. $(x + 1)^6 + (x - 1)^6 = 2x^6 + 30x^4 + 30x^2 + 2$

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 198$

OR

$\frac{16}{x^4} - \frac{32}{x^3} + \frac{8}{x^2} + \frac{16}{x} - 5 - 4x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$

29. $7x^2 + 15y^2 = 247$

OR

$\frac{y^2}{36} - \frac{x^2}{108} = 1 \quad i.e., 3y^2 - x^2 = 108$

30. Minimum length is 41 cm

32. Mean = 27

Variance = 132

OR

4 and 9

33. $-\frac{1}{16}$

35. (i) $(ax+b)^{n-1}(cx+d)^{m-1}\{(m+n)acx + mcb + nad\}$

(ii) $\frac{\sin x - nx\cos x}{(\sin x)^{n+1}}$

36. (i) 360 (ii) 120

(iii) (a) 240 (iii) (b) N A A A H R Y

37. (i) 5 sq. units

(ii) 2 units

(iii) (a) $4x - 3y = 0$

(iv) (b) $D\left(\frac{6}{5}, \frac{8}{5}\right)$

38. (i) 0.87

(ii) 0.11

Notes

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